SHELDONS' TWO-BOOK SERIES.

SHELDONS'

COMPLETE ARITHMETIC,

WITH

ORAL AND WRITTEN EXERCISES.

SHELDON & COMPANY

NEW YORK AND CHICAGO.

THE SHELDON SERIES

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SHELDONS' ELEMENTARY ARITHMETIC,

SHELDONS' COMPLETE ARITHMETIC,

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PUBLISHERS' PREFACE.

SEVERAL new works on the subject of Arithmetic have been issued by different publishing houses during the past few years, and for each is claimed some special merit. In each some particular idea or method has been given prominence, and other important features have been lightly treated or entirely ignored. In the preparation of the series of which this book is the second or more advanced, care has been exercised to present at every point those methods and to introduce those exercises which have met with the most general approval of prominent and progressive instructors.

The ELEMENTARY book will be appreciated by those teachers who have broken away from the old mechanical methods and are looking for a book which, when placed in the hands of the pupils, will assist in accomplishing the results aimed at by the better methods of the present day.

The scope of this, the COMPLETE or advanced book, will be most readily comprehended by glancing at the "Table of Contents" on the following pages.

No attempt has been made to introduce elementary lessons in the fundamental rules. It is taken for granted that the pupils have studied an elementary book, or have been instructed orally. The fundamental definitions and rules are reached by the inductive method; drill tables are provided for the purpose of perfecting the pupils in the different processes; and a sufficient number of examples and problems, oral and written, are given to afford a thorough review.

Properties of Numbers and Fractions do not immediately follow Division, but are preceded by a series of lessons designed not only to prepare the pupils for the study of these more difficult topics, but also to familiarize them with practical methods of performing many operations in numbers as they are performed in every-day life. The lessons on Equal Parts of Units clear the way for the exercises in Mixed Numbers. Here the fractional parts of the numbers are simple, and the result is determined by inspection. The pupils are thus trained to work a large class of examples in the short and easy way adopted by business men. The

lessons on Decimal Parts of Units prepare the pupils for a thorough study of United States Money as presented in the lessons that follow; and the whole is supplemented and completed by giving the more commonly used tables of Denominate Numbers and a large number of practical examples. This arrangement must prove to be of great assistance not only to those who continue the study of Arithmetic, but also to many pupils who would otherwise leave school without the information and discipline thus attained.

The chapters on Properties of Numbers, Fractions, Decimals, and Denominate Numbers are given in the usual order. Here, as in the preceding chapters, the definitions, principles and rules are reached by the inductive method, and the explanations of processes are models of simplicity and clearness. Due attention has been given to the Metric System of Weights and Measures, but instead of treating it as a separate topic, the tables have been introduced among the other tables of Denominate Numbers in their natural order, thus affording an opportunity for a comparison of units of measure.

In the chapters on Percentage and the Applications of Percentage, those processes have been adopted which the experience of business men has proved to be the most practical, and the problems introduced are simple statements of commercial transactions as they actually occur.

Especial attention is called to the comparatively large number of concrete problems, oral and written, which have been introduced throughout the book. All puzzles have been excluded, and the practical and natural character of the problems, as well as their abundance, makes them an important feature of the book.

The Appendix, in addition to notes and tables of reference, contains a full presentation of the Greatest Common Divisor and Least Common Multiple of Fractions, Circulating Decimals, Exchange, Alligation, Arithmetical and Geometrical Progressions, and other minor topics needed in order to make the book complete in every particular.

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CONTENTS.

SHELDONS'

COMPLETE ARITHMETIC.

DEFINITIONS.

1. A **unit** is a *single thing* or *one;* as one apple, one dollar, one hour, one.

A *group* of things if considered as a single thing or *one* is also a unit; as one class, one dozen, one ten.

2. A **number** is a unit or a collection of units; as seven apples, five dollars, six classes, eight.

3. The **unit of a number** is one of the collection. Thus, *one* apple is the unit of *seven* apples; *one* dollar of *five* dollars; *one* of *eight.*

4. A **concrete** or **denominate** number is a number whose units are named; as five *pounds,* seven *books.*

5. An **abstract** number is a number whose units are not named; as three, six, eleven.

A *concrete* or *denominate* number expresses some particular kind of quantity. An *abstract* number expresses one or more units without reference to any particular object or objects.

6. **Arithmetic** is the science of numbers and the art of computation.

As a science, arithmetic treats of the properties and relations of numbers; as an art, it treats of their use.

NOTATION AND NUMERATION.

7. Numbers are expressed in three ways: 1. By *words;* 2. By *figures;* 3. By *letters.*

8. Notation is the art of expressing numbers by figures or letters.

9. Numeration is the art of reading numbers expressed by figures or letters.

ARABIC SYSTEM.

10. In the Arabic System of Notation ten **figures** are employed to express numbers, viz.:

1	2	3	4	5	6	7	8	9	0
One,	**Two,**	**Three,**	**Four,**	**Five,**	**Six,**	**Seven,**	**Eight,**	**Nine,**	**Naught.**

11. The first nine of these figures are called **significant figures** or **digits,** and each expresses the number of units indicated by its name.

12. The last figure, **naught**, also called **cipher** or **zero,** stands for *nothing.* It is used with the other figures in expressing numbers larger than **9**.

Figures are not numbers but are **characters** used to express numbers. By using them in accordance with certain principles, any number may be expressed.

13. Nine (**9**) is the largest number that can be expressed by a single figure. To express **ten** units, or **1 ten,** the figure **1** is written at the left of the cipher, thus, **10**.

Twenty,	or 2 tens,	is	written	**20.**		**Sixty,**	or 6 tens,	is	written	**60.**
Thirty,	" 3	"	"	"	**30.**	**Seventy,**	" 7	"	"	"	**70.**
Forty,	" 4	"	"	"	**40.**	**Eighty,**	" 8	"	"	"	**80.**
Fifty,	" 5	"	"	"	**50.**	**Ninety,**	" 9	"	"	"	**90.**

14. The numbers between **10** and **20, 20** and **30, 30** and **40**, etc., are written by putting in place of the cipher the significant figure which will express the number of units named. Thus,

Eleven,	or 1 ten	and	1 unit,	is	written	**11.**
Twelve,	" 1 ten	"	2 units,	"	"	**12.**
Thirteen,	" 1 ten	"	3 units,	"	"	**13.**
Fifteen,	" 1 ten	"	5 units,	"	"	**15.**
Seventeen	" 1 ten	"	7 units,	"	"	**17.**
Twenty-one,	" 2 tens	"	1 unit,	"	"	**21.**
Twenty-five,	" 2 tens	"	5 units,	"	"	**25.**
Thirty-seven,	" 3 tens	"	7 units,	"	"	**37.**
Fifty-six,	" 5 tens	"	6 units,	"	"	**56.**
Sixty-eight,	" 6 tens	"	8 units,	"	"	**68.**
Eighty-six,	" 8 tens	"	6 units,	"	"	**86.**
Ninety-nine,	" 9 tens	"	9 units,	"	"	**99.**

15. Ninety-nine (**99**) is the largest number that can be expressed by two figures. To express **one hundred** units, or **10 tens,** the figure **1** is written in the third place, or at the left of two ciphers, thus, **100.**

Two hundred	is	written	**200.**	**Six hundred**	is	written	**600.**
Three hundred	"	"	**300.**	**Seven hundred**	"	"	**700.**
Four hundred	"	"	**400.**	**Eight hundred**	"	"	**800.**
Five hundred	"	"	**500.**	**Nine hundred**	"	"	**900.**

16. The numbers between **100** and **200**, **200** and **300**, etc., are written by substituting, for the ciphers, the figures required to express the tens and units named. Thus,

One hundred one,	or 1 hund., 0 tens, 1 unit,	is	written	**101.**
One hundred two,	" 1 hund., 0 tens, 2 units,	"	"	**102.**
One hundred eleven,	" 1 hund., 1 ten, 1 unit,	"	"	**111.**
One hundred twelve,	" 1 hund., 1 ten, 2 units,	"	"	**112.**
One hundred seventeen,	" 1 hund., 1 ten, 7 units,	"	"	**117.**
Two hundred twenty-seven,	" 2 hund., 2 tens, 7 units,	"	"	**227.**
Three hundred thirty-six,	" 3 hund., 3 tens, 6 units,	"	"	**336.**
Five hundred fifty-five,	" 5 hund., 5 tens, 5 units,	"	"	**555.**
Seven hundred eighty-nine,	" 7 hund., 8 tens, 9 units,	"	"	**789.**
Nine hundred ninety-nine,	" 9 hund., 9 tens, 9 units,	"	"	**999.**

17. Principle.—*When a number is expressed by three figures, the first or right-hand figure expresses units, the second figure expresses tens, and the third expresses hundreds.*

18. ***Read the following numbers:***

1. 9. 10. 17. 20. 33. 78. 99. 100.

2. 111. 112. 113. 120. 167. 200. 223. 304.

3. 317. 502. 570. 600. 611. 660. 666. 699.

4. 700. 812. 878. 880. 944. 909. 999.

19. ***Express the following numbers by figures:***

1. Five.
2. Ten.
3. Twelve.
4. Thirteen.
5. Fifteen.
6. Twenty.
7. Thirty-five.
8. Eighty-three.
9. Ninety-nine.
10. One hundred.
11. One hundred twenty.
12. Two hundred twenty.
13. Four hundred four.
14. Five hundred.
15. Five hundred fifty.
16. Five hundred fifty-five.
17. Five hundred five.
18. Eight hundred eighty.
19. Nine hundred ninety-nine.

20. ***Write from dictation the numbers in Article 18.***

21. Numbers of more than three figures are divided into periods of three figures each, commencing at the right hand.

TABLE OF NOTATION AND NUMERATION.

	6th.		5th.		4th.		3d.		2d.		1st.
NAMES OF PERIODS.	Quadrillions.		Trillions.		Billions.		Millions.		Thousands.		Units.
Number.	64	,	305	,	080	,	700	,	520	,	234
ORDERS.	17th. 16th.		15th. 14th. 13th.		12th. 11th. 10th.		9th. 8th. 7th.		6th. 5th. 4th.		3d. 2d. 1st.

For the names of higher periods, see Appendix, Art. 704.

22. Principles.—I. *Every full period expresses hundreds, tens, and units of the denomination indicated by its name.*

II. *Commencing with the first or units' place, the different orders of units are numbered in succession.*

III. *Vacant orders and periods are filled with ciphers.*

IV. *Ten units of any order in a number make one unit of the next higher order.*

23. The number in the preceding table is read by commencing at the left-hand and reading each period as if it stood alone, and then adding the name of the period. Thus,

Sixty-four quadrillion, three hundred five trillion, eighty billion, seven hundred million, five hundred twenty thousand, two hundred thirty-four.

NOTE.—Observe that the final *s* is omitted from the names of the periods, and that the name of the last period is not given.

24. ***Read the following numbers, omitting the name of any period composed entirely of ciphers:***

	Trillions.	Billions.	Millions.	Thousands.	Units.
1.		24,	700,	560,	008.
2.	55,	000,	408,	723,	600.
3.	123,	080,	000,	040,	817.
4.	12,	120,	102,	000,	748.
5.	400,	342,	070,	411,	000.

25. To read and write numbers with rapidity, it is necessary to memorize the names and numbers of the periods.

26. ***Give the names of the following periods:***

4th period.	3d period.	5th period.
2d period.	5th period.	2d period.
6th period.	4th period.	3d period.

27. ***Give the numbers of the following periods:***

Thousands.	Trillions.	Quadrillions.
Billions.	Millions.	Billions.
Quadrillions.	Units.	Millions.

28. Rule for Notation.—*Begin at the left-hand and write the hundreds, tens, and units of each period in succession, filling vacant orders and periods with ciphers.*

29. Rule for Numeration.—I. *Separate the number into periods, beginning at the right.*

II. *Begin at the left and read each period as if it stood alone, adding its name.*

30. ***Read the following numbers:***

1. 8,962.	*6.* 512,875.	*11.* 357,065,423.
2. 9,012.	*7.* 670,020.	*12.* 980,006,097.
3. 10,907.	*8.* 807,905.	*13.* 3,057,305,435.
4. 13,090.	*9.* 5,835,627.	*14.* 24,300,010,170.
5. 20,005.	*10.* 23,070,900.	*15.* 37,563,004,128.

31. ***Express the following numbers in figures:***

1. Five thousand four hundred sixty-eight.

2. Ten thousand seven hundred seventy-five.

3. Thirty-seven thousand thirty-seven.

4. Five hundred twenty-four thousand twenty-nine.

5. Nine hundred nine thousand four hundred forty.

6. Seven hundred thousand seventy-four.

7. One million seven hundred twenty-one thousand eight hundred forty-seven.

8. Six million fourteen thousand five.

9. Sixty-one million five hundred thirteen thousand four.

10. One hundred ninety-three million two hundred five.

11. Four hundred three million five hundred thousand fifty-eight.

12. Nine billion one hundred eighteen million six hundred fifty-three thousand seven hundred twelve.

13. Seventy-five billion eighty-three million seven hundred four thousand.

14. One hundred forty-five billion three hundred million forty-three thousand four hundred seven.

32. ***Write in words the numbers in Article 30.***

33. ***Write from dictation the numbers in Articles 30 and 31.***

ROMAN SYSTEM.

34. In the **Roman System of Notation** seven letters are employed to represent numbers, viz.:

LETTERS.	I.	V.	X.	L.	C.	D.	M.
VALUES.	1	5	10	50	100	500	1000

35. In combination, their significance is in accordance with the following

PRINCIPLES.

I. *To repeat a letter repeats its value.*

Thus, II. is 2, and XXX. is 30.

No letter except C and M is ever used more than three times in any one combination, and V, D and L are never repeated.

II. *When a letter follows one of greater value, their sum is denoted; when it precedes, their difference is denoted.*

Thus, XI. is 11, and IX. is 9.

III. *A letter between two of greater value diminishes by its value the sum of the other two.*

Thus, XIX. is 19, and LIV. is 54.

IV. *A bar over a letter increases its value one thousand times.*

Thus, $\overline{\text{V}}$. is 5000, and $\overline{\text{M}}$. is 1000000.

NOTE.—Roman numbers are always followed by a period.

36. ***Express the following numbers by the Arabic Notation:***

1. XXIV.	*9.* XCIX.	*17.* DLXII.
2. XXIX.	*10.* CVIII.	*18.* DXCVIII.
3. XXXVIII.	*11.* CCXIX.	*19.* DCCCLXXXIX.
4. XLV.	*12.* CCCC.	*20.* MC.
5. XLIV.	*13.* CCCLXIX.	*21.* MCCCC.
6. XLIX.	*14.* CCCCXLIX.	*22.* MDCXIX.
7. LXIX.	*15.* DIV.	*23.* MMMM.
8. XCIV.	*16.* DIX.	*24.* $\overline{\text{L}}$DCXIX.

37. ***Express the following numbers by the Roman Notation:***

1. 23.	*5.* 42.	*9.* 89.	*13.* 239.	*17.* 506.
2. 15.	*6.* 48.	*10.* 113.	*14.* 365.	*18.* 923.
3. 29.	*7.* 67.	*11.* 104.	*15.* 449.	*19.* 949.
4. 37.	*8.* 93.	*12.* 250.	*16.* 487.	*20.* 999.

21. 1019.	*23.* 1469.	*25.* 1659.	*27.* 10000.
22. 1198.	*24.* 1499.	*26.* 2025.	*28.* 50000.

REVIEW QUESTIONS.

38. What is a unit? A number? The unit of a number? A concrete or denominate number? An abstract number? Give an illustration of each.

Mention the different ways in which a number may be expressed. Illustrate. What is the difference between figures and numbers? What are significant figures? For what are ciphers used? Illustrate. How are numbers less than 10 expressed by the Arabic System? How are numbers from 9 to 100 expressed? Illustrate. From 99 to 1000? Illustrate.

Give in reverse order the names and numbers of the first six periods. How many units of the first order are expressed by the figure 7 if written in tens' place? If written in hundreds' place? In thousands' place? How many units of the fourth order are equal to 7 units of the fifth order? To 9 units of the sixth order?

By what are numbers expressed in the Roman System? State the principles. Illustrate.

ADDITION.

1. If there are 5 apples in a dish and you put in 7 more, how many apples will there be in the dish? 5 apples and 7 apples are how many apples? 5 units and 7 units are how many units? 5 and 7 are how many?

39. The **sum** or **amount** of two or more numbers is a number which contains as many units as the numbers taken together.

Thus, the sum of 5 apples and 7 apples is 12 apples, because 12 contains as many units as 5 and 7 together.

40. Addition is the process of finding the *sum* of two or more numbers.

The numbers added are called **addends.**

41. The **sign of addition** is +, and is read *plus.*

Thus, 8 + 9 means that 8 and 9 are to be added, and may be read the *sum of* 8 *and* 9, or 8 *plus* 9.

42. The **sign of equality** is =, and is read *equals*, or *is equal to.*

Thus, 4 + 5 = 9 is read 4 *plus* 5 *equals* 9, or the *sum of* 4 *and* 5 *is equal to* 9.

43. An expression in which the sign of equality is used between two numbers or sets of numbers is called an **equation.**

Thus, IV. = 4, and 5 + 3 = 1 + 7 are equations.

44. The interrogation point (?) is sometimes used instead of the words *how many* or *what number.*

Thus, 4 + 7 = ? is read, the sum of 4 and 5 is equal to *how many*? 4 and 5 are *how many*? 4 plus 5 equals *what number*?

36. ***Express the following numbers by the Arabic Notation:***

1. XXIV.	*9.* XCIX.	*17.* DLXII.
2. XXIX.	*10.* CVIII.	*18.* DXCVIII.
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1. 23.	*5.* 42.	*9.* 89.	*13.* 239.	*17.* 506.
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3. 29.	*7.* 67.	*11.* 104.	*15.* 449.	*19.* 949.
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38. What is a unit? A number? The unit of a number? A concretē or denominate number? An abstract number? Give an illustration of each.

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By what are numbers expressed in the Roman System? State the principles. Illustrate.

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Thus, $4+5=9$ is read 4 *plus* 5 *equals* 9, or the *sum of* 4 *and* 5 *is equal to* 9.

43. An expression in which the sign of equality is used between two numbers or sets of numbers is called an **equation.**

Thus, IV. = 4, and $5+3=1+7$ are equations.

44. The interrogation point (?) is sometimes used instead of the words *how many* or *what number.*

Thus, $4+7=?$ is read, the sum of 4 and 5 is equal to *how many*? 4 and 5 are *how many?* 4 plus 5 equals *what number?*

45. The **sign of dollars** is $; and **cents** are denoted by *ct.*, *¢*, or *cts.*, and by placing a period between dollars and cents.

Thus, $24 and 17 ct.; $24 and 17¢, or $24 and 17 cts.; and $24.17.

Cents must always occupy two places. Thus, 98 cts. is written $.98; 5¢ is written $.05; and $3 and 5 cts. is written $3.05.

46. Like numbers are numbers whose units are the same; as $7 and $9.

47. Unlike numbers are numbers whose units are different; as 8 lb. and 12 cts.

2. Can you add 8 cts. and 7 cts.? What kind of numbers are they? Can you add $5 and 5 lb.? What kind of numbers are they?

48. Principles.—I. *Only like numbers, and units of the same order, can be added.*

II. *The sum is like the numbers added.*

49. To find the sum of any two numbers expressed by single figures.

50. The sum of any two numbers expressed by single figures should first be learned by counting objects.

51. The **addition table** is given in the Appendix, p. 318.

52. To find the sum of more than two numbers expressed by single figures.

I. Required the sum of 5, 7, 4, 8 and 9.

Explanation.—9 units and 8 units are 17 units, equal to 1 ten and 7 units. The 1 ten and 7 units added to the 4 units gives 1 ten and 11 units, equal to 2 tens and 1 unit. The 2 tens and 1 unit added to the 7 units gives 2 tens and 8 units. The 2 tens and 8 units added to the 5 units gives 2 tens and 13 units, equal to 3 tens and 3 units or 33. Hence, 33 is the sum of 5, 7, 4, 8 and 9.

In practice, the numbers given are added by simply saying or thinking, 17, 21, 28, 33.

Process.

$$\begin{array}{r} 5 \\ 7 \\ 4 \\ 8 \\ 9 \\ \hline 33 \end{array}$$

53. To find the sum of two or more numbers expressed by any number of figures.

I. Required the sum of 56789, 78597, and 45887.

Explanation.—1. The numbers are written so that units of the same order stand in the same column.

2. The sum of the units, 7, 7, and 9, is 23, equal to 2 tens and 3 units; the 3 is written under the units' column, and the 2 tens added to the tens' column.

3. The sum of the tens, including those from the units' column, is 27, equal to 2 hundreds and 7 tens; the 7 is written under the tens' column, and the 2 hundreds is added to the hundreds' column.

4. In the same manner the sum of each successive column is found.

Process.

56789
78597
45887
181273

54. Rule.—I. *Write the numbers so that units of the same order shall stand in the same column.*

II. *Begin at the right and add each column separately. When the sum is less than ten, write it under the column added; when the sum is ten or more than ten, write the units' figure under the column added and add the ten or tens to the next column.*

Proof.—*Find the sum by adding the columns in the opposite direction; if the results agree the work may be considered correct.*

55. ***Write from dictation and find the sum of:***

1.	*2.*	*3.*	*4.*	*5.*
234	347	549	857	2398
461	456	876	5346	5647
748	1638	3985	492	895

6.	*7.*	*8.*	*9.*
598	3964	432	59608
4385	8576	5891	43295
976	769	642	384
7897	5387	17608	296871

59. The following is a model for a series of exercises to supplement the one on page 18.

60. ***Announce sums at sight:***

Read answers: 1. From left to right; 2. From right to left; 3. From top to bottom; 4. From bottom to top; 5. In any order suggested by the teacher.

	1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*	*8th.*	*9th.*
A	7	7	7	7	7	7	7	7	7
	10	30	60	90	20	40	70	80	50
B	7	7	7	7	7	7	7	7	7
	41	21	31	11	91	71	61	51	81
C	7	7	7	7	7	7	7	7	7
	22	12	52	42	32	62	92	82	72
D	7	7	7	7	7	7	7	7	7
	93	63	33	13	23	53	83	43	73
E	7	7	7	7	7	7	7	7	7
	84	94	54	44	34	24	14	64	74
F	7	7	7	7	7	7	7	7	7
	65	45	25	35	15	55	95	85	75
G	7	7	7	7	7	7	7	7	7
	16	46	76	96	26	36	56	66	86
H	7	7	7	7	7	7	7	7	7
	27	57	87	17	37	67	47	77	97
I	7	7	7	7	7	7	7	7	7
	58	48	68	38	78	28	88	18	98
J	7	7	7	7	7	7	7	7	7
	19	49	99	29	89	39	79	59	69

ORAL EXERCISES.

58. *1.* John spends 9 cts. for oranges and 8 cts. for apples; how much does he spend? 9 and 8 are how many?

2. A boy has 13 cts. and finds 8 cts. more; how many has he then?

3. Mary picked 8 quarts of berries, Jane 7 quarts, and Kate 5 quarts; how many quarts did all pick?

4. To reach home I rode 19 miles in the cars, 9 miles by stage, and walked 7 miles; how far from home had I been?

5. What is the sum of the numbers on the face of an ordinary clock?

6. How many apples are 14 apples, 9 apples, 7 apples and 8 apples?

7. A gentleman pays board as follows: for himself and wife $21 per week, for his son $9 per week, for his daughter $8, and for the servant $6 per week; what is his bill by the week?

8. I spend $43 for a suit of clothes, $8 for a pair of boots, $9 for a cane, and $7 for a hat; how much do all cost?

9. Commencing at 1, add by 2's to 99.

10. Commencing at 2, add by 2's to 100.

11. Commencing at 3, add by 3's to 99.

12. Commencing at 4, add by 3's to 100.

13. Commencing at 2, add by 4's to 98.

14. Commencing at 3, add by 4's to 99.

15. Add by 5's from 3 to 48; from 4 to 39; from 2 to 67.

16. Add by 6's from 5 to 77; from 3 to 69; from 1 to 73.

17. Add by 7's from 3 to 66; from 2 to 79; from 5 to 75.

18. Add by 8's from 5 to 77; from 7 to 103; from 3 to 83

19. Add by 9's from 4 to 103; from 8 to 98; from 1 to 91

20. $17+5+3+8+7=?$ $48+9+8+7+6+5=?$

21. Find the sum of $15+1+2+3$, etc., to 9.

22. Find the sum of $37+9+8+7+6$, etc., to 0.

59. The following is a model for a series of exercises to supplement the one on page 18.

60. ***Announce sums at sight:***

Read answers: 1. From left to right; 2. From right to left; 3. From top to bottom; 4. From bottom to top; 5. In any order suggested by the teacher.

	1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*	*8th.*	*9th.*
A	7	7	7	7	7	7	7	7	7
	10	30	60	90	20	40	70	80	50
B	7	7	7	7	7	7	7	7	7
	41	21	31	11	91	71	61	51	81
C	7	7	7	7	7	7	7	7	7
	22	12	52	42	32	62	92	82	72
D	7	7	7	7	7	7	7	7	7
	93	63	33	13	23	53	83	43	73
E	7	7	7	7	7	7	7	7	7
	84	94	54	44	34	24	14	64	74
F	7	7	7	7	7	7	7	7	7
	65	45	25	35	15	55	95	85	75
G	7	7	7	7	7	7	7	7	7
	16	46	76	96	26	36	56	66	86
H	7	7	7	7	7	7	7	7	7
	27	57	87	17	37	67	47	77	97
I	7	7	7	7	7	7	7	7	7
	58	48	68	38	78	28	88	18	98
J	7	7	7	7	7	7	7	7	7
	19	49	99	29	89	39	79	59	69

61. Each of the other exercises in the series for which the preceding is a model, is made by substituting for **7** one of the other digits. When prepared, each exercise should be faithfully practiced. The utility of these exercises appears when the operation of adding a long column is considered. Thus,

To add the first of the following columns, 5+7=12, 12+4=16, 16+9=25, 25+8=33, 33+5=38, 38+7=45, 45+8=53, 53+6=59, 59+2=61, 61+9=70, 70+4=74, 74+5=79, 79+3=82. In each instance, a number composed of tens and units is added to a number expressed by one of the digits. The ability to add with accuracy and rapidity is more surely acquired by practice on tables like the preceding than in any other way.

62. ***Find the sum of the following columns:***

1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*	*8th.*	*9th.*	*10th.*
3	4	5	9	8	9	6	7	5	8
5	6	9	3	5	5	5	8	9	3
4	8	8	8	4	7	9	9	3	9
9	2	7	6	6	8	8	6	7	6
2	9	2	4	9	3	4	5	4	5
6	7	6	7	3	6	3	4	9	8
8	5	5	5	1	2	9	1	5	6
7	6	9	6	8	9	6	3	9	7
5	8	4	8	7	8	7	5	2	5
8	7	7	3	6	4	5	7	6	9
9	3	3	9	9	3	8	2	8	2
4	9	8	7	8	7	3	4	7	8
7	8	6	6	3	2	7	6	5	7
5	5	9	5	8	7	6	8	9	5

63. In practice, all that should be said or thought at each step, in the addition of a column of figures, is simply the amount. Thus,

The first of the above columns is added by thinking or saying 12, 16, 25, 33, 38, 45, 53, 59, 61, 70, 74, 79, 82; and one proficient in adding will add quite as fast as he can read.

64. ***Write from dictation and add the following:***

1.	2.	3.	4.	5.
$827.	$426.	$645.	$9.23	$4.28
384.	938.	547.	3.85	3.57
496.	872.	359.	9.28	9.42
828.	768.	926.	4.56	7.64
798.	594.	834.	8.65	4.56
345.	678.	387.	9.37	9.87
567.	592.	649.	8.42	1.65
$	$	$	$	$

6.	7.	8.	9.
31	2587	$965.48	$454.63
847	369	26.95	82.79
9	3954	4.87	368.47
6482	829	658.29	.85
228	487	5.39	3.76
1736	6984	18.68	48.29
8592	765	692.87	634.78

65. ***Find the value of each of the following expressions:***

1. 839 + 3187 + 62 + 86945 + 8238 + 19 + 826957.
2. 73429 + 87 + 264 + 4146 + 53257 + 75 + 345.
3. 38597 + 29 + 187 + 6928 + 31 + 15625 + 19375.
4. 1206 + 3598 + 4679 + 5807 + 7563 + 6954.
5. 48267 + 70965 + 32968 + 59437 + 73865.
6. 386742 + 59305 + 7647 + 859 + 36 + 9.
7. 19 + 168 + 37 + 647 + 38 + 74 + 96 + 59 + 1378.
8. $93.86 + $829.65 + $2250.25 + $3.16 + $9614.37.
9. $68.43 + $9.26 + $835.48 + $7.09 + $50683.27 + $43.86.
10. $325 + $86 + $89587 + $29 + $1685 + $906 + $360275.
11. $57.59 + $6137.85 + $8.37 + $296.83 + $0.39 + $0.67 + $8.97.

NOTE.—Longer examples may be made by combining two or more examples.

CONCRETE ORAL PROBLEMS.

66. *1.* Mary bought a pound of candy for 28 cts., some nuts for 8 cts., and some dates for 14 cts.; find the cost of all.

2. Find the sum of 5 dimes 3 cents, 2 dimes 9 cents, and 3 dimes 5 cents. (1 dime = 10 cents.)

3. John paid 53 cts. for a comb, 29 cts. for a tooth-brush, and 35 cts. for soap; find the cost of all.

4. A man travels four days as follows: the first day 18 miles, the second 23 miles, the third 25 miles, the fourth 31 miles; how far does he travel in all?

5. John caught 13 fish, William 17, Harry 25, and James 36; how many did they all catch?

NOTE.—The pupils should be required to compose oral problems, and to write them neatly in correct language. These may frequently be used with profit for a class exercise.

CONCRETE WRITTEN PROBLEMS.

67. *1.* A man receives $5895 for his farm and $3956 for his cattle and farming utensils; how much does he get for all?

2. I own a house and lot worth $9650, and a mill worth $5875; how much are both worth?

3. I deposit in a bank at one time $1389, at another $2869, at a third $3675, and at a fourth $4867; how much have I in the bank?

4. I buy a farm for $3896. After expending $297 for fences and $837 for new buildings, I sell it so as to gain $1987; how much do I get for it?

5. A nurseryman sold 2845 apple trees, 3687 peach trees, 389 apricot trees, 9673 plum trees, and 1166 pear trees; how many trees in all did he sell?

6. A butcher sold 3967 lb. of beef, 2869 lb. of pork, 1368 lb. of mutton, and 965 lb. of veal; how many pounds of meat did he sell?

7. I pay in one year $875 for rent, $1385 for groceries, $687 for meat, $878 for clothing, $339 for servants' hire, and $466 for incidentals; how much do I spend in one year?

8. A father gives to his oldest son $18395, to his second son $15680, to his daughter $12575, and to his wife $5825 more than to his oldest son; how much does he give to all?

9. I pay $175 for a sleigh, for a carriage $137 more than for the sleigh, and for a span of horses $365 more than for the sleigh and carriage together; find the cost of all.

10. A speculator bought a drove of cattle for $36895 and sold so as to gain $15868; find his selling price.

11. Three brothers buy a farm together; the first pays $3560, the second pays $1875 more than the first, and the third pays $755 more than the first and second together; how much does the farm cost?

12. I begin business with a store worth $5600, fixtures worth $386, and goods worth $35875. During the first year I gain $5875; how much am I worth?

13. An oil-well produced in January 3589 barrels of petroleum, in February 5675 barrels, in March 6844 barrels, and in April 7660 barrels; how many barrels did it produce during the four months?

14. A coasting vessel brought 83965 pine-apples from Havana to New York City on her first trip; on the second, 75698; and on the third, 90877; how many did she bring in all?

15. Rhode Island contains 1250 square miles, Connecticut 3740 more than Rhode Island, Massachusetts 2075 more than Rhode Island and Connecticut together, and New Hampshire 990 more than Massachusetts; find the area of the four states together.

16. A, B, and C go into business together; A invests $35650, B $13875 more than A, and C $1875 more than A and B together; how much do they all invest in the business?

SUBTRACTION.

1. Henry caught 15 fish and James caught 8 fish; how many more did Henry catch than James? 15 fish are how many more than 8 fish? 15 is how many more than 8? What number added to 8 will make 15?

68. The **difference** between two numbers is a number which, added to the smaller, will give a result equal to the greater.

Thus, the difference between 15 and 8 is 7, because 7 added to 8 makes 15.

69. **Subtraction** is the process of finding the *difference* between two numbers.

70. The *greater* of two numbers whose difference is to be found is called the **minuend.**

71. The *smaller* of two numbers whose difference is to be found is called the **subtrahend.**

72. The *result* in subtraction is called the **remainder.**

73. The **sign of subtraction** is —, and is read *minus* or *less.*

Thus, 14—6 means that 6 is to be subtracted from 14, and may be read, 14 *minus* 6, or 14 *less* 6.

2. Can you subtract 7 cts. from 19 cts.? What kind of numbers are they?

3. Can 8 marbles be subtracted from 12 eggs? What kind of numbers are they?

74. Principles.—I. *Only like numbers, and units of the same order, can be subtracted.*

II. *The remainder is always like the minuend and subtrahend.*

75. To find the difference between two numbers, the minuend being less than 20 and the subtrahend less than 10.

4. If $6 is subtracted from $15, what is the *remainder?* What is the *subtrahend?* What is the *minuend?* What number added to the subtrahend will give the minuend?

76. The *remainder* added to the *subtrahend* always gives the *minuend* for the sum; hence, to determine the difference between two numbers, it is necessary to find what number added to the smaller will make the larger.

77. The difference between any two numbers, the larger of which is less than 20 and the smaller less than 10, should first be learned objectively.

78. The **subtraction table** is given in the Appendix, page 319.

79. To find the difference between numbers expressed by two or more figures.

I. Required the difference between 40462 and 19287.

Process.

$$\begin{array}{r} 40462 \\ 19287 \\ \hline 21175 \end{array}$$

Explanation.—1. Beginning with units, 7 cannot be subtracted from 2; 1 ten is therefore taken from the 6 tens of the minuend, changed to units and added to the 2, making 12 units; 7 units subtracted from 12 units gives 5 units, which is written in units' place in the remainder.

2. Taking 1 ten from the 6 tens of the minuend left 5 tens; the 8 tens of the subtrahend cannot be subtracted from 5 tens, and hence 1 of the 4 hundreds of the minuend is taken and changed to tens, making 15 tens in all; 8 tens from 15 tens gives 7 tens, which is written in tens' place in the remainder.

3. 2 hundreds subtracted from the 3 hundreds left after changing 1 of the hundreds to tens, gives 1 hundred, which is written in hundreds' place in the remainder.

4. Since there are no thousands in the minuend, 1 ten-thousand is changed to thousands, and the 9 thousands of the subtrahend subtracted from the 10 thousands gives 1 thousand for thousands' place in the remainder.

5. From the 3 ten-thousands left in the minuend the 1 ten-thousand is subtracted, giving 2 ten-thousands for the highest order in the remainder.

NOTE.—In practice the following is all that needs to be said or thought: 7 from 12 is 5, 8 from 15 is 7, 2 from 3 is 1; 9 from 10 is 1; 1 from 3 is 2.

II. Required to find the difference between 6000 and 2347.

Explanation.—Since there are no hundreds, tens, or units in the minuend from which to subtract the corresponding orders of the subtrahend, 1 of the thousands is taken and changed to hundreds, 1 of the 10 hundreds thus obtained is changed to tens, and 1 of the 10 tens is changed to 10 units. 7 units from 10 units gives 3 units, 4 tens from 9 tens gives 5 tens, 3 hundreds from 9 hundreds gives 6 hundreds, and 2 thousands from 5 thousands gives 3 thousands.

Process.

$$\begin{array}{r} 6000 \\ \underline{2347} \\ 3653 \end{array}$$

80. Rule.—I. *Write the subtrahend under the minuend, so that units of the same order shall stand in the same column.*

II. *Begin at the right and subtract each figure of the subtrahend from that above in the same column.*

III. *When any figure of the minuend is less in value than that of the same order in the subtrahend, add* 10 *to it before subtracting; and then consider the next figure of the minuend as diminished by* 1.

Proof.—*Add the remainder to the subtrahend, and if their sum is equal to the minuend, the work is correct.*

NOTE.—For a different method in subtraction, see Appendix, page 319.

81. ***Write from dictation and find the difference between:***

1.	2.	3.	4.	5.
7596	9658	18741	70813	80715
3485	2356	9298	15358	70118

6.	7.	8.	9.
170560	200504	487564	897003
85452	79623	287664	799452

82. The following exercise, which requires pupils to give remainders at sight, is designed to facilitate written work in subtraction.

83. ***Announce remainders at sight:***

Read answers: 1. From left to right; 2. From right to left; 3. From top to bottom; 4. From bottom to top; 5. As directed by the teacher.

	1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*	*8th.*	*9th.*	*10th.*
A	11	13	4	6	8	10	13	14	10	11
	6	5	3	2	1	8	9	7	9	3
B	9	8	12	12	12	9	5	3	4	10
	0	8	5	9	6	4	5	1	2	1
C	4	8	9	6	3	7	9	10	5	7
	4	2	6	1	2	5	3	7	4	3
D	10	12	5	13	11	15	14	7	6	5
	6	3	2	8	5	6	5	7	0	3
E	4	9	6	5	13	10	12	9	8	11
	1	2	4	1	7	3	4	1	3	7
F	16	14	11	12	6	7	17	11	9	10
	8	6	9	7	3	0	8	4	9	4
G	9	7	18	8	6	8	9	16	12	8
	5	6	9	5	6	4	7	9	8	0
H	7	6	13	11	7	10	15	7	8	9
	1	5	6	8	2	5	7	4	6	8
I	10	16	14	11	15	13	12	15	8	14
	2	7	9	2	9	4	9	8	7	8

NOTE.—This table, as well as those already given, should be frequently practiced upon.

84. ***Subtract as indicated:***

1. 175364 − 94872.
2. 231562 − 87968.
3. 412653 − 298387.
4. 628341 − 598389.
5. 932467 − 369849.
6. 745354 − 698746.
7. 392738 − 108009.
8. 842005 − 608927.
9. 803064 − 487675.
10. 238013 − 199815.
11. 721010 − 354019.
12. 936061 − 847079.
13. 1623802 − 908939.
14. 1010304 − 841705.
15. 2005021 − 1983037.
16. 2103000 − 837501.
17. 2420030 − 1940041.
18. 3010007 − 1012008.
19. 4201021 − 3302032.
20. 5300060 − 3099909.

85. When several numbers are connected by the signs + and −, the operations indicated must be performed in the order of the signs, commencing at the left.

Thus, to find the value of 6 + 9 − 5 + 7, add 6 and 9, subtract 5 from the sum, and add 7 to the remainder.

86. The **parenthesis,** ( ), is used to include several quantities to be treated as one. A **vinculum,** $\overline{\quad}$, serves the same purpose.

Thus, (6 + 9) − (5 + 7) denotes the difference between the sum of 6 and 9 and the sum of 5 and 7. The same is denoted by $\overline{6+9}-\overline{5+7}$.

87. ***Find the value of:***

1. 25 − 11 − 7 + 3.
2. 25 − (11 − 7) + 3.
3. 25 − 11 − (7 + 3).
4. 35 + 14 − 9 − 5.
5. 35 + 14 − (9 − 5).
6. 33 − (24 − 13) − 9.
7. (75 + 33) − (36 + 40).
8. 18 − (3 − 2) + 17 − 5.
9. 1225 − 106 − 708.
10. $1875-\overline{927+465}$.
11. $2349-\overline{902-367}$.
12. 187 − (68 + 41) + 47.
13. $236-\overline{92-49}-98$.
14. $387-\overline{83-46-28}$.
15. 708 + 405 − 98 − 125.
16. 708 − 405 + 98 − 125.
17. 708 − (405 + 98) − 125.
18. 708 − (405 − 98 − 125).

ORAL EXERCISES.

88. *1.* Subtract by 2's from 48 to 0; from 49 to 1.

2. By 3's from 60 to 0; from 61 to 1; from 62 to 2.

In like manner, subtract

3. By 4's from 64, from 65, from 66, and from 67; by 5's from 70, 71, 72, 73, and 74.

4. By 6's from 72, 73, 74, 75, 76, and 77.

5. By 7's from 77, and from each successive number up to 83.

6. By 8's from 80, and from each successive number up to 87.

7. By 9's from 90, and from each successive number up to 98.

8. Subtract 3 from 91, 81, 71, 61, 51, 41, 31, 21, and 11.

9. Subtract 4 from 90, 80, 70, etc.; from 91, 81, 71, etc.; from 92, 82, 72, etc.; from 93, 83, 73, etc.

10. In like manner, subtract 5, 6, 7, 8, 9 from 90, 80, 70, etc.; from 91, 81, 71, etc.; from 92, 82, 72, etc.; from 93, 83, 73, etc.

CONCRETE ORAL PROBLEMS.

89. *1.* I buy a comb for 15 cents; how much change should I receive out of 25 cents?

2. A boy has 17 marbles in one pocket and 10 in another; how many more does he have in the one than in the other?

3. John earns 14 dimes in one day, and his brother earns 7 dimes; how many dimes per day does John earn more than his brother?

4. Mary earned 13 cents and found 7 cents, and her father gave her enough to make 50 cents; how much did she receive from her father?

5. Henry had \$17, and after earning \$20, he deposited \$25 in the bank and loaned \$5 to his brother; how many dollars had he left?

6. My coat cost $17, my hat $8, and my boots $5; how much more did my coat cost than my hat and boots together?

7. A lady purchased a dress for $43 and a shawl for $17; how much change should she get from a hundred-dollar bill?

8. A farmer goes to market with a load of potatoes and apples; for the potatoes he gets $32, for the apples $18; with the proceeds he buys groceries costing $14, and dry-goods for $19; how much has he left?

9. 23 of 50 seats in a railroad car are occupied. At a stopping place, 7 gentlemen and 9 ladies get in; how many seats are still vacant?

10. A grocer bought 5 dozen eggs, of which he sold 26 and broke 9 in handling; how many were left?

CONCRETE WRITTEN PROBLEMS.

90. *1.* A public school contains 1155 pupils; 578 are girls; how many are boys?

2. A man that was born in 1839, was married in 1872; how old was he at his marriage?

3. From a flock of sheep numbering 912 there were sold 485; how many remained?

4. A farmer put into his cellar 832 bushels of apples, and they were all frozen but 387 bushels; how many bushels were frozen?

5. I have $2150 in cash, all of which is in the bank except $378; find how much is in the bank.

6. A man buys a farm for $5895, and sells it for $8340; find his gain.

7. A merchant gained $1199 by selling a lot of goods for $7465; how much did they cost?

8. A drover has 3250 sheep and 2873 hogs; how many more sheep than hogs has he?

9. A father leaves $30500 to his son and $18950 to his daughter; how much more does the son receive than the daughter?

10. I buy a house for $35425, and pay all but $8768; how much do I pay?

11. The last payment on a farm costing $13325, was $4876; how much had been paid before?

12. The distance around the earth is 24899 miles, and the distance around the moon is 6786 miles; how much farther is it around the earth than around the moon?

13. The highest mountain in the world is Mt. Everest, in Asia, 29002 ft. high; the highest in North America is Mt. St. Elias, 17900 ft.; how much higher is Mt. Everest?

14. I have a certain sum of money; after saving $3575 more, I can buy a property I want for $8420, and have $1328 left; how much have I?

15. The area of England is 50922 sq. miles, of Wales 7398 sq. miles, of Scotland 29819 sq. miles, and of Ireland 32531 sq. miles; which is larger, and how much, all these together or California, whose area is 158360 sq. miles?

16. A merchant bought a store and fixtures for $11875, and laid in goods costing $17835; during the year he sold goods amounting to $8568, after which he disposed of store and all for $23975; did he gain or lose, and how much?

17. Bought a farm for $20875 and made on it two payments, one of $5832, the other of $12968; what is still due?

18. Three men invested $185625 in business; the first put in $61235, the second $9876 less than the first; what was the investment of the third, who contributed the remainder?

19. A father gave to the younger of two sons $15875 with which to begin business; to the older he gave $3738 less than to the younger; he then had left $45673. How much was he worth at first?

20. According to the census of 1880, New York City contained 1206299 inhabitants, Brooklyn 566663, Jersey City 120722, and Hoboken 30999; if they were all united under one city government, how much larger would London be, whose population is 3814571?

MULTIPLICATION.

1. If a man earns \$7 in one day, how much will he earn at the same rate in 5 days? \$7 + \$7 + \$7 + \$7 + \$7 are how many dollars?

2. 9 + 9 + 9 + 9 are how many? *Four* 9's are how many? 4 times 9 are how many? *Five* 9's are how many? 5 times 9 are how many? Six 9's are how many? 6 times 9 are how many?

91. If a number is repeated several times the sum may be found by addition, or given from memory. When given from memory, the process is called *multiplication* and the sum is called a *product.*

When the numbers are large, partial products are given from memory and the entire product is found by uniting the partial products.

92. Multiplication is the process of taking one number as many times as there are units in another.

93. The number repeated or multiplied is called the **multiplicand.**

94. The number which indicates how many times the multiplicand is to be taken is called the **multiplier.**

95. The result obtained by multiplication is called the **product.**

96. The **factors** of a number are the numbers which multiplied together will produce it.

The multiplicand and multiplier are factors of the product.

97. When the product of more than two numbers is found by multiplying the first by the second, that result by the third, etc., the process is called **continuous multiplication,** and the final result the **continued product.**

3. 4 times 5 are how many? 5 times 4 are how many? What is the continued product of 2, 3 and 4? Of 4, 3 and 2? Of 4, 2 and 3?

98. The product of two or more factors will be the same in whatever order they are used.

99. The **sign of multiplication** is ×, and is read *multiplied by*, or *times*.

Thus, $7 × 5 is read *$7 multiplied by 5*, or *5 times $7*. 7 × 5, two abstract numbers, may be read, *7 times 5*, or *5 times 7*. 2 × 3 × 5 is read, *the continued product of 2, 3 and 5.*

4. Can you multiply $5 by 4? Is the multiplicand concrete or abstract? The multiplier? The product?

5. Can you multiply 5 by 4? What kind of a number is the multiplicand? The multiplier? The product?

6. Can you multiply 5 by $4? $5 by $4?

100. Principles.—I. *The multiplicand may be either concrete or abstract.*

II. *The multiplier is always abstract.*

III. *The product is always like the multiplicand.*

101. To find the product of any two numbers from 1 to 12.

102. The product of any two numbers from 1 to 12 should first be found by addition, and then memorized.

103. The **multiplication table** is given in the Appendix, page 320.

104. To multiply with accuracy and rapidity, one must know at sight the product of any two numbers from 2 to 12. The exercise on the following page contains the combinations of these numbers, and should be practiced until pupils can read it in any direction correctly, and without hesitation.

105. *Announce products at sight:*

Read answers: 1. From left to right; 2. From right to left; 3. From top to bottom; 4. From bottom to top; 5. In any order suggested by the teacher.

	1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*	*8th.*	*9th.*	*10th*
A	6	8	10	9	5	4	2	9	11	7
	3	7	6	3	9	11	7	4	2	5
B	6	5	3	12	11	9	8	6	5	7
	7	4	5	4	11	7	5	4	6	8
C	5	4	10	12	9	4	5	3	6	8
	11	7	7	3	2	4	10	6	8	9
D	9	11	12	8	10	11	4	9	12	7
	11	6	2	10	11	9	12	6	5	6
E	11	3	7	10	11	4	11	4	3	4
	3	7	10	4	12	10	4	8	10	9
F	6	8	2	8	9	6	10	12	7	4
	5	11	10	3	5	11	8	6	3	6
G	8	5	3	5	7	11	12	6	5	9
	8	5	12	7	9	10	7	6	12	9
H	3	6	2	8	12	10	8	10	9	11
	8	10	9	6	12	10	4	5	8	8
I	2	4	5	12	11	7	11	12	2	6
	12	5	8	11	7	12	5	10	11	12
J	3	12	8	10	7	9	7	12	9	10
	11	9	12	9	7	12	4	8	10	12

106. ***Read and answer:***

1. 5+5+5 is equal to how many times 5? 4 times 5 = ? 5 times 5 = ? 5 times 6 = ? 5×7 = ? 5×8 = ? 5×9 = ? 5×11 = ? 11×5 = ? 9×5 = ? 8×5 = ?

2. $9×5 is equal to how many dollars? $9 × 6 = ? $9×8 = ? 8 times $9 = ? 6 times 9 = ?

3. 2×4×7 = ? 7×4×2 = ? 4×7×2 = ?

4. What is the continued product of 2, 3, 4 and 5? Of 3, 4, 3 and 2? Of 2, 3, 5 and 7?

5. What is the number whose factors are 3, 5 and 2? 3, 5 and 3? 5, 3 and 4? 6, 2 and 3? 6, 2 and 5? 6, 3 and 4? 6, 3 and 5?

107. To find the product of two numbers when the multiplier is expressed by a single figure.

I. Required the product of 14378 multiplied by 6.

Process.

$$\begin{array}{r} 14378 \\ 6 \\ \hline 86268 \end{array}$$

Analysis.

$$\begin{array}{rl} 8 \times 6 = & 48 \\ 70 \times 6 = & 420 \\ 300 \times 6 = & 1800 \\ 4000 \times 6 = & 24000 \\ 10000 \times 6 = & 60000 \\ \hline 14378 \times 6 = & 86268 \end{array}$$

Explanation.—1. The product of 8 units multiplied by 6 is 48 units, or 4 tens and 8 units; the 8 units are written in the units' place of the product and the 4 tens reserved to be added to the tens.

2. The product of 7 tens multiplied by 6 is 42 tens, or 4 hundreds and 2 tens; the 4 tens reserved when added make 6 tens, which are written in tens' place; and the 4 hundreds are reserved to be added to the hundreds.

3. In the same manner the hundreds, thousands, etc., are multiplied, giving 86268 for the entire product.

108. ***Find the product of:***

1.	1728×2.	*7.*	6007×8.	*13.*	800499×9.
2.	4862×3.	*8.*	3068×9.	*14.*	307090×7.
3.	8324×4.	*9.*	13875×8.	*15.*	836592×4.
4.	5680×5.	*10.*	37659×6.	*16.*	376059×8.
5.	7359×6.	*11.*	23865×5.	*17.*	865497×6.
6.	8000×7.	*12.*	756830×7.	*18.*	708096×9.

109. To find the product of two numbers when the multiplier is expressed by two or more figures.

I. Required the product of 573 multiplied by 264.

Explanation.—1. The multiplicand is multiplied by the 4 units of the multiplier, giving for a product 2292, which is so written that the right-hand figure shall stand under the units' figure of the multiplier.

2. The multiplicand is multiplied by the 6 tens of the multiplier, giving for a product 3438 tens, which is so written that the right-hand figure shall stand under the tens of the multiplier.

3. In the same manner, the multiplicand is multiplied by the 2 hundreds of the multiplier.

4. The partial products are added, giving 151272 for the entire product.

Process.

$$\begin{array}{r} 573 \\ \underline{264} \\ 2292 \\ 3438 \\ \underline{1146} \\ 151272 \end{array}$$

Analysis.

$$\begin{array}{rcr} 573 \times 4 &=& 2292 \\ 573 \times 60 &=& 34380 \\ 573 \times \underline{200} &=& \underline{114600} \\ 573 \times 264 &=& 151272 \end{array}$$

110. Rule.—I. *Write the multiplicand and multiplier so that units of the same order shall stand in the same column.*

II. *Multiply the multiplicand by each significant figure of the multiplier, writing the first or right-hand figure of each partial product under the figure of the multiplier used.*

III. *Add the partial products.*

Proof.—*Multiply the multiplier by the multiplicand, and if the product is the same, the work is correct.*

111. ***Find the product of:***

1.	143 × 24.	*7.*	746 × 972.	*13.*	65498 × 7009.
2.	167 × 35.	*8.*	789 × 795.	*14.*	71487 × 4567.
3.	326 × 29.	*9.*	987 × 687.	*15.*	98547 × 5678.
4.	387 × 48.	*10.*	1847 × 876.	*16.*	86456 × 6789.
5.	409 × 64.	*11.*	3959 × 987.	*17.*	59387 × 7890.
6.	532 × 75.	*12.*	4584 × 703.	*18.*	58973 × 5789.

112. To find the product of two factors, when there are ciphers at the right of the significant figures of one or both.

I. Required to multiply 375×100.

Explanation.—375 multiplied by 100 is equal to 375 times 100, or 375 hundreds. Hence, to multiply a number by 10, 100, 1000, etc., annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

Process.

$$\begin{array}{r} 375 \\ \underline{100} \\ 37500 \end{array}$$

II. Required to multiply 870 by 2500.

Explanation.—870 multiplied by 2500 equals $87 \times 10 \times 25 \times 100$; and since the factors may be taken in any order, 87 may be multiplied by 25 and the product by 10×100, or 1000. Hence, to find the product of two factors, one or both having ciphers at the right of the significant figures, first find the product of the numbers expressed by the figures at the left of the ciphers, and then annex to it as many ciphers as there are at the right in both of the factors.

Process.

$$\begin{array}{r} 870 \\ \underline{2500} \\ 435 \\ \underline{174} \\ 2175000 \end{array}$$

113. Rule.—*Find the product of the parts of the multiplicand and multiplier expressed by the significant figures at the left of the ciphers, and then annex as many ciphers as there are at the right in both factors.*

114. ***Find the product of:***

1. 457×30.
2. 1325×60.
3. 2386×50.
4. 4096×70.
5. 5836×80.
6. 3600×800.
7. 7600×900.
8. 5000×400.
9. 8300×700.
10. 3600×500.
11. 7500×600.
12. 8310×7500.
13. 9250×8600.
14. 3060×3900.
15. 8970×9800.
16. 8500×6000.
17. 9000×4000.
18. 7000×5000.

115. ***Find the continued product of:***

1. $784 \times 420 \times 500$.
2. $367 \times 408 \times 700$.
3. $598 \times 750 \times 260$.
4. $400 \times 900 \times 287$.
5. $650 \times 750 \times 850$.
6. $327 \times 456 \times 500 \times 700$.
7. $439 \times 548 \times 600 \times 800$.
8. $300 \times 790 \times 850 \times 740$.
9. $900 \times 900 \times 425 \times 985$.
10. $789 \times 697 \times 600 \times 900$.

116. When the sign of multiplication is used with the signs of addition and subtraction, the indicated products must be found first, except when the quantities joined by the latter are united by the parenthesis or vinculum.

Thus, to find the value of $5+4\times 3-8$, multiply 4 by 3, add the product to 5, and subtract 8 from the sum.

117. ***Perform the operations indicated:***

1.	$564\times 987-456\times 25.$	*6.*	$8987-764\times 5.$
2.	$\overline{17584-9869}\times 8009.$	*7.*	$(8987-764)\times 5.$
3.	$3869\times 234+756.$	*8.*	$(8987+764)\times 5.$
4.	$3869\times(234+756).$	*9.*	$95-50+35\times 7.$
5.	$17560-648\times 25.$	*10.*	$\overline{95-50}\times\overline{35-7}.$

11. $89763\times 687+45983\times 603-984\times 789.$

12. $75\times 84\times 93\times 60-23\times 32\times 90\times 80+750\times 90.$

13. $\overline{75486-250}\times\overline{875+1400}+187500+94302.$

14. $\overline{875\times 234-98+65}\times\overline{98+8760}-(8794-789).$

CONCRETE ORAL PROBLEMS.

118. *1.* What is the value of 9 tons of hay at 7 dollars per ton?

2. What is paid for 8 oranges at 5 cents each?

3. How much do 9 sacks of salt cost at 3 dollars per sack?

4. If a man walk 7 miles an hour, how far will he walk in 12 hours?

5. In 8 weeks how much more can a man earn at 12 dollars a week than at 7 dollars a week?

6. If I buy sheep at 6 dollars a head and sell them at 8 dollars a head, what do I make on 20 sheep?

7. How many hours in one week?

8. How many minutes in 11 hours?

9. In one bushel there are 4 pecks; what will 4 bushels of potatoes cost at 20 cents a peck?

10. In one gallon there are 4 quarts; what will 3 gallons of milk cost at 8 cents a quart?

11. Henry has 12 marbles and John has twice as many; how many have they together?

12. I buy 5 quarts of chestnuts at 8 cents a quart, and 5 quarts at 5 cents a quart; what is my profit, if I sell all at 9 cents a quart?

13. If I exchange 6 tons of hay at $9 a ton for 20 cords of wood at $3 a cord, how much cash must I pay?

14. Two ships sail from the same place, one east at 8 miles an hour, the other west at 9 miles an hour; how far apart are they in three hours?

15. A boy earns 5 dollars a week; his brother earns three times as much, and his father earns four times as much; what would be their united wages for 4 weeks?

CONCRETE WRITTEN PROBLEMS.

119. *1.* Find the cost of 346 acres of land at $165 per acre.

2. What will 756 tons of hay cost at $27 per ton?

3. On a steamer from New York to Aspinwall there were 378 passengers, each of whom paid $83 fare; how much did all pay?

4. A sailing vessel that averages 8 miles per hour, requires just 25 days of 24 hours each to go from San Francisco to Japan; how far is it?

5. Find the cost of 43 sleeping-cars at $8975 each.

6. What cost 375 cords of hickory wood at $7.75 per cord?

7. A steamer that averages 267 miles per day can go from New York to the Cape of Good Hope in 25 days; find the distance.

8. To build a certain railroad 189 miles long cost $47847 per mile; what did the road cost?

9. In one mile there are 1760 yards; how many yards in 827 miles?

10. A man buys a drove of 369 horses, for which he pays on an average $175 each; how much do they all cost?

11. Find the cost of 875 bushels of wheat at $1.35 per bushel.

12. A farmer plants on each of 87 acres 8956 hills of corn; how many does he plant in all?

13. An importer sold 23 rolls of Axminster carpet at $4.37 per yard; what did he receive for it, if each roll contained 165 yards?

14. A drover traded 465 cattle worth $49 a head for a farm of 389 acres worth $85 per acre, and paid the difference in cash; how much cash did he pay?

15. A man bought one farm of 325 acres at $63 an acre, and another of 247 acres at $87 per acre; he sold both for $76 per acre. Did he gain or lose, and how much?

16. I bought 36 car-loads of wheat at $1.53 per bushel, and sold it at $1.71 per bushel; find my gain, if each car contained 413 bushels.

17. A contractor built a row of 23 houses all alike, each of which cost him $2375.74; he sold them at $2735 each; find his gain.

18. A miller owed $15000; he gave in payment 975 barrels of flour at $7.63 per barrel, 1235 bushels of oats at $.37, and some cash, after which he still owed $1828.80; how much cash did he pay?

19. According to the best authority, light travels at the rate of 185172 miles per second, and in reaching the earth from the sun requires 493 seconds; how far away is the sun?

20. A passenger train running at the average rate of 34 miles, requires 28 hours to go from New York to Chicago; what is the distance?

21. A speculator bought 15 car-loads of potatoes at $1.15 per bushel; he sold 3756 bushels at $1.38 per bushel, and the rest at $.98 per bushel; if each car contained 375 bushels, did he gain or lose, and how much?

DIVISION.

1. How many 7's are there in 35? 5 times 7 are how many? 7 is contained how many times in 35?

2. \$5 is contained how many times in \$20? In \$25? In \$30? In \$50? In \$75?

3. If \$24 is divided into 4 equal parts, how many dollars will there be in each part? How many dollars will there be in each part, if \$24 is divided into 6 equal parts?

120. Division is the process of finding how many times one number is contained in another of the same kind; or of dividing a number into equal parts, the number of parts being given.

121. The number divided is called the **dividend.**

122. The number by which the dividend is divided is called the **divisor.**

123. The result is called the **quotient.**

The quotient indicates the number of times one number is contained in another; or it is one of the equal parts into which a number is divided.

124. The **remainder** is the part left over when the divisor is not contained an exact number of times in the dividend.

Thus, \$8 is contained 6 times in \$55 and \$7 is left over; \$7 in this case is the remainder. The remainder is a part of the dividend, and therefore of the same denomination as the dividend.

125. The **sign of division** is $\div$, and is read *divided by.* Division is also indicated by writing the dividend above and the divisor below a short horizontal line, or by writing the divisor before the dividend with a curved line between.

Thus, each of the expressions, $45 \div 9$, $\frac{45}{9}$, and $9)45$ means that 45 is to be divided by 9.

126. The signs of multiplication and division take precedence over those of addition and subtraction, except when the quantities joined by the latter are united by the parenthesis or vinculum. Thus,

$14-3\times2+8\div4=14-6+2=8+2=10.$
$(14-3)\times2+8\div4=11\times2+8\div4=22+2=24.$

When the signs of multiplication and division are used in immediate succession, the parenthesis or vinculum is needed to indicate which operation is to be performed first.

Thus, $5\times8\div4\times2$ is indefinite. It may mean $(5\times8)\div(4\times2)$, or $(5\times\overline{8\div4})\times2$, or $5\times(8\div\overline{4\times2})$, etc.

127. ***Read and answer:***

1. If $18 is divided equally among 3 boys, how many dollars will each boy receive?

2. If the dividend is 51 and the divisor 9, what are the quotient and remainder?

3. $2+3+4-5+6\div2=?$ $8-3+10\div2-7=?$

4. $9\div3+12\div2=?$ $3\times6-4\times2=?$ $\overline{4\times6}\div\overline{3\times4}=?$

5. $2+3+4-\overline{5+9}\div2=?$ $\overline{4+3}\times4-\overline{5+6}\times2=?$

6. The divisor is 8 and the dividend 65; what are the quotient and the remainder?

7. The remainder is 3, the divisor 9, and the quotient 7; what is the dividend?

8. The quotient is 7, the remainder 5, and the dividend 47; what is the divisor?

9. How many equal parts, each containing 9 units, are there in 63? 36? 81? 45? 54?

10. If there are 27 eggs in a basket and you take out 9, how many will be left? If you take out 9 more, how many will be left? How many times can you take 9 eggs out of a basket containing 27 eggs?

11. Find by subtracting how many 9's there are in 36.

12. How many 7's in 42? How many in 63? 7 is contained in 42 how many times? In 63?

13. \$5 is contained how many times in \$30? What kind of a number is the divisor—abstract or concrete? The dividend? The quotient?

14. If a man earns \$35 in 5 days, how much does he earn each day? If \$35 is divided into five equal parts, how many dollars in each part? What kind of a number is the divisor? The dividend? The quotient?

128. Principles.—I. *If the division consists in finding how many times one number is contained in another, the dividend and divisor are like numbers, and the quotient is abstract.*

II. *If the division consists in dividing a number into equal parts, the dividend and quotient are like numbers, and the divisor is abstract.*

III. *The divisor and quotient are factors of the dividend.*

IV. *The remainder is always less than the divisor and of the same denomination as the dividend.*

129. To divide when the divisor is expressed by one figure and the dividend is less than 10 times the divisor.

130. In this case, since division is the reverse of multiplication, to know how many times the divisor is contained in the dividend, it is necessary to remember by what number the former must be multiplied to give the latter.

Thus, to know that 7 is contained 6 times in 42, it is necessary to remember that there are six 7's in 42, or that 6 times 7 are 42.

131. The following are models for oral exercises, designed to teach pupils how to find the number of times one number is contained in another.

1. $5 \times 6 = 30$; therefore $30 \div 5 = 6$, and $30 \div 6 = 5$.
2. $7 \times 5 = 35$; therefore $35 \div 7 = 5$, and $35 \div 5 = 7$.
3. $7 \times 9 = 63$; therefore $63 \div 7 = 9$, and $63 \div 9 = 7$.

132. To find the quotient of one number divided by another when the divisor is expressed by one figure.

I. Required the quotient of 9698 divided by 7.

Process.

$$7\,\underline{)\,9698}$$
$$1385\tfrac{3}{7}$$

Analysis.

$$\begin{aligned} 7000\div7 &= 1000\\ 2100\div7 &= 300\\ 560\div7 &= 80\\ 35\div7 &= 5\\ 3\div7 &= \tfrac{3}{7}\\ \hline 9698\div7 &= 1385\tfrac{3}{7} \end{aligned}$$

Explanation.—1. The 9 thousands of the dividend divided by 7 gives 1 thousand for a quotient and a remainder of 2 thousand, or 20 hundred.

2. The 1 thousand is written in thousands' place in the quotient, and the 20 hundreds added to the 6 hundred for the next partial dividend. The divisor is contained in 26 hundred, 3 hundred times, with a remainder of 5 hundred, or 50 tens.

3. The 3 hundreds is written in hundreds' place in the quotient and the 50 tens added to the 9 tens for the next partial dividend. 7 is contained in 59 tens, 8 tens times with a remainder of 3 tens or 30 units.

4. The 8 tens is written in tens' place in the quotient and the 30 units added to the 8 units gives 38 for the last partial dividend. 7 is contained in 38 units, 5 times with a remainder of 3 units; the 5 is written in units' place in the quotient and the remainder, written over the divisor to indicate the division, is annexed to the quotient.

NOTE.—In practice, the different orders of the dividend are regarded as units and the division is made as follows: 7 in 9 once; in 26 three times; in 59 eight times; in 38 five times with a remainder of 3, which is written over the dividend and annexed to the quotient.

133. ***Find the quotient of:***

1. 5) 256784.	*5.* 9) 583497.	*9.* 8) 1598623.
2. 9) 358602.	*6.* 7) 605830.	*10.* 7) 3865462.
3. 7) 730125.	*7.* 8) 5867431.	*11.* 9) 9287536.
4. 8) 843592.	*8.* 6) 6280347.	*12.* 8) 7569873.

13. $\overline{875\times250}\div5.$	*17.* $\overline{7642\div2}\div\overline{28\div4}.$
14. $\overline{960-360}\div4.$	*18.* $\overline{17960\div5}\div\overline{4+4}.$
15. $\overline{8000+2948}\div6.$	*19.* $\overline{42\div7+8492}\div\overline{7+2}.$
16. $\overline{325\times27}\div7.$	*20.* $\overline{849\times678}\div\overline{96\div12}.$

134. Accuracy and rapidity in division are determined largely by the pupil's ability to give quotients at sight. The following exercise should be faithfully practiced.

135. ***Announce quotients at sight:***

Read in different directions, as directed by the teacher, naming only the quotients.

	1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*
A	8) 80	6) 42	4) 28	6) 30	8) 48	4) 12	3) 24
B	6) 54	8) 64	9) 45	8) 24	5) 20	4) 24	6) 18
C	4) 16	6) 30	8) 32	9) 54	2) 18	6) 36	3) 27
D	5) 45	7) 56	3) 21	2) 16	8) 40	5) 25	9) 27
E	7) 14	6) 48	3) 12	5) 15	2) 14	6) 6	4) 32
F	3) 3	2) 12	9) 72	8) 16	7) 63	5) 40	7) 7
G	4) 36	3) 9	8) 16	9) 36	2) 10	6) 12	3) 15
H	9) 9	5) 20	7) 35	8) 72	9) 18	5) 5	4) 8
I	3) 15	6) 24	8) 56	7) 28	5) 35	3) 12	7) 21
J	7) 49	9) 63	5) 10	4) 20	7) 42	3) 18	9) 81

136. In the above exercise the divisions are exact. In practice there is usually a remainder, and the exercise on the following page is given as a model for a series of exercises, the full series to be constructed by using successively each of the other digits in place of the **9**. The exercises may be arranged on the blackboard by the teacher, or the pupils may be required to write them out on slates or paper.

137. ***Announce at sight quotients and remainders:***

Read in different directions, naming only quotients and remainders.

	1st.	*2d.*	*3d.*	*4th.*	*5th.*	*6th.*	*7th.*
A	9) 56	9) 31	9) 46	9) 83	9) 95	9) 11	9) 22
B	9) 13	9) 82	9) 71	9) 10	9) 70	9) 91	9) 40
C	9) 93	9) 53	9) 41	9) 80	9) 16	9) 21	9) 65
D	9) 12	9) 79	9) 42	9) 66	9) 69	9) 44	9) 33
E	9) 20	9) 14	9) 67	9) 92	9) 17	9) 50	9) 24
F	9) 57	9) 19	4) 89	9) 15	9) 23	9) 61	9) 85
G	9) 25	9) 49	9) 38	9) 88	9) 59	9) 29	9) 94
H	9) 68	9) 55	9) 43	9) 73	9) 86	9) 32	9) 62
I	9) 48	9) 77	9) 35	9) 75	9) 52	9) 26	9) 74
J	9) 39	9) 87	9) 30	9) 37	9) 58	9) 34	9) 84
K	9) 60	9) 78	9) 51	9) 64	9) 76	9) 47	9) 28

138. ***Find the quotient of:***

1. 45,708,622,317 ÷ 3.
2. 69,285,926,837 ÷ 4.
3. 59,836,274,159 ÷ 5.
4. 36,492,631,864 ÷ 6.
5. 89,536,823,598 ÷ 7.
6. 62,304,600,181 ÷ 6.
7. 59,862,349,286 ÷ 9.
8. 64,632,567,814 ÷ 5.
9. 386,928,165,432 ÷ 9.
10. 729,834,649,847 ÷ 8.
11. 326,874,923,864 ÷ 7.
12. 420,068,000,621 ÷ 9.
13. 300,168,427,368 ÷ 6.
14. 222,333,444,555 ÷ 8.
15. 807,000,111,000 ÷ 9.
16. 315,678,911,625 ÷ 7.

139. To find the quotient of one number divided by another when the divisor contains two or more figures.

I. Required the quotient of 151353 divided by 24.

Process.

$$
\begin{array}{r}
24\,)\,151353\,(\,6306\tfrac{9}{24} \\
\underline{144} \\
73 \\
\underline{72} \\
153 \\
\underline{144} \\
9
\end{array}
$$

Analysis.

$$
\begin{aligned}
144000 \div 24 &= 6000 \\
7200 \div 24 &= 300 \\
144 \div 24 &= 6 \\
9 \div 24 &= \tfrac{9}{24} \\
\hline
151353 \div 24 &= 6306\tfrac{9}{24}
\end{aligned}
$$

Explanation.—1. The least number of left-hand figures that will contain the divisor is *three*, and hence the *first* partial dividend is 151 thousands.

2. 24 is contained in 151 thousands, 6 thousand times with a remainder. The 6 is written in thousands' place in the quotient, the divisor multiplied by it, and the product subtracted from the partial dividend.

3. To the remainder, 7 thousands, is annexed the 3 hundreds of the dividend, giving 73 hundreds for the *second* partial dividend. 24 is contained in 73 hundreds, 3 hundred times with a remainder. The 3 is written in hundreds' place in the quotient, the divisor multiplied by it, and the product subtracted from the partial dividend.

4. To the remainder is annexed the 5 tens of the dividend, giving 15 tens for the *third* partial dividend. 24 is not contained in 15; a cipher is therefore written in the quotient and the 3 units of the dividend annexed to the 15 tens, giving 153 units for the *fourth* partial dividend. 24 is contained in 153 units, 6 times with a remainder. The 6 is written in units' place in the quotient, the divisor multiplied by it, the product subtracted, and the remainder written over the divisor and annexed to the quotient.

140. When the operation of division is fully expressed, as above, the process is called **long division**; where a part of the operation is performed mentally, as in Art. 131, the process is called **short division.**

141. Rule for Long Division.—I. *Write the divisor at the left of the dividend with a curved line between.*

II. *Use as the first partial dividend the fewest left-hand figures of the dividend that will contain the*

divisor. Divide, and write the quotient at the right of the dividend with a curved line between.

III. *Multiply the divisor by this quotient figure, subtract the result from the partial dividend, and to the remainder annex the next figure of the dividend.*

IV. *Divide the number thus obtained by the divisor, write the result in the quotient, multiply the divisor by it, and subtract as before.*

V. *So continue until all the figures of the dividend have been annexed.*

VI. *Should any partial dividend not contain the divisor, place a cipher in the quotient, annex another figure to the partial dividend and proceed as before.*

Proof.—*Multiply the divisor and quotient together and to the product add the remainder; if the work is correct the sum will equal the dividend.*

142. ***Find the quotient of:***

1.	$13567 \div 27$.	*8.*	$830685 \div 81$.	*15.*	$569834 \div 346$.
2.	$28693 \div 29$.	*9.*	$927359 \div 93$.	*16.*	$361589 \div 169$.
3.	$50657 \div 61$.	*10.*	$361456 \div 75$.	*17.*	$398234 \div 458$.
4.	$38753 \div 42$.	*11.*	$856001 \div 87$.	*18.*	$789567 \div 387$.
5.	$79685 \div 83$.	*12.*	$953213 \div 98$.	*19.*	$657984 \div 656$.
6.	$43061 \div 52$.	*13.*	$732509 \div 123$.	*20.*	$987654 \div 237$.
7.	$72086 \div 48$.	*14.*	$528037 \div 234$.	*21.*	$432864 \div 978$.

22.	$\overline{1387 \times 235} \div 196$.	*26.*	$(110459 + 138967) \div 837$.
23.	$\overline{4836 \times 4096} \div 512$.	*27.*	$(349290 - 96586) \div 298$.
24.	$\overline{\$18575 \times 25025} \div 775$.	*28.*	$\overline{156875 \div 125} \times 15625$.
25.	$(4577296 \times 8368) \div 496$.	*29.*	$\overline{37323 \div 87} \div \overline{9114 \div 93}$.

30. The continued product of 17, 23, 34, and 56, divided by the difference between 897 and 659.

31. The difference between 859684 and 759609 divided by the product of 87 and 78.

CONCRETE ORAL PROBLEMS.

143. *1.* If 9 barrels of flour cost $63, what does 1 barrel cost?

2. If 8 tons of coal cost $56, what do 2 tons cost?

3. How many rods of ditch must a man dig for $54, if he gets $6 per rod?

4. A box contains 60 oranges, which are sold 3 for 5 cents; how much is received for all?

5. I buy 3 dozen eggs at 20 cents a dozen, and sell them 9 for 25 cents; find my gain.

6. A man walks 4 miles per hour; how long will it take him to go to a place 30 miles away and return?

7. A boy found $1, and after buying a top for 13 cents and a book for 37 cents, he expended the rest for apples, paying 5 cents for 3 apples; how many apples did he get?

8. A grocer paid $96 for 12 barrels of flour and sold it at $10 per barrel; how much did he gain?

9. A coal merchant bought some coal for $80 and sold it for $120, gaining $2 on each ton; how many tons were there?

10. A boy has some marbles, and after reserving 13 for himself, he divides the rest among his 5 playmates, each of whom receives 12 marbles; how many had he at first?

11. Mary divides a string of 119 buttons among 9 of her friends, keeping 11 buttons for herself; how many does she give to each friend?

12. How many more oranges can be bought with $1 at 3 for 5 cents than at 4 for 10 cents?

13. John earns $3 per week, and Arthur earns $8 per week; how long will it take Arthur to earn $100 more than John?

14. Henry can gather 3 quarts of berries in an hour, Oscar 4 quarts, and their father 10 quarts; in how many hours can the father gather 24 quarts more than Henry and Oscar together?

CONCRETE WRITTEN PROBLEMS.

144. *1.* A farmer raises 8091 bushels of corn on 93 acres of land; find the yield per acre.

2. A coal dealer paid $1066.28 for 244 tons of coal; find the price per ton.

3. It is 24899 miles around the world; how long would it take a man to make the trip traveling at the average daily rate of 39 miles?

4. A steamer burns 1786 tons of coal in a voyage of 47 days; how much does she burn daily?

5. A manufacturer pays for labor $83675 yearly, for material $17587, and for freight and incidental expenses, $11561; find his average weekly outlay, allowing 52 weeks to the year.

6. A contractor of St. Louis sold the English Government 694518 pounds of canned beef; how many cattle were slaughtered to supply it, if an average of 561 pounds was obtained from each animal?

7. A certain city owes a debt of $3026925 and decides to pay the debt in 99 years, paying each year the same amount; what will be the annual payment?

8. A stock farmer has 5989 bushels of corn; how long will it last, if he feeds 53 bushels to his cattle daily?

9. The President of the United States receives $50000 annual salary; how much is that per day, counting 365 days in the year?

10. By selling a farm of 165 acres for $13530, I gained $1155; how much did it cost me per acre?

11. A drover paid for horses and cattle $17696; there were 68 horses, and 138 cattle at $42 each; how much were the horses apiece?

12. A miller buys wheat for which he pays $1023.70; he sells it at $1.37 per bushel and gains $17.50; how many bushels are there?

13. A field of 35 acres produces 43 bushels of oats per acre, another of 56 acres produces 47 bushels per acre, and a third of 78 acres produces 39 bushels per acre; the grain is all sold for $12563.25; what is the price per bushel?

REVIEW QUESTIONS.

145. Addition. What is the sum or amount of two or more numbers? Illustrate. What is the sign of addition? Of equality? Illustrate each. What is the sign of dollars? How may cents be denoted? Illustrate. Can unlike numbers be added? Why? Can units be added to tens? Why? Explain the process of finding the sum of more than two numbers: 1. When each is expressed by a single figure; 2. When each is expressed by two or more figures.

146. Subtraction. What is the difference between two numbers? Define and illustrate: Subtraction, minuend, subtrahend, remainder. What is the sign of subtraction? Illustrate. Give an example illustrating the use of the parenthesis. Must the subtrahend be like the minuend? Why? Explain the process of finding the difference between two numbers when each is expressed by two or more figures.

147. Multiplication. Show that multiplication is a short process of finding the sum of two or more equal numbers. Define and illustrate: Multiplicand, multiplier, product, continued product, factors. Give the sign of multiplication and illustrate its use. When may the multiplicand and multiplier be like numbers? Why? When is the product abstract? When concrete? Explain the process of finding the product of two numbers: 1. When the multiplier is expressed by one figure; 2. When the multiplier is expressed by two or more figures.

148. Division. What is division? Show how it is the reverse of multiplication? Define and illustrate: Dividend, divisor, quotient. Illustrate the different ways in which division is indicated. Give an example which will show how the signs of multiplication and division take precedence over those of addition and subtraction. When are the dividend and divisor like numbers? Illustrate. When are the dividend and quotient like numbers? Illustrate. When is the divisor abstract? When is the quotient abstract? Why is the remainder always like the dividend? Explain the process of short division. How does long division differ from short division?

EQUAL PARTS OF UNITS.

1. If an apple, a piece of paper, or any unit is divided into *two* equal parts, what is one of the parts called?

2. If a unit is divided into *three* equal parts, what is each of the parts called?

3. What is each of the parts called when a unit is divided into *four* equal parts? When a unit is divided into *five* equal parts? Into *six* equal parts?

4. How many halves are there in a unit? How many thirds? Fourths? Fifths? Sixths? Sevenths? Tenths? Thirteenths? Fifteenths?

149. Two numbers, one written above the other with a short horizontal line between, are used to express one or more of the equal parts into which a unit is divided. Thus,

$\frac{1}{2}$,	read	one half,	expresses	1	of	2	equal	parts.
$\frac{1}{3}$,	"	one third,	"	1	"	3	"	"
$\frac{1}{4}$,	"	one fourth,	"	1	"	4	"	"
$\frac{3}{4}$	"	three fourths,	"	3	"	4	"	"
$\frac{1}{5}$,	"	one fifth,	"	1	"	5	"	"
$\frac{4}{5}$,	"	four fifths,	"	4	"	5	"	"

150. The number below the line is called the **denominator,** and indicates the number of equal parts into which the unit is divided. The number above the line is called the **numerator,** and shows how many of the equal parts are taken. Thus,

$\frac{1}{6}$	means	1	of	6	equal	parts.
$\frac{2}{6}$	"	2	"	6	"	"
$\frac{3}{6}$	"	3	"	6	"	"
$\frac{4}{6}$	"	4	"	6	"	"
$\frac{3}{7}$	means	3	of	7	equal	parts.
$\frac{8}{9}$	"	8	"	9	"	"
$\frac{5}{12}$	"	5	"	12	"	"
$\frac{13}{20}$	"	13	"	20	"	"

ORAL EXERCISES.

151. *1.* How many thirds are $\frac{1}{3}$ and $\frac{1}{3}$? $\frac{1}{3}$ and $\frac{2}{3}$?

2. How many thirds in one apple? $\frac{3}{3}$ of an apple is how much of it? Which is greater, $\frac{1}{3}$ of an apple or $\frac{2}{3}$ of it?

3. How many fourths are $\frac{1}{4}$ and $\frac{2}{4}$? $\frac{1}{4}$ and $\frac{3}{4}$? $\frac{2}{4}$ and $\frac{2}{4}$?

4. $\frac{1}{5}+\frac{2}{5}=?$ $\frac{1}{5}+\frac{3}{5}=?$ $\frac{2}{5}+\frac{2}{5}=?$ $\frac{4}{5}+\frac{1}{5}=?$

5. How many sixths in 1? In 2? In 3? In 4?

6. How many units in $\frac{6}{6}$? In $\frac{12}{6}$? In $\frac{18}{6}$? In $\frac{24}{6}$?

7. How many units and thirds in $\frac{4}{3}$? In $\frac{5}{3}$? In $\frac{7}{3}$? In $\frac{10}{3}$? In $\frac{16}{3}$? In $\frac{20}{3}$?

8. How many units and fifths in $\frac{6}{5}$? In $\frac{8}{5}$? In $\frac{12}{5}$? In $\frac{17}{5}$? In $\frac{22}{5}$?

9. What is the sum of $\frac{1}{7}$, $\frac{2}{7}$ and $\frac{3}{7}$? Of $\frac{1}{8}$, $\frac{4}{8}$ and $\frac{6}{8}$?

10. $\frac{5}{4}+\frac{2}{4}+\frac{6}{4}=?$ $\frac{8}{3}+\frac{7}{3}+\frac{6}{3}=?$ $\frac{6}{5}+\frac{9}{5}+\frac{6}{5}=?$

11. $\frac{9}{8}+\frac{12}{8}+\frac{3}{8}=?$ $\frac{13}{6}+\frac{11}{6}+\frac{7}{6}=?$ $\frac{5}{2}+\frac{13}{2}+\frac{9}{2}=?$

12. How many fourths in 1? In $\frac{1}{2}$ of 1?

13. Into how many halves may a unit be divided? Into how many fourths? How many fourths in $\frac{1}{2}$? In $\frac{2}{2}$? In 1? How many halves in 1? In $\frac{4}{4}$? In $\frac{2}{4}$?

14. How many sixths in 1? In $\frac{1}{2}$ of 1? In $\frac{1}{3}$ of 1? In of 1? In $\frac{3}{3}$?

15. How many eighths in 1? In $\frac{1}{2}$ of 1? In $\frac{1}{4}$ of 1?

16. Into how many eighths may a unit be divided? How many eighths in $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{2}{4}$? In $\frac{3}{4}$? In $\frac{4}{4}$?

17. $\frac{4}{4}$ equals how many eighths? $\frac{3}{3}$ equals how many sixths? $\frac{2}{2}$ equals how many fourths?

18. How many halves in 1? In $\frac{2}{4}$? In $\frac{3}{6}$? In $\frac{4}{8}$? In $\frac{5}{10}$? In $\frac{6}{12}$?

19. $\frac{1}{3}$ equals how many sixths? Ninths? Twelfths? $\frac{2}{3}$ equals how many sixths? Ninths? Twelfths?

20. How many thirds in $\frac{2}{6}$? In $\frac{4}{6}$? In $\frac{6}{6}$? In $\frac{3}{9}$? In $\frac{6}{9}$? In $\frac{9}{9}$? In $\frac{4}{12}$? In $\frac{8}{12}$? In $\frac{12}{12}$? In $\frac{8}{6}$? In $\frac{12}{9}$? In $\frac{15}{9}$?

21. How many fourths in $\frac{2}{8}$? In $\frac{4}{8}$? In $\frac{6}{8}$? In $\frac{8}{8}$? In $\frac{12}{8}$? In $\frac{3}{12}$? In $\frac{9}{12}$? In $\frac{12}{12}$? In $\frac{15}{12}$? In $\frac{18}{12}$?

152. Like parts of the same or of equal units are added by finding the sum of the numerators.

Thus, $\frac{1}{4}+\frac{2}{4}+\frac{3}{4}=\frac{6}{4}=1\frac{2}{4}=1\frac{1}{2}$.

153. In order to add unlike parts of the same or of equal units, such as halves and fourths, it is necessary to reduce them to like parts. Thus,

1. To find the sum of $\frac{1}{2}$ and $\frac{1}{4}$, the one half is reduced to fourths. $\frac{1}{2}=\frac{2}{4}$; $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$.

2. To find the sum of $\frac{2}{3}$ and $\frac{5}{6}$, the two thirds are reduced to sixths. $\frac{2}{3}=\frac{4}{6}$; $\frac{4}{6}+\frac{5}{6}=\frac{9}{6}=1\frac{3}{6}=1\frac{1}{2}$.

3. To find the sum of $\frac{1}{2}$ and $\frac{1}{3}$, both are reduced to sixths. $\frac{1}{2}=\frac{3}{6}$; $\frac{1}{3}=\frac{2}{6}$. $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

154. When unlike parts of the same or of equal units are to be reduced to like parts, the number selected for the denominator should be the smallest number that is exactly divisible by each of the given denominators. Thus,

4. To find the sum of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$, the thirds, fourths and sixths are reduced to twelfths, 12 being the smallest number exactly divisible by 3 4 and 6. $\frac{2}{3}=\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$; $\frac{5}{6}=\frac{10}{12}$. $\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$.

ORAL EXERCISES.

155. *1.* How many eighths in 1? In $\frac{1}{2}$ of 1? In $\frac{1}{4}$ of 1? $\frac{1}{2}$ is equal to how many eighths? $\frac{1}{4}$ is equal to how many eighths? The sum of $\frac{1}{2}$ and $\frac{1}{4}$ is equal to how many eighths?

2. Add $\frac{1}{2}$ and $\frac{3}{4}$; $\frac{1}{4}$ and $\frac{1}{8}$; $\frac{3}{8}$ and $\frac{1}{2}$; $\frac{3}{4}$ and $\frac{4}{8}$.

3. How many ninths in 1? In $\frac{1}{3}$ of 1? $\frac{1}{3}$ equals how many ninths? The sum of $\frac{1}{3}$ and $\frac{1}{9}$ is equal to how many ninths? $\frac{1}{3}+\frac{3}{9}$ is equal to how many ninths? $\frac{2}{3}+\frac{4}{9}$ is equal to how many ninths?

4. How many twentieths in $\frac{1}{4}$? In $\frac{1}{5}$? $\frac{1}{4}+\frac{1}{5}$ makes how many twentieths? What is the sum of $\frac{2}{4}$ and $\frac{3}{5}$.

5. How many fourths in $\frac{1}{2}+\frac{3}{4}+\frac{1}{2}+\frac{1}{4}+\frac{2}{4}+\frac{1}{2}$?

6. How many eighths in the sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$?

7. How many halves in $\frac{4}{8}$? In $\frac{2}{4}$? In $\frac{3}{6}$? In $\frac{6}{8}$? In $\frac{12}{8}$?

8. To what like parts may halves, thirds and sixths be reduced? Halves, fourths and eighths? Halves, fifths and tenths? Thirds, fourths and sixths? Thirds, sixths and ninths?

9. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$? Of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$? Of $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{4}$? Of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{6}$? Of $\frac{1}{2}$, $\frac{1}{5}$ and $\frac{1}{10}$?

10. $\frac{1}{2}+\frac{2}{5}+\frac{3}{10}=?$ $\frac{1}{2}+\frac{1}{3}+\frac{5}{6}=?$ $\frac{1}{2}+\frac{3}{5}+\frac{7}{10}=?$

11. $\frac{3}{5}+\frac{2}{3}=?$ $\frac{8}{9}+\frac{5}{6}=?$ $\frac{3}{4}+\frac{5}{6}=?$ $\frac{3}{5}+\frac{7}{10}=?$

12. How many dollars in $\$\frac{3}{4}+\$\frac{4}{5}+\$\frac{7}{10}$? In $\$\frac{2}{3}+\$\frac{3}{4}+\$\frac{5}{8}$?

13. Find the sum of $\frac{5}{6}$ of a bushel, $\frac{2}{3}$ of a bushel, and $\frac{1}{4}$ of a bushel. Of $\frac{3}{5}$ of a bushel, $\frac{1}{2}$ of a bushel, and $\frac{3}{10}$ of a bushel.

14. $\frac{5}{12}$ of a foot, $\frac{3}{8}$ of a foot, and $\frac{1}{6}$ of a foot equal how many feet?

156. A unit has been defined as a single thing or one. A unit, however, may be divided into equal parts, and each of these parts becomes a single thing or a unit. In order to distinguish between these two kinds of units, the first is called an *integral unit,* and the second a *fractional unit.*

157. An **integral unit** is a whole or undivided unit.

Thus, 1 foot, 1 pound, $1, 1, are integral units.

158. A **fractional unit** is one of the equal parts of an integral unit.

Thus, $\frac{1}{2}$ of a foot, $\frac{1}{4}$ of a pound, $\frac{1}{10}$ of a dollar, $\frac{1}{7}$, are fractional units.

159. A **whole number,** or **integer,** is an integral unit, or a collection of integral units.

Thus, 5 feet, 7 pounds, $15, 27, are integers.

160. A **fractional number,** or **fraction,** is a fractional unit, or a collection of fractional units.

Thus, $\frac{1}{4}$ of a pound, $\frac{4}{5}$ of a dollar, $\frac{1}{2}$, $\frac{7}{16}$, are fractions.

161. A **mixed number** is a whole number and a fraction written together.

Thus, $3\frac{1}{2}$ feet, $7\frac{3}{4}$ lb., $12\frac{4}{9}$, are mixed numbers.

ADDITION OF MIXED NUMBERS.

162. Mixed numbers are added by finding the sum of the fractional and the integral parts separately, and uniting the results.

I. Required the sum of $317\frac{1}{2}$, $579\frac{1}{4}$, $863\frac{3}{4}$ and $745\frac{1}{4}$.

Process.

$$\begin{array}{r} 317\frac{1}{2} \\ 579\frac{1}{4} \\ 863\frac{3}{4} \\ \underline{745\frac{1}{4}} \\ 2505\frac{3}{4} \end{array}$$

$$\frac{1}{4}+\frac{3}{4}+\frac{1}{4}+\frac{1}{2}=\frac{7}{4}$$

$$\frac{7}{4}=1\frac{3}{4}$$

Explanation.—1. Adding the column of fractions, the sum of $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{1}{2}$ is found to be $\frac{7}{4}$, equal to $1\frac{3}{4}$. The $\frac{3}{4}$ is written under the column of fractions and the 1 carried to the units' column of the whole numbers.

2. The whole numbers, including the 1 from the fractional column, are added in the usual manner and written before the fractional part, giving $2505\frac{3}{4}$ for the entire amount.

163. ***Find the sum of:***

1. $18\frac{1}{2}+23\frac{1}{4}+17\frac{2}{4}+31\frac{3}{4}$.

2. $19\frac{1}{5}+61\frac{2}{5}+83\frac{3}{5}+94\frac{4}{5}$.

3. $12\frac{1}{2}+27\frac{3}{4}+45\frac{1}{4}+17\frac{1}{2}$.

4. $15\frac{2}{3}+18\frac{1}{3}+71\frac{5}{6}+38\frac{1}{6}$.

5. $\$29\frac{3}{8}+\$61\frac{3}{4}+\$85\frac{1}{2}$.

6. $\$148\frac{1}{4}+\$237\frac{2}{3}+\$69\frac{1}{2}$.

7.	*8.*	*9.*	*10.*	*11.*
$632\frac{1}{5}$	$\$345\frac{1}{5}$	$\$732\frac{5}{8}$	$2401\frac{5}{6}$	$\$59683\frac{4}{5}$
$598\frac{4}{5}$	$\$269\frac{1}{2}$	$\$217\frac{1}{2}$	$5632\frac{2}{3}$	$\$42751\frac{2}{5}$
$671\frac{2}{5}$	$\$107\frac{3}{5}$	$\$418\frac{7}{8}$	$8594\frac{5}{6}$	$\$36875\frac{1}{3}$
$838\frac{3}{5}$	$\$424\frac{4}{5}$	$\$131\frac{3}{4}$	$1357\frac{2}{6}$	$\$58326\frac{3}{5}$
$908\frac{4}{5}$	$\$963\frac{9}{10}$	$\$337\frac{1}{8}$	$6982\frac{1}{3}$	$\$73592\frac{3}{5}$
$659\frac{2}{5}$	$\$859\frac{7}{10}$	$\$685\frac{1}{4}$	$5678\frac{1}{2}$	$\$43601\frac{1}{15}$

12. $183\frac{11}{16}$ lb., $938\frac{5}{8}$ lb., $1284\frac{1}{2}$ lb., $524\frac{3}{4}$ lb. and $223\frac{3}{8}$ lb.

13. $49\frac{2}{5}$ miles, $84\frac{1}{5}$ miles, $68\frac{7}{10}$ miles, $53\frac{1}{2}$ miles, $31\frac{4}{5}$ miles and 875 miles.

14. $3825\frac{1}{2}$ feet, $4361\frac{3}{4}$ feet, $8756\frac{5}{6}$ feet, $7301\frac{2}{3}$ feet, $9356\frac{7}{12}$ feet and $5061\frac{1}{3}$ feet.

CONCRETE ORAL PROBLEMS.

164. *1.* A man paid at one time $\frac{1}{3}$ of a bill and at another $\frac{1}{5}$ of it; what part of the whole bill did he pay?

2. Bought at a grocer's 12 eggs for $\frac{3}{10}$ of a dollar, 1 pound of butter for $\frac{3}{8}$ of a dollar, and 2 pounds of coffee for $\frac{1}{2}$ of a dollar; how much did all cost?

3. A farmer sold to one man $3\frac{1}{2}$ tons of hay, to another $4\frac{1}{4}$ tons, and to another $2\frac{3}{4}$ tons; how much hay did he sell to all?

4. During a certain month my expenses the first week amounted to $5\frac{3}{4}$ dollars, the second week to $6\frac{1}{2}$ dollars, the third to $6\frac{5}{8}$ dollars, and the fourth to $7\frac{1}{4}$ dollars; find my expenses for the four weeks.

5. A farmer has $5\frac{3}{4}$ acres of potatoes, $6\frac{3}{8}$ acres of corn, and $7\frac{1}{2}$ acres of oats; how many acres in all?

6. In one piece of velvet there are $4\frac{3}{4}$ yards, in another $7\frac{1}{2}$ yards, and in a third $5\frac{1}{4}$ yards; how many yards in all?

7. Mary gathered $9\frac{3}{4}$ quarts of berries, Kitty $8\frac{1}{4}$ quarts, and Lizzie $10\frac{1}{2}$ quarts; how many quarts did they all gather?

8. A fisherman caught three trout weighing respectively $5\frac{7}{16}$ pounds, $3\frac{3}{4}$ pounds, and $4\frac{3}{8}$ pounds; what was their united weight?

9. A boy paid \$$1\frac{3}{8}$ for a pair of skates, \$$2\frac{3}{4}$ for a sled, \$$\frac{1}{2}$ for a pair of mittens, and \$$1\frac{5}{8}$ for a pair of rubber boots; what did he pay for all?

10. A farmer cleared one year $7\frac{3}{8}$ acres, the next year $5\frac{1}{2}$ acres, and the third year $6\frac{5}{16}$ acres, and then enclosed the land in one field; how many acres did it contain?

11. A grocer sold from a roll of butter $1\frac{3}{4}$ lb. to one customer and $2\frac{1}{2}$ lb. to another; he then had $3\frac{3}{4}$ lb. left; how many pounds were there in the roll?

12. A boy bought some almonds, of which he ate $\frac{3}{8}$ of a pound, gave his sister $\frac{1}{4}$ of a pound, and had $\frac{1}{2}$ of a pound left; what quantity did he buy?

CONCRETE WRITTEN PROBLEMS.

165. *1.* In the first of five piles of wood there are $27\frac{3}{8}$ cords, in the second $45\frac{3}{4}$ cords, in the third $36\frac{1}{2}$ cords, in the fourth $52\frac{7}{8}$ cords, and in the fifth $96\frac{1}{4}$ cords; how many cords in all?

2. A steamer burned $2161\frac{1}{4}$ tons of coal while going from London to Suez, $1608\frac{1}{2}$ tons from Suez to Calcutta, $2249\frac{4}{5}$ tons from Calcutta to Honolulu, and $987\frac{7}{10}$ tons from Honolulu to San Francisco; how many tons were burned during the voyage?

3. I used $34\frac{7}{8}$ yards of carpet for one bed-room, $29\frac{1}{2}$ yards for another, $61\frac{3}{4}$ yards for the parlor, $21\frac{2}{3}$ yards for the hall, $48\frac{5}{8}$ for the dining-room, and $36\frac{3}{4}$ yards for the stairs; how many yards did I use in all?

4. The four sides of a field are $29\frac{3}{8}$ rods, $31\frac{3}{4}$ rods, $43\frac{1}{2}$ rods, and $23\frac{5}{8}$ rods; how far is it around the field?

5. A merchant cut at one time from a piece of cloth $12\frac{3}{4}$ yds., at another $19\frac{2}{3}$ yds., and at another $36\frac{1}{2}$ yds. He had $71\frac{5}{6}$ yds. left; how many yards were in the piece at first?

6. A grocer sold $95\frac{1}{2}$ quarts of chestnuts, $292\frac{5}{8}$ quarts of hickory nuts, $356\frac{1}{4}$ quarts of walnuts, and $39\frac{3}{4}$ quarts of hazel nuts; how many quarts of nuts did he sell in all?

7. A man paid $\$39\frac{3}{5}$ for a watch, $\$11\frac{4}{5}$ for a chain, $\$7\frac{3}{4}$ for a gold pencil, and $\$15\frac{7}{10}$ for a silver pitcher; he then bought for his wife a pair of bracelets for $\$65\frac{1}{2}$ and had $\$160\frac{4}{5}$ left in his purse; how much had he at first?

8. The authorities at the mint in Philadelphia coined at one time $2375\frac{4}{5}$ ounces of gold, at a second $8306\frac{1}{4}$ oz., at a third $13837\frac{7}{10}$ oz., at a fourth $3659\frac{3}{4}$ oz., and at a fifth $21651\frac{1}{2}$ oz.; how many ounces were coined altogether?

9. Of a certain bill I paid $\$25\frac{4}{5}$ at one time, $\$38\frac{3}{4}$ at another, and still owed $\$47\frac{3}{5}$; how much was the bill?

SUBTRACTION OF MIXED NUMBERS.

1. Which is the larger, $\frac{1}{2}$ of an apple or $\frac{1}{3}$ of it? $\frac{1}{2}$ of \$1 or $\frac{1}{4}$ of \$1? $\frac{1}{4}$ of \$1 or $\frac{1}{5}$ of \$1? $\frac{1}{5}$ of a unit or $\frac{1}{6}$ of the same unit? $\frac{1}{6}$ or $\frac{1}{7}$? $\frac{1}{7}$ or $\frac{1}{8}$? $\frac{1}{8}$ or $\frac{1}{9}$?

166. The larger the number of parts into which a unit is divided, the smaller will each part be.

2. Which is the larger, $\frac{3}{4}$ of \$1 or $\frac{3}{5}$ of \$1? Which is the larger, $\frac{7}{8}$ of an inch or $\frac{7}{10}$ of an inch?

167. When the numerators of two fractions are the same, the larger fraction is the one having the smaller denominator.

3. Which is the greater, $\frac{3}{5}$ of \$1 or $\frac{4}{5}$ of \$1? $\frac{7}{12}$ of 1 foot or $\frac{9}{12}$ of 1 foot?

168. When the denominators of two fractions are the same, the larger fraction is the one having the larger numerator.

169. When two fractions have equal denominators, or denominators expressing like parts of a unit, the difference between the fractions is found by subtracting the smaller numerator from the larger, and writing the remainder over the common denominator. Thus,

The difference between $\frac{7}{8}$ and $\frac{5}{8}$ is $\frac{2}{8}$.
The difference between $\frac{14}{17}$ and $\frac{9}{17}$ is $\frac{5}{17}$.

170. When two fractions have unequal denominators, before the difference of the fractions can be found they must be reduced to equivalent fractions having equal denominators. Thus,

To find the difference between $\frac{3}{4}$ and $\frac{2}{3}$, both are reduced to twelfths. $\frac{3}{4} = \frac{9}{12}$. $\frac{2}{3} = \frac{8}{12}$. $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$.

ORAL EXERCISES.

171. *1.* What is the difference between $\frac{1}{2}$ and $\frac{1}{3}$? $\frac{1}{2}$ and $\frac{1}{4}$? and $\frac{1}{4}$? $\frac{1}{2}$ and $\frac{1}{5}$? $\frac{1}{3}$ and $\frac{1}{5}$? $\frac{1}{4}$ and $\frac{1}{5}$?

2. $\frac{1}{2}-\frac{1}{6}=?$ $\frac{1}{3}-\frac{1}{6}=?$ $\frac{1}{4}-\frac{1}{6}=?$ $\frac{1}{5}-\frac{1}{6}=?$

3. How much greater is $\frac{1}{2}$ than $\frac{1}{7}$? $\frac{1}{2}$ than $\frac{1}{8}$? $\frac{1}{2}$ than $\frac{1}{10}$? than $\frac{1}{12}$?

4. Which is greater, and how much, $\frac{1}{3}$ or $\frac{1}{8}$? $\frac{1}{9}$ or $\frac{1}{3}$? $\frac{1}{12}$ or $\frac{1}{3}$? $\frac{1}{4}$ or $\frac{1}{12}$? $\frac{1}{5}$ or $\frac{1}{10}$?

5. $\frac{5}{8}-\frac{3}{8}=?$ $\frac{11}{12}-\frac{5}{12}=?$ $\frac{7}{10}-\frac{3}{10}=?$ $\frac{7}{12}-\frac{4}{12}=?$

6. Subtract $\frac{1}{2}$ from $\frac{2}{3}$; $\frac{1}{2}$ from $\frac{7}{10}$; $\frac{1}{2}$ from $\frac{5}{8}$; $\frac{1}{2}$ from $1\frac{1}{2}$.

7. $\frac{7}{8}-\frac{3}{4}=?$ $\frac{5}{6}-\frac{3}{4}=?$ $\frac{8}{9}-\frac{2}{3}=?$ $\frac{7}{10}-\frac{2}{5}=?$ $\frac{5}{12}-\frac{1}{3}=?$

8. $\frac{5}{12}-\frac{1}{4}=?$ $\frac{1}{2}+\frac{1}{3}-\frac{5}{6}=?$ $\frac{1}{2}+\frac{1}{4}-\frac{3}{8}=?$ $\frac{1}{3}+\frac{1}{4}-\frac{5}{12}=?$ $\frac{1}{2}+\frac{2}{3}-\frac{5}{6}=?$

9. Find how much greater $\frac{1}{2}+\frac{1}{3}$ is than $\frac{1}{3}+\frac{1}{4}$. [illegible] than $\frac{3}{4}-\frac{1}{3}$. $\frac{1}{2}-\frac{1}{12}$ than $\frac{1}{3}-\frac{1}{8}$.

10. $\frac{3}{2}-\frac{4}{3}=?$ $\frac{11}{8}-\frac{3}{4}=?$ $\frac{7}{6}-\frac{2}{3}=?$ $\frac{13}{10}-\frac{1}{2}-\frac{2}{5}=?$ $\frac{13}{12}-\frac{5}{6}-\frac{1}{4}=?$ $\frac{7}{6}-\frac{7}{12}-\frac{1}{3}=?$

11. $1\frac{1}{2}-\frac{3}{4}=?$ $2\frac{1}{3}-1\frac{1}{6}=?$ $3\frac{3}{4}-2\frac{1}{2}=?$ $4\frac{1}{2}-2\frac{3}{10}=?$

12. How much must be added to $\frac{1}{3}$ to give $\frac{1}{2}$? To $\frac{1}{4}$ to give $\frac{5}{8}$? To $\frac{2}{5}$ to give $\frac{7}{10}$? To $\frac{3}{8}$ to give $\frac{3}{4}$? To $\frac{5}{12}$ to give $\frac{3}{4}$?

172. To subtract a mixed number from a whole number, or from a mixed number.

I. Required the difference between 1273 and $648\frac{2}{3}$.

Process.

$$\begin{array}{r} 1273 \\ 648\frac{2}{3} \\ \hline 624\frac{1}{3} \end{array}$$

$1=\frac{3}{3}$; $\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$
$1272-648=624$

Explanation.—1. There being no thirds in the minuend from which to subtract the $\frac{2}{3}$ in the subtrahend, one of the units in the minuend is reduced to thirds and the $\frac{2}{3}$ subtracted from the $\frac{3}{3}$, giving $\frac{1}{3}$ for the fractional part of the remainder.

2. The integral part of the subtrahend is then subtracted from 1272, the minuend less the one unit which was reduced to thirds; and the remainder thus obtained is written before $\frac{1}{3}$, the fractional part of the remainder giving $624\frac{1}{3}$ for the complete remainder.

II. Required the difference between $1685\frac{2}{3}$ and $897\frac{3}{4}$.

Explanation.—1. The fractional parts of the minuend and subtrahend, having different numbers for denominators, are reduced to equivalent fractions having equal denominators. $\frac{2}{3}=\frac{8}{12}$ and $\frac{3}{4}=\frac{9}{12}$. Since the fractional part of the minuend is the smaller it is necessary to reduce one of the units of the minuend to twelfths; $\frac{12}{12}+\frac{8}{12}$ make $\frac{20}{12}$, from which the $\frac{9}{12}$ are subtracted, giving $\frac{11}{12}$ for the fractional part of the remainder.

Process.

$$\begin{array}{r} 1685\frac{2}{3} \\ 897\frac{3}{4} \\ \hline 787\frac{11}{12} \end{array}$$

$$\tfrac{2}{3}=\tfrac{8}{12} \quad \tfrac{3}{4}=\tfrac{9}{12}$$
$$1=\tfrac{12}{12} \quad \tfrac{12}{12}+\tfrac{8}{12}=\tfrac{20}{12}$$
$$\tfrac{20}{12}-\tfrac{9}{12}=\tfrac{11}{12}$$

2. The integral part of the subtrahend is then subtracted, giving 787 for the integral part of the remainder, which written before the fractional part, giving $787\frac{11}{12}$ for the complete remainder.

NOTE.—When the fractional part of the minuend is larger than that of the subtrahend, it is unnecessary to reduce to fractional parts one of the units of the minuend.

173. ***Find remainders:***

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*
$43\frac{1}{2}$	$85\frac{1}{3}$	$69\frac{3}{4}$	$93\frac{2}{3}$	$81\frac{4}{5}$	$185\frac{2}{3}$	$232\frac{3}{4}$
$29\frac{1}{3}$	$47\frac{1}{4}$	$30\frac{5}{8}$	$50\frac{1}{4}$	$27\frac{7}{10}$	$79\frac{1}{6}$	$157\frac{1}{3}$

8.	*9.*	*10.*	*11.*	*12.*
$839\frac{3}{5}$	$1596\frac{1}{12}$	$3879\frac{7}{9}$	$6831\frac{7}{10}$	$5623\frac{1}{3}$
$178\frac{1}{3}$	$888\frac{3}{4}$	$1998\frac{2}{3}$	$5879\frac{3}{5}$	$3333\frac{5}{6}$

174. ***Find the value of:***

1. $897\frac{1}{4}+489\frac{1}{3}-106\frac{5}{6}$.

2. $983\frac{2}{3}-\overline{68\frac{3}{4}+147\frac{1}{2}}$.

3. $431\frac{2}{3}+817\frac{3}{4}-1006\frac{1}{2}$.

4. $487\frac{5}{6}+(568\frac{2}{3}-199\frac{4}{5})$.

5. $1021\frac{1}{3}-(87\frac{5}{9}+99\frac{2}{3})$.

6. $851\frac{3}{5}+(982\frac{1}{2}-797\frac{4}{5})$.

7. $544\frac{5}{12}-(187\frac{1}{2}+313\frac{5}{6})$.

8. $432\frac{5}{8}+(964\frac{1}{2}-698\frac{3}{4})$.

9. $83\frac{5}{6}+97\frac{11}{12}-\overline{72\frac{2}{3}+81\frac{3}{4}}$.

10. $196\frac{5}{8}+(91\frac{3}{4}-\overline{97\frac{2}{3}-85\frac{3}{8}})$.

11. $\overline{183\frac{1}{2}+235\frac{1}{3}}-\overline{69\frac{1}{2}+83\frac{3}{8}}$.

12. $23\frac{4}{5}+61\frac{1}{2}-(17\frac{3}{5}+25\frac{7}{10})$.

13. $875\frac{1}{4}-(369\frac{2}{3}-87\frac{1}{2}+171\frac{5}{6})$

14. $43\frac{5}{8}-(81\frac{1}{2}+61\frac{3}{4}-129\frac{3}{8})$.

15. $61\frac{1}{5}+(186\frac{1}{2}-97\frac{2}{3}-79\frac{3}{5})$

16. $275\frac{1}{9}-102\frac{5}{6}+89\frac{2}{3}-61$[illegible].

CONCRETE ORAL PROBLEMS.

175. *1.* I pay $\$1\frac{3}{5}$ for a bushel of apples; how much change should I receive out of a five-dollar bill?

2. A man starts to walk to a town 10 miles distant; after going $4\frac{5}{8}$ miles, how far has he yet to travel?

3. A cistern that will hold 9 hogsheads of water lacks $3\frac{1}{6}$ hogsheads of being full; how much water is in it?

4. A man owing $\$8\frac{7}{10}$ pays $\$3\frac{1}{2}$; what does he still owe?

5. George is now $10\frac{5}{12}$ years old; how old was he $4\frac{2}{3}$ years ago?

6. A tailor used $5\frac{3}{4}$ yards of cloth to make a suit, of which $1\frac{2}{3}$ yards were used for the pants, $\frac{5}{8}$ of a yard for the vest, and the remainder for the coat; how many yards were used for the coat?

7. A criminal was sent to prison for $7\frac{1}{2}$ years; after having served $4\frac{2}{3}$ years how long did he still have to remain?

8. A grocer has $5\frac{7}{8}$ bushels of walnuts; he sells $1\frac{3}{4}$ bushels to one man, and $1\frac{5}{8}$ bushels to another man; how many bushels remain unsold?

9. John earns $\$1\frac{4}{5}$ per day, and he and Henry together earn $\$4\frac{1}{2}$ per day; how much does Henry earn in a day?

CONCRETE WRITTEN PROBLEMS.

176. *1.* A farmer has $428\frac{5}{8}$ bushels of wheat and sells all but $109\frac{3}{4}$ bushels; how much does he sell?

2. A man sets out to walk from New York to Philadelphia, 90 miles; the first day he travels $23\frac{1}{2}$ miles, and the second $28\frac{2}{3}$ miles; how far has he yet to go?

3. A man makes a profit of $\$135\frac{4}{5}$ on fish and oysters, $\$75\frac{9}{10}$ of which he makes on the fish; how much is his profit on the oysters?

4. A farmer sells from his farm of $140\frac{9}{16}$ acres, $15\frac{3}{4}$ acres to one man and $60\frac{7}{8}$ acres to another; how many acres are left?

5. A grocer has two casks of syrup, one containing $41\frac{3}{4}$ gallons and the other $43\frac{7}{8}$ gallons. From one he sells $15\frac{5}{8}$ gallons and from the other $19\frac{1}{2}$ gallons; how much remains?

6. A German after coming to this country lived $5\frac{3}{4}$ years in Boston, $8\frac{2}{3}$ years in New York City and $28\frac{5}{6}$ years in Philadelphia; he then moved to Chicago, where he lived for $9\frac{7}{12}$ years, and died at the age of $71\frac{11}{12}$ years; how old was he when he reached this country?

7. A wholesale grocer the first quarter of a year sells goods to the amount of \$$35683\frac{4}{5}$, the second quarter his sales amount to \$$41361\frac{7}{10}$, the third to \$$39365\frac{9}{10}$, and the fourth to \$$46515\frac{3}{4}$. The goods cost \$$103561\frac{11}{20}$, he paid \$$1155\frac{1}{2}$ rent, \$$11341\frac{13}{20}$ clerk hire, and \$$5465\frac{3}{4}$ for cartage, freight and incidental expenses; find his profit for the year.

MULTIPLICATION OF MIXED NUMBERS.

1. 4 times \$3 are how many dollars? \$3 is what part of \$12? $\frac{1}{4}$ of \$12 is how many dollars? $\frac{2}{4}$ of \$12 are how many dollars? $\frac{3}{4}$ of \$12 are how many dollars?

2. 5 times 4 are how many? 4 is what part of 20? What is $\frac{1}{5}$ of 20? $\frac{2}{5}$ of 20? $\frac{3}{5}$ of 20? $\frac{4}{5}$ of 20? $\frac{5}{5}$ of 20?

177. Finding such a part of a number as is indicated by a fraction is called **multiplying the number by the fraction.**

Multiplying a number by 4 means taking the number 4 times. Multiplying a number by $\frac{1}{4}$ means taking one of the four equal parts into which the number may be divided; multiplying by $\frac{2}{4}$ means taking two of the four equal parts; by $\frac{3}{4}$, three of the four equal parts, etc.

3. Multiply 12 by $\frac{1}{2}$; by $\frac{1}{3}$; by $\frac{1}{4}$.

4. Multiply 12 by $\frac{1}{6}$; by $\frac{2}{6}$; by $\frac{3}{6}$; by $\frac{4}{6}$.

5. Multiply 16 by $\frac{1}{2}$; by $\frac{1}{4}$; by $\frac{1}{8}$; by $\frac{3}{8}$.

6. Multiply 20 by $\frac{1}{2}$; by $\frac{1}{4}$; by $\frac{1}{5}$; by $\frac{3}{5}$; by $\frac{7}{10}$.

178. An integer may be multiplied by a fraction by dividing the integer by the denominator of the fraction and multiplying the quotient by the numerator, or by multiplying the integer by the numerator of the fraction and dividing the product by the denominator. Thus,

1. To multiply 12 by $\frac{5}{6}$, or to find $\frac{5}{6}$ of 12, the 12 is divided by 6 and the quotient multiplied by 5. Dividing 12 by 6 gives $\frac{1}{6}$ of 12 or 2; and multiplying 2 by 5 gives 5 times $\frac{1}{6}$ of 12, or $\frac{5}{6}$ of 12, equal to 10.

2. The same result is obtained by first multiplying the 12 by the 5 and then dividing the product by 6. $12 \times 5 = 60$, and $60 \div 6 = 10$; or 12 times $\frac{5}{6} = \frac{60}{6} = 10$.

Observe that in both cases the numerator is used as a multiplier and the denominator as a divisor.

ORAL EXERCISES.

179. *1.* What is $\frac{1}{4}$ of 8? $\frac{1}{5}$ of 10? $\frac{1}{6}$ of 12? $\frac{1}{7}$ of 21?

2. Multiply 12, 16, 20, 24, 32, 40 and 60, each by $\frac{1}{4}$.

3. Multiply the numbers ending in 2, 4, 6, 8 and 0, from 2 to 48, by $\frac{1}{2}$.

4. Multiply the numbers ending in 0 from 10 to 100 by $\frac{1}{5}$; also by $\frac{1}{10}$.

5. Multiply the numbers ending in 5 from 25 to 75 by $\frac{1}{5}$.

6. Find the product of the following: $18 \times \frac{1}{3}$, $27 \times \frac{1}{9}$, $\frac{1}{9} \times 27$, $\frac{1}{7} \times 35$, $48 \times \frac{1}{6}$.

7. $6 \times \frac{1}{3} = ?$ $6 \times \frac{2}{3} = ?$ $10 \times \frac{2}{5} = ?$ $15 \times \frac{2}{3} = ?$ $16 \times \frac{3}{4} = ?$

8. $\frac{3}{5} \times 20 = ?$ $24 \times \frac{5}{8} = ?$ $\frac{5}{9} \times 27 = ?$ $36 \times \frac{7}{9} = ?$

9. What is the product of $\frac{2}{3}$ and 21? Of 36 and $\frac{5}{9}$? Of 60 and $\frac{3}{5}$? Of 84 and $\frac{5}{7}$? Of $\frac{5}{6}$ and 30? Of $\frac{6}{7}$ and 35?

10. Multiply 40 by $\frac{1}{4}$, by $\frac{3}{4}$, by $\frac{1}{5}$, by $\frac{4}{5}$, by $\frac{5}{8}$, by $\frac{7}{10}$.

11. Find the sum of $\frac{2}{3}$ of 24 and $\frac{3}{4}$ of 12.

12. $\frac{3}{7}$ of 21 plus $\frac{4}{9}$ of $27 = ?$ $\frac{5}{12}$ of 36 minus $\frac{3}{8}$ of $32 = ?$

13. $20 \times \frac{4}{5} - 9 \times \frac{2}{3} = ?$ $\frac{2}{5} \times 60 + \frac{5}{8} \times 40 = ?$

14. Find $\frac{1}{3}$ of 8, $\frac{1}{4}$ of 9, $\frac{1}{5}$ of 17, $\frac{1}{6}$ of 20.

15. $12 \times \frac{1}{5} = ?$ $19 \times \frac{1}{3} = ?$ $18 \times \frac{1}{5} = ?$ $\frac{1}{4} \times 11 = ?$

16. What is $\frac{2}{3}$ of 8? $\frac{3}{4}$ of 11? $\frac{3}{5}$ of 12? $\frac{5}{6}$ of 8? $\frac{3}{7}$ of 20?

17. Multiply 28 by $\frac{2}{3}$, 40 by $\frac{2}{7}$, 21 by $\frac{3}{4}$, 17 by $\frac{5}{6}$.

5. A grocer has two casks of syrup, one containing $41\frac{3}{4}$ gallons and the other $43\frac{7}{8}$ gallons. From one he sells $15\frac{5}{8}$ gallons and from the other $19\frac{1}{2}$ gallons; how much remains?

6. A German after coming to this country lived $5\frac{3}{4}$ years in Boston, $8\frac{2}{3}$ years in New York City and $28\frac{5}{6}$ years in Philadelphia; he then moved to Chicago, where he lived for $9\frac{7}{12}$ years, and died at the age of $71\frac{11}{12}$ years; how old was he when he reached this country?

7. A wholesale grocer the first quarter of a year sells goods to the amount of \$$35683\frac{4}{5}$, the second quarter his sales amount to \$$41361\frac{7}{10}$, the third to \$$39365\frac{9}{10}$, and the fourth to \$$46515\frac{3}{4}$. The goods cost \$$103561\frac{11}{20}$, he paid \$$1155\frac{1}{2}$ rent, \$$11341\frac{13}{20}$ clerk hire, and \$$5465\frac{3}{4}$ for cartage, freight and incidental expenses; find his profit for the year.

MULTIPLICATION OF MIXED NUMBERS.

1. 4 times \$3 are how many dollars? \$3 is what part of \$12? $\frac{1}{4}$ of \$12 is how many dollars? $\frac{2}{4}$ of \$12 are how many dollars? $\frac{3}{4}$ of \$12 are how many dollars?

2. 5 times 4 are how many? 4 is what part of 20? What is $\frac{1}{5}$ of 20? $\frac{2}{5}$ of 20? $\frac{3}{5}$ of 20? $\frac{4}{5}$ of 20? $\frac{5}{5}$ of 20?

177. Finding such a part of a number as is indicated by a fraction is called multiplying the number by the fraction.

Multiplying a number by 4 means taking the number 4 times. Multiplying a number by $\frac{1}{4}$ means taking one of the four equal parts into which the number may be divided; multiplying by $\frac{2}{4}$ means taking two of the four equal parts; by $\frac{3}{4}$, three of the four equal parts, etc.

3. Multiply 12 by $\frac{1}{2}$; by $\frac{1}{3}$; by $\frac{1}{4}$.
4. Multiply 12 by $\frac{1}{6}$; by $\frac{2}{6}$; by $\frac{3}{6}$; by $\frac{4}{6}$.
5. Multiply 16 by $\frac{1}{2}$; by $\frac{1}{4}$; by $\frac{1}{8}$; by $\frac{3}{8}$.
6. Multiply 20 by $\frac{1}{2}$; by $\frac{1}{4}$; by $\frac{1}{5}$; by $\frac{3}{5}$; by $\frac{7}{10}$.

178. An integer may be multiplied by a fraction by dividing the integer by the denominator of the fraction and multiplying the quotient by the numerator, or by multiplying the integer by the numerator of the fraction and dividing the product by the denominator. Thus,

1. To multiply 12 by $\frac{5}{6}$, or to find $\frac{5}{6}$ of 12, the 12 is divided by 6 and the quotient multiplied by 5. Dividing 12 by 6 gives $\frac{1}{6}$ of 12 or 2; and multiplying 2 by 5 gives 5 times $\frac{1}{6}$ of 12, or $\frac{5}{6}$ of 12, equal to 10.

2. The same result is obtained by first multiplying the 12 by the 5 and then dividing the product by 6. $12 \times 5 = 60$, and $60 \div 6 = 10$; or 12 times $\frac{5}{6} = \frac{60}{6} = 10$.

Observe that in both cases the numerator is used as a multiplier and the denominator as a divisor.

ORAL EXERCISES.

179. *1.* What is $\frac{1}{4}$ of 8? $\frac{1}{5}$ of 10? $\frac{1}{6}$ of 12? $\frac{1}{7}$ of 21?

2. Multiply 12, 16, 20, 24, 32, 40 and 60, each by $\frac{1}{4}$.

3. Multiply the numbers ending in 2, 4, 6, 8 and 0, from 2 to 48, by $\frac{1}{2}$.

4. Multiply the numbers ending in 0 from 10 to 100 by $\frac{1}{5}$; also by $\frac{1}{10}$.

5. Multiply the numbers ending in 5 from 25 to 75 by $\frac{1}{5}$.

6. Find the product of the following: $18 \times \frac{1}{3}$, $27 \times \frac{1}{9}$, $\frac{1}{9} \times 27$, $\frac{1}{7} \times 35$, $48 \times \frac{1}{6}$.

7. $6 \times \frac{1}{3} = ?$ $6 \times \frac{2}{3} = ?$ $10 \times \frac{2}{5} = ?$ $15 \times \frac{2}{3} = ?$ $16 \times \frac{3}{4} = ?$

8. $\frac{3}{5} \times 20 = ?$ $24 \times \frac{5}{8} = ?$ $\frac{5}{9} \times 27 = ?$ $36 \times \frac{7}{9} = ?$

9. What is the product of $\frac{2}{3}$ and 21? Of 36 and $\frac{5}{9}$? Of 60 and $\frac{3}{5}$? Of 84 and $\frac{5}{7}$? Of $\frac{5}{6}$ and 30? Of $\frac{6}{7}$ and 35?

10. Multiply 40 by $\frac{1}{4}$, by $\frac{3}{4}$, by $\frac{1}{5}$, by $\frac{4}{5}$, by $\frac{5}{8}$, by $\frac{7}{10}$.

11. Find the sum of $\frac{2}{3}$ of 24 and $\frac{3}{4}$ of 12.

12. $\frac{3}{7}$ of 21 plus $\frac{4}{9}$ of $27 = ?$ $\frac{5}{12}$ of 36 minus $\frac{3}{8}$ of $32 = ?$

13. $20 \times \frac{4}{5} - 9 \times \frac{2}{3} = ?$ $\frac{2}{5} \times 60 + \frac{5}{8} \times 40 = ?$

14. Find $\frac{1}{3}$ of 8, $\frac{1}{4}$ of 9, $\frac{1}{5}$ of 17, $\frac{1}{6}$ of 20.

15. $12 \times \frac{1}{5} = ?$ $19 \times \frac{1}{3} = ?$ $18 \times \frac{1}{5} = ?$ $\frac{1}{4} \times 11 = ?$

16. What is $\frac{2}{3}$ of 8? $\frac{3}{4}$ of 11? $\frac{3}{5}$ of 12? $\frac{5}{6}$ of 8? $\frac{3}{7}$ of 20?

17. Multiply 28 by $\frac{2}{3}$, 40 by $\frac{2}{7}$, 21 by $\frac{3}{4}$, 17 by $\frac{5}{6}$.

180. To find the product of an integer multiplied by a mixed number, or of a mixed number multiplied by an integer.

I. Required the product of 1275 multiplied by $65\frac{3}{4}$.

Explanation.—1. The multiplicand is multiplied by $\frac{3}{4}$, the fractional part of the multiplier, and the product is written for the first partial product.

2. The multiplicand is multiplied by the integral part of the multiplier, each partial product being written so that units of the same order stand in the same column.

3. The partial products are added, giving $83831\frac{1}{4}$ for the entire product.

Process.

$$\begin{array}{r} 1275 \\ \underline{65\frac{3}{4}} \\ 956\frac{1}{4} \\ 6375 \\ \underline{7650} \\ 83831\frac{1}{4} \end{array}$$

$1275 \times 3 = 3825$
$3825 \div 4 = 956\frac{1}{4}$

II. Required the product of $785\frac{3}{4}$ by 69.

Explanation.—1. The fractional part of the multiplicand multiplied by 69 gives $51\frac{3}{4}$ for the first partial product.

2. The integral part of the multiplicand, multiplied by 69 gives the other partial products.

3. The partial products are added, giving $54216\frac{3}{4}$ for the entire product.

Process.

$$\begin{array}{r} 785\frac{3}{4} \\ \underline{69} \\ 51\frac{3}{4} \\ 7065 \\ \underline{4710} \\ 54216\frac{3}{4} \end{array}$$

$69 \times 3 = 207$
$207 \div 4 = 51\frac{3}{4}$

181. For the multiplication of one mixed number by another, see Art. 352.

182. ***Find the product of:***

1.	*2.*	*3.*	*4.*	*5.*	*6.*
28	35	54	365	429	$827\frac{3}{4}$
$6\frac{1}{4}$	$9\frac{4}{5}$	$12\frac{3}{5}$	$29\frac{7}{8}$	$47\frac{3}{7}$	63

7.	*8.*	*9.*	*10.*
$5986\frac{7}{8}$	13865	$35896\frac{2}{3}$	$75804\frac{3}{8}$
409	$3005\frac{5}{6}$	1507	6008

183. ***Perform the operations indicated:***

1. $(123\frac{1}{2} + 245\frac{1}{4} + 756\frac{1}{8}) \times 807$.

2. $(\$237\frac{1}{4} - \$109\frac{1}{8}) \times 1600$.

3. $(\$23\frac{1}{2} + \$45\frac{1}{4} + \$67\frac{1}{4} + \$45\frac{1}{8} + \$67\frac{7}{8}) \times 95\frac{1}{2}$.

4. $(\$1753\frac{3}{4} - \$608\frac{1}{2}) \times (806\frac{1}{2} - 307\frac{1}{2})$.

5. $(1808 \times 40\frac{1}{4}) \times (320\frac{1}{4} - 160\frac{1}{2} + 560\frac{1}{6})$.

CONCRETE ORAL PROBLEMS.

184. *1.* Find the cost of $4\frac{1}{2}$ pounds of sugar at 8 cents a pound.

2. How much will $7\frac{1}{3}$ dozen eggs cost at 12 cents a dozen?

3. A man earns $\$2\frac{1}{4}$ per day; how much will he earn in 12 days?

4. To fill a tub with water requires 9 pailfuls of $2\frac{3}{4}$ gallons each; how many gallons does the tub hold?

5. John can walk $2\frac{3}{4}$ miles an hour; how far can he walk in 8 hours?

6. Mary can gather $3\frac{3}{4}$ quarts of berries per hour; how many can she gather in 6 hours?

7. A horse eats $8\frac{1}{2}$ quarts of oats daily for 7 days; how much does he eat in all?

8. A baker uses $7\frac{1}{2}$ ounces of flour to make a loaf of bread; how much flour does he use in a dozen loaves?

9. My hat cost $\$3\frac{3}{4}$ and my coat 5 times as much; what was the cost of my coat?

10. A locomotive burns $10\frac{4}{5}$ tons of coal in one month; how much will it burn in a year?

11. How much will $7\frac{3}{4}$ lb. of almonds cost at 20 cents a pound?

12. Find the cost of 8 tons of coal at $5\frac{3}{4}$ dollars per ton.

13. How much must I pay for a ticket to ride $13\frac{1}{2}$ miles on a railroad if the fare is 4 cents a mile?

14. How much will one dozen eggs cost at the rate of $3\frac{3}{4}$ cents a piece?

CONCRETE WRITTEN PROBLEMS.

185. *1.* A farmer sold a carload of watermelons at $\$3\frac{1}{8}$ per dozen. How much did he receive for them if the car contained 168 dozen ?

2. Find the cost of $98\frac{5}{8}$ bu. of wheat at \$1.75 per bushel.

3. How much would an invoice of 837 barrels of flour cost at $\$9\frac{3}{4}$ per barrel?

4. The area of Rhode Island is 1250 square miles, and that of Dakota is $119\frac{7}{25}$ times as great; find the area of Dakota.

5. I buy a house for \$3750 and sell it for $\frac{4}{5}$ of its cost; how much do I get for it ?

6. In a field of $47\frac{5}{8}$ acres there were raised on an average 75 bushels of potatoes to the acre; how many bushels were raised on the entire field?

7. An object starting at the source of the Missouri River, and floating at the rate of $3\frac{3}{5}$ miles per hour, would require 1165 hours to reach the Gulf of Mexico; how far is it?

8. Brazil is $666\frac{2}{3}$ square miles more than $62\frac{2}{3}$ times the size of Alabama, and the area of the latter is 52250 square miles; find the area of Brazil.

9. There are $16\frac{1}{2}$ feet in a rod and 320 rods in a mile; how many feet in $25\frac{7}{8}$ miles ?

10. A miller received from a farmer 168 bushels of wheat at $\$1\frac{4}{5}$ per bushel, and gave in return 31 barrels of flour at $\$6\frac{3}{4}$ per barrel, paying the remainder in cash; how much cash did the farmer receive?

11. A father earns $\$4\frac{4}{5}$ per day and his son $\$1\frac{1}{2}$; how much do they earn together in a month of 26 working days ?

12. I owe a bill of $\$190\frac{3}{4}$ to Mr. Brown; in payment I work for him 42 days at $\$3\frac{1}{2}$ per day; how much do I still owe him ?

13. A steamer must average $17\frac{19}{24}$ miles per hour, day and night, in order to go from New York to Liverpool in $7\frac{1}{8}$ days; how far is it?

DIVISION OF MIXED NUMBERS.

1. $\frac{3}{6}+\frac{3}{6}+\frac{3}{6}+\frac{3}{6}$ are how many sixths? $\frac{3}{6}$ is contained how many times in $\frac{12}{6}$? In 2? $\frac{4}{6}+\frac{4}{6}+\frac{4}{6}$ are how many sixths? $\frac{4}{6}$ is contained how many times in $\frac{12}{6}$? In 2?

2. $\frac{2}{8}$ is contained how many times in $\frac{4}{8}$? In $\frac{6}{8}$? In $\frac{8}{8}$? In 1? In 2? In 3?

186. An integer is divided by a fraction by multiplying the integer by the denominator of the fraction and dividing the product by the numerator. Thus,

To divide 10 by $\frac{5}{6}$, the 10 is multiplied by 6 and the product divided by 5. $10 \times 6 = 60$; $60 \div 5 = 12$. Multiplying 10 by 6 gives the number of sixths in 10; $\frac{5}{6}$ is contained in $\frac{60}{6}$ twelve times.

3. How many times is 4 contained in 8? How many times is 4 contained in 4? What part of 4 is equal to 3?

187. In the division of integers, the quotient indicates how many times the divisor is contained in the dividend; when the dividend is less than the divisor, the quotient is a fraction, indicating what part of the divisor is equal to the dividend. Thus,

2 divided by 5 means, what part of 5 is equal to 2; and the quotient $\frac{2}{5}$ means that $\frac{2}{5}$ of 5 is equal to 2.

4. What part of 3 is 2? What part of 5 is 4? What part of 6 is 5? What part of 8 is 3? What part of 10 is 7?

5. What is the quotient of 2 divided by 3? Of $4 \div 5$? Of $5 \div 6$? Of $3 \div 8$? Of $7 \div 10$?

188. When the dividend is less than the divisor, the quotient is expressed by writing the dividend over the divisor in the form of a fraction.

Thus, the quotient of 7 divided by 8 is $\frac{7}{8}$.

ORAL EXERCISES.

189. *1.* How many times is $\frac{1}{4}$ contained in $\frac{2}{4}$? In $\frac{3}{4}$? In $\frac{4}{4}$? In $\frac{5}{4}$? In $1\frac{1}{4}$? In $1\frac{3}{4}$? In $3\frac{2}{4}$?

2. $\frac{2}{3}$ is contained how many times in $\frac{2}{3}$? In $\frac{4}{3}$? In $1\frac{1}{3}$? In 2? In $3\frac{1}{3}$?

3. Divide 4 by $\frac{1}{2}$; by $\frac{1}{3}$; by $\frac{2}{3}$; by $\frac{1}{4}$; by $\frac{1}{5}$; by $\frac{2}{5}$; by $\frac{3}{5}$.

4. Divide 5 by $\frac{1}{4}$; by $\frac{2}{4}$; by $\frac{1}{3}$; by $\frac{1}{5}$; by $\frac{1}{6}$; by $\frac{5}{6}$.

5. $6 \div \frac{2}{3} = ?$ $6 \div \frac{3}{4} = ?$ $6 \div \frac{3}{5} = ?$ $7 \div \frac{2}{7} = ?$

6. How many halves in $2\frac{1}{2}$? $2\frac{1}{2}$ is contained how many times in $\frac{5}{2}$? In $\frac{15}{2}$? In $\frac{20}{2}$? In 10? In 15? In 20?

7. How many thirds in $1\frac{1}{3}$? In 4? Divide 4 by $1\frac{1}{3}$; 5 by $2\frac{1}{2}$; 5 by $1\frac{1}{4}$.

8. Divide 16 by $\frac{1}{2}$; by $\frac{1}{3}$; by $1\frac{1}{3}$; by $5\frac{1}{3}$.

9. What part of 3 is 2? What is the quotient of 2 divided by 3? Of 7 divided by 9?

10. What is the quotient of 6 divided by 2? Of 6 divided by 3? Of 6 divided by 6? Of 6 divided by 8? Of 6 divided by 9? Of 6 divided by 12?

11. What part of 15 is 5? $\frac{1}{3}$ is equal to how many fifteenths? $\frac{5}{15}$ is equal to how many thirds? What is the quotient of 5 divided by 15? Of 4 divided by 12? Of 3 divided by 6?

12. Of what number is 4 one third? What is $\frac{1}{3}$ of 12? $\frac{2}{3}$ of 12? $\frac{3}{4}$ of 12? $\frac{5}{6}$ of 12.

13. 8 is $\frac{2}{3}$ of what number? 12 is $\frac{3}{5}$ of what number? 15 is $\frac{5}{6}$ of what number?

14. If $\frac{2}{3}$ of a number is 20, what is the number? If $\frac{3}{7}$ of a number is 27, what is the number?

15. $\frac{5}{6}$ of a number is 10; what is the number? What is $\frac{3}{4}$ of the number?

16. $\frac{2}{3}$ of a number is 24; what is $\frac{4}{5}$ of the number? $\frac{4}{5}$ cf a number is 28; what is $\frac{5}{7}$ of the number?

17. $\frac{4}{9}$ of a number is 8; what is $\frac{5}{6}$ of the number? $\frac{2}{7}$ of a number is 6; what is $\frac{2}{3}$ of the number?

190. To find the quotient of an integer divided by a mixed number, or of a mixed number divided by an integer.

A number expressing fractional units is divided by another expressing like fractional units in the same manner as one integer is divided by another. Hence, before dividing, the dividend and divisor must be changed to the same fractional unit.

I. Required the quotient of 168 divided by $7\frac{2}{3}$.

Process.

$$\begin{array}{r|l} 7\frac{2}{3} & 168 \\ \underline{3} & \underline{3} \\ 23\ ) & 504\ (\ 21\frac{21}{23} \\ & \underline{46} \\ & 44 \\ & \underline{23} \\ & 21 \end{array}$$

Explanation.—1. The divisor and dividend are both reduced to thirds. In 1 there are 3 thirds; multiplying the 7 by 3 and adding in the 2 gives 23 thirds for the divisor; and multiplying the dividend by 3 gives the number of thirds in the dividend.

2. Having reduced both dividend and divisor to the same fractional unit, the operation is performed as in ordinary division.

Both divisor and dividend are reduced to the fractional unit indicated by the fractional part of the divisor.

$$7\frac{2}{3} = \frac{23}{3}$$
$$168 = \frac{504}{3}$$
$$\frac{504}{3} \div \frac{23}{3} = 21\frac{21}{23}$$

II. Required the quotient of $475\frac{3}{4}$ divided by 18.

Process.

$$\begin{array}{r|l} 18 & 475\frac{3}{4} \\ \underline{4} & \underline{4} \\ 72\ ) & 1903\ (\ 26\frac{31}{72} \\ & \underline{144} \\ & 463 \\ & \underline{432} \\ & 31 \end{array}$$

Explanation.—1. The divisor and dividend are both reduced to fourths.

2. The operations are then performed as in ordinary division.

If the fractional part of the dividend had been fifths, then both divisor and dividend would have been reduced to fifths; if sixths, both would have been reduced to sixths, etc.

Both divisor and dividend are reduced to the fractional unit indicated by the fractional part of the dividend.

$$18 = \frac{72}{4}$$
$$475\frac{3}{4} = \frac{1903}{4}$$
$$\frac{1903}{4} \div \frac{72}{4} = 26\frac{31}{72}$$

191. For the division of one mixed number by another see Art. 362.

192. ***Find the quotient of:***

1. $75 \div 3\frac{1}{4}$.
2. $87 \div 9\frac{1}{2}$.
3. $178 \div 8\frac{2}{3}$.
4. $225 \div 20\frac{1}{5}$.
5. $376 \div 14\frac{3}{7}$.
6. $87\frac{1}{2} \div 12$.
7. $153\frac{2}{3} \div 15$.
8. $864\frac{4}{5} \div 24$.
9. $575\frac{3}{4} \div 51$.
10. $738\frac{5}{6} \div 18$.
11. $5695 \div 125\frac{5}{8}$.
12. $13864\frac{3}{8} \div 87$.
13. $87598\frac{7}{12} \div 137$
14. $76834 \div 293\frac{7}{10}$
15. $91007 \div 48\frac{5}{12}$.

CONCRETE ORAL PROBLEMS.

193. *1.* At $\frac{1}{4}$ of a dollar each, how many books could I buy for \$2 ?

2. How many pounds of cheese at $\frac{1}{8}$ of a dollar a pound could I buy for \$3 ? For \$4 ? For \$5 ?

3. The water in a certain river moves at the rate of $\frac{2}{3}$ of a mile per hour ; how long will it take to go 4 miles ?

4. I eat $\frac{5}{8}$ of a pound of meat daily ; how long would 10 pounds last me ? 12 pounds ?

5. My range consumes on an average one ton of coal in $\frac{5}{6}$ of a month ; how many tons would last a year ?

6. A pail containing 3 gallons of water leaks $\frac{3}{4}$ of a gallon each hour ; in how many hours will it be emptied ?

7. A farmer agrees to deliver to Mr. Brown 8 tons of hay if he takes $\frac{4}{5}$ of a ton at a load, how many loads will he deliver ?

8. A baker puts $\frac{3}{8}$ of a pound of flour in each loaf ; how many loaves can he make of 24 pounds?

9. A man earns \$$2\frac{3}{4}$ per day ; how many days' work will be required to pay for a suit of clothes costing \$22 ?

10. A wood-cutter can cut $3\frac{1}{2}$ cords of wood daily ; how long will it take him to cut 14 cords ?

11. Harry walks $15\frac{3}{4}$ miles in 7 hours ; how much is that an hour ?

12. How many shirts will 38 yards of muslin make if each requires $2\frac{3}{8}$ yards ?

13. If $\frac{3}{4}$ of a lb. of dates cost 12 cents, what will $1\frac{1}{2}$ lb. cost?

14. What will $5\frac{1}{2}$ cords of wood cost, if $1\frac{1}{2}$ cords cost \$9?

15. What is the cost of $7\frac{3}{4}$ lb. of rice, if $1\frac{1}{2}$ lb. cost 15 cents?

CONCRETE WRITTEN PROBLEMS.

194. *1.* There are $511\frac{7}{8}$ pounds of tea in 9 chests; how much in each?

2. If a horse travels 78 miles in $8\frac{2}{3}$ hours, how much does he average per hour?

3. In $1\frac{7}{9}$ miles there are 112640 inches; how many inches in one mile?

4. A farmer pays \$2292 for $31\frac{5}{6}$ acres of land; how much does it cost him per acre?

5. From New York to Havana is 1240 miles; how many hours will a steamer require for the journey if she makes $15\frac{1}{2}$ miles per hour?

6. A merchant buys $158\frac{5}{8}$ yards of silk velvet for \$846 how much does it cost per yard?

7. A carriage advances $14\frac{2}{3}$ feet each time one of the hind wheels makes one revolution; how many times will the wheel turn while the carriage goes a mile, or 5280 feet?

8. A farmer sold $\frac{5}{8}$ of his farm for \$4795; how much was the whole farm worth at that rate?

9. At \$$1\frac{3}{4}$ per bushel, how many bushels of apples can be bought for \$5285?

10. At \$$5\frac{1}{4}$ per ton, how many tons of coal can be bought for \$4200? For \$10500?

11. Mr. Edwards bought two pieces of land, one at \$41 per acre, the other at \$$63\frac{5}{8}$ per acre; the first piece cost \$$2808\frac{1}{2}$, and the second \$6108; how many acres did he buy?

12. A farm is worth \$19850; how many acres does it contain if the land is worth \$$99\frac{1}{4}$ per acre?

13. A mason earned \$23 more in $18\frac{1}{2}$ days than in $12\frac{3}{4}$ days; what did he receive per day?

14. How many cords of wood can be bought for \$620, if 9 cords cost \$$69\frac{3}{4}$?

15. If $5\frac{3}{8}$ yds. of silk brocade cost \$53, how many yards will \$$654\frac{3}{4}$ pay for?

DECIMAL PARTS OF UNITS.

1. If a unit is divided into 10 equal parts, what are the parts called? What does $\frac{1}{10}$ mean? $\frac{3}{10}$?

2. If a unit is divided into tenths and each of the tenths into ten equal parts, into how many equal parts will the unit be divided? What does $\frac{1}{100}$ mean? What does $\frac{37}{100}$ mean?

3. If a unit is divided into hundredths, and each of the hundredths into ten equal parts, into how many equal parts will the unit be divided? What does $\frac{1}{1000}$ mean? $\frac{27}{1000}$?

195. Decimal parts of units are fractional units obtained by dividing an integral unit into tenths, hundredths, thousandths, etc.

The word decimal is derived from the Latin word *decem*, which signifies *ten*.

196. A fraction composed of one or more of the decimal parts of a unit is called a **decimal fraction**, or a **decimal**.

197. For convenience, only the numerator of a decimal fraction is written, and the denominator is indicated by placing a period, or a period and one or more ciphers, before the numerator. Thus,

One tenth	or	$\frac{1}{10}$	is	written	.1.
Three tenths	"	$\frac{3}{10}$	"	"	.3.
One hundredth	"	$\frac{1}{100}$	"	"	.01.
Twenty-five hundredths	"	$\frac{25}{100}$	"	"	.25.
One thousandth	"	$\frac{1}{1000}$	"	"	.001.
Twenty-five thousandths	"	$\frac{25}{1000}$	"	"	.025.
Two hundred twenty-five thousandths	"	$\frac{225}{1000}$	"	"	.225.
One ten-thousandth	"	$\frac{1}{10000}$	"	"	.0001.
Sixty-five ten-thousandths	"	$\frac{65}{10000}$	"	"	.0065.

198. The *period* is called the **decimal point**; places to the right of the decimal point are called **decimal places** or **orders**; and ciphers between the decimal point and first significant figure are called **decimal ciphers.**

199. The number of decimal places is always equal to the number of ciphers which would be in the denominator if the decimal were written as a common fraction.

200. ***Express decimally the following fractions:***

1.	$\frac{5}{10}$.	*6.*	$\frac{55}{100}$.	*11.*	$\frac{75}{1000}$.
2.	$\frac{5}{100}$.	*7.*	$\frac{155}{1000}$.	*12.*	$\frac{75}{10000}$.
3.	$\frac{5}{1000}$.	*8.*	$\frac{155}{10000}$.	*13.*	$\frac{705}{1000}$.
4.	$\frac{5}{10000}$.	*9.*	$\frac{123}{1000}$.	*14.*	$\frac{45}{1000}$.
5.	$\frac{15}{10000}$.	*10.*	$\frac{505}{1000}$.	*15.*	$\frac{7}{1000}$.

4. How many hundreds in 1 thousand? How many tens in 1 hundred? How many units in 1 ten? How many tenths in 1 unit? How many hundredths in 1 tenth? How many thousandths in 1 hundredth?

201. Since the different orders of decimals decrease from left to right in the same ratio as in the different orders of integral units, a decimal and an integer may be written together in a single number. Thus,

7 and $\frac{7}{10}$	is written	7.7		27 and $\frac{27}{1000}$	is written	27.027
7 and $\frac{7}{100}$	" "	7.07		27 and $\frac{127}{10000}$	" "	27.0127
27 and $\frac{27}{100}$	" "	27.27		27 and $\frac{127}{100000}$	" "	27.00127

7000, 700, 70, 7, $\frac{7}{10}$, $\frac{7}{100}$, $\frac{7}{1000}$, may all be expressed as a single number thus, 7777.777.

202. Numbers composed of integers and decimal fractions, are called **decimals,** or **mixed decimals.**

203. ***Write the following mixed numbers as decimals:***

1.	$127\frac{7}{10}$.	*5.*	$1728\frac{7}{10}$.	*9.*	$4006\frac{6}{1000}$.
2.	$45\frac{45}{100}$.	*6.*	$1728\frac{17}{100}$.	*10.*	$4000\frac{4}{1000}$.
3.	$45\frac{5}{100}$.	*7.*	$1728\frac{172}{1000}$.	*11.*	$404\frac{404}{10000}$.
4.	$36\frac{66}{1000}$.	*8.*	$1728\frac{1728}{10000}$.	*12.*	$4040\frac{4004}{10000}$.

5. The first decimal place expresses what? The second? The third? The fourth?

204. To read a decimal, read the number expressed by the figures at the right of the decimal ciphers, and then give the name of the last decimal figure.

Thus, .025 is read 25 thousandths; .0017 is read 17 ten-thousandths.

205. In reading a number composed of an integer and a decimal the decimal point is read *and*.

Thus, 5234.2027 is read five thousand two hundred thirty-four *and* two thousand twenty seven ten-thousandths.

See note in Appendix, Art. 709.

206. To add and subtract decimals.

207. Write the decimals so that units of the same order shall stand in the same column, and then proceed as in addition and subtraction of whole numbers.

208. ***Write from dictation and find the sum of.***

1.	*2.*	*3.*	*4.*
$27.40	147.13	$327.17	82.014
$48.07	750.009	$450.25	93.715
$97.16	94.97	$670.50	42.0093
$54.87	863.075	$945.37	51.0985
$83.90	400.04	$415.75	27.7002
$45.73	908.873	$750.33	82.9703

209. ***Perform the subtractions indicated:***

1. $203.75—$87.15.
2. $100—$18.375.
3. $65.375—$26.31¼.
4. 987.9—908.009.
5. 101—87.087.
6. 1001.01—96.009.
7. 1621.023—835.095.
8. $75.62½—$18.66⅔.
9. $50.125—$30.33⅓.
10. 2000.001—999.99.

210. **To multiply or divide by 10, 100, 1000, etc.**

211. When the multiplicand consists of an integer and a decimal written together, the product is found by moving the decimal point to the right as many places as there are ciphers in the multiplier, annexing ciphers when necessary.

Thus, to multiply 178.25 by 1000, annex one cipher, move the decimal point three places to the right, and the result 178250 is the product.

212. When the dividend is an integer, the quotient is found by pointing off from the right of the dividend as many places as there are ciphers in the divisor.

Thus, to divide 17568 by 100, point off two places, and the result, 175.68, is the quotient.

Explanation.—Dividing in the regular way gives a quotient of 175 and a remainder of 68, which, when written over the divisor and annexed to the quotient, gives for the entire quotient $175\frac{68}{100}$, or 175.68.

The number of places pointed off in the quotient, 175.68, is the same as the number of ciphers in the divisor. This will evidently be true whatever may be the number of ciphers in the divisor.

Process.

$$\begin{array}{r|l} 100\ )\ 17568\ (\ 175\frac{68}{100} \\ \underline{100} \\ 756 \\ \underline{700} \\ 568 \\ \underline{500} \\ 68 \end{array}$$

213. When the dividend is an integer and a decimal written together, the quotient is found by moving the decimal point as many places to the left as there are ciphers in the divisor.

Thus, to divide 2143.17 by 100, the decimal point is moved two places to the left, giving 21.4317 for the quotient.

214. ***Find the value of:***

1.	875×100.	*5.*	$9456 \div 10$.	*9.*	$3875.5 \div 100$.
2.	98.35×1000.	*6.*	$7456 \div 100$.	*10.*	$83176.2 \div 1000$
3.	687.5×1000.	*7.*	$61327 \div 100$.	*11.*	$5184.23 \div 1000$.
4.	29.265×100.	*8.*	$97500 \div 1000$.	*12.*	$3230.00 \div 1000$.

215. For Multiplication and Division of Decimals, see Articles 394 and 398.

UNITED STATES MONEY.

216. The different denominations of United States Money, together with their signs, are given in the following

TABLE.

10 mills (*m.*)	=	1 cent, . . .	¢ or *ct.*
10 cents	=	1 dime, . . .	*d.*
10 dimes	=	1 dollar, . . .	$.
10 dollars	=	1 eagle, . . .	*E.*

1. 1 dime is what part of 1 dollar?

2. 1 cent is what part of 1 dime? Of 1 dollar?

3. 1 mill is what part of 1 cent? Of 1 dime? Of 1 dollar?

4. How many dollars in 1 eagle?

217. United States Money is a decimal currency. The *unit* is 1 dollar. Cents are expressed as hundredths of 1 dollar and mills as thousandths.

Thus, forty-five dollars, seventy-five cents, three mills, is written $45.753. The decimal point is placed between dollars and cents. Since there are 100 cents in 1 dollar, two decimal places are given to cents, and the third decimal place, or thousandths, is given to mills.

Mills may be regarded as tenths of cents, dimes as tenths of dollars and eagles as tens of dollars.

218. ***Express in figures the following amounts:***

1. Seventy-five dollars, fifteen cents, one mill.

2. Seventy-three dollars, forty-five cents, one mill.

3. Four hundred dollars, forty cents, four mills.

4. One thousand dollars, four cents, four mills.

5. Forty-five thousand dollars, fifty cents.

6. Four hundred dollars, five cents.

7. One hundred eighty dollars, six and one-half cents.

219. To reduce cents and mills to dollars, or dollars to cents and mills.

220. Cents are reduced to dollars by dividing by 100, or by pointing off two places at the right; and mills are reduced to dollars by dividing by 1000, or by pointing off three places.

Thus, 1756 cents = $17.56, and 1756 mills = $1.756.

221. Dollars are reduced to cents by multiplying by 100, or by annexing two ciphers; dollars are reduced to mills by multiplying by 1000, or by annexing three ciphers; dollars and cents are reduced to mills by annexing one cipher and dropping the decimal point.

Thus, $25 = 2500 cents, or 25000 mills; $17.28 = 17280 mills.

222. ***Express as dollars and cents:***

1.	1275 cts.	*4.*	$36805\frac{1}{2}$ cts.	*7.*	300125 m.
2.	7080 cts.	*5.*	125000 m.	*8.*	$590505\frac{1}{2}$ cts.
3.	15025 m.	*6.*	$16262\frac{1}{2}$ cts.	*9.*	$188183\frac{3}{4}$ cts.

223. ***Express as cents, or as mills:***

1.	$75.	*4.*	$\$18.18\frac{3}{4}$.	*7.*	$\$8375\frac{1}{3}$.
2.	$87.56.	*5.*	$\$106.06\frac{1}{4}$.	*8.*	$\$1875\frac{1}{2}$.
3.	$\$93.62\frac{1}{2}$.	*6.*	$\$157\frac{2}{3}$.	*9.*	$\$9362\frac{3}{4}$.

224. To add or subtract in United States Money.

225. Numbers expressing United States Money are added or subtracted the same as decimals.

226. ***Write from dictation and add:***

1.	*2.*	*3.*	*4.*
$300.62	$659.87	$1850.	$427.312
425.56	566.66	2937.06	625.625
837.50	30.31	5628.25	107.333
920.08	860.43	4320.33	400.062
1212.12	87.18	9008.93	366.666

227. ***Perform the subtractions indicated:***

1. $90.40 — $57.17.
2. $212.10 — $125.31.
3. $300.62 — $150.18.
4. $106.06 — $87.87.
5. $200.12 — $85.875.
6. $100 — $33.33.
7. $1000 — $575.18.
8. $837.312½ — $575.62.
9. $587.33⅓ — $309.666.
10. $87.875 — $43.43.

228. To multiply in United States Money.

I. Required to multiply $56.17 by 45.

Explanation.—The multiplicand is considered as expressing 5617 cents, the multiplication is performed as in whole numbers, and two places are pointed off from the product to reduce it to dollars.

Process.

```
  $56.17
      45
  ------
   28085
  22468
 -------
$2527.65
```

II. Required to multiply $87.12½ by 37.

Explanation.—The multiplicand may be considered as expressing 87125 mills, and multiplied as in whole numbers. Three places are then pointed off to reduce the product to dollars. Or,

The same result will be obtained if the multiplicand is considered as expressing 8712½ cents, the multiplication performed as in mixed numbers, and two places pointed off from the product to reduce it to dollars.

Process.

```
  $87.125
       37
  -------
   609875
  261375
 ---------
$3223.625
```

229. ***Find the value of:***

1. $30.25 × 57.
2. $56.50 × 89.
3. $93.25 × 50.
4. $875.15 × 100.
5. $171.125 × 32.
6. $16.18¾ × 75.
7. $31.31¼ × 500.
8. $15.66⅔ × 36.
9. $18.75 × 14⅔.
10. $29.45 × 18⅘.
11. $25.375 × 37⅗.
12. $90.67½ × 85.
13. ($175.62½ — $87.93¾) × 64 — ($73.06¼ + $68.66).
14. ($43.33 — $9.87½) × 186 — ($133.43¾ — $117.66).
15. ($75.62½ — $5.18¾) × 175 — ($337.66 — $227.18¾).
16. ($198.33 — $89.43¾) × 96 — ($56.56 — $17.875).

230. To divide in United States Money.

I. Required to divide \$17.45 by 75 cts.

Explanation.—The dividend is considered as expressing 1745 cents and the division is performed as in whole numbers. The dividend and divisor are both concrete, and the quotient is therefore abstract.

Whatever the dividend expresses, whether dollars, cents, or mills, the divisor, if concrete, must be made to express the same before dividing.

Process.

```
$.75 ) $17.45 ( 23 4/15
        150
        ---
         245
         225
         ---
          20
```

II. Required to divide \$93.825 by 45.

Explanation.—The dividend is considered as expressing 93825 mills, and the division is performed as in whole numbers. The divisor is abstract, hence the quotient is like the dividend and expresses 2085 mills. To reduce it to dollars three places are pointed off.

Process.

```
45 ) $93.825 ( $2.085
      90
      ---
       382
       360
       ---
        225
        225
        ---
```

NOTE.—When the dividend expresses simply dollars and does not contain the divisor an exact number of times, reduce the dividend to cents before dividing.

231. ***Perform the operations indicated:***

1. $\$2925 \div \$45$.

2. $\$1687.50 \div \$13.50$.

3. $\$115.92 \div \$.84$.

4. $\$1416.20 \div \$.97$.

5. $\$32706 \div \$23.70$.

6. $\$750 \div 625$ mills.

7. $\$975\frac{1}{2} \div 1225$ cents.

8. $\$172.80 \div 28\frac{4}{5}$.

9. $\$1562.50 \div 31\frac{1}{4}$.

10. $\$1550 \div \$24.80$.

11. 410375 mills $\div \$8.37\frac{1}{2}$

12. 776175 cents $\div \$59\frac{1}{4}$

13. $\$1139\frac{7}{8} \div \$1.37\frac{1}{2}$.

14. $\$2547\frac{1}{2} \div 203\frac{4}{5}$.

15. $\$1875 \div 1125$ mills.

16. $\$156.25 \div \$1\frac{1}{4}$.

17. $\$625.25 \div 101\frac{2}{3}$.

18. $\$409.60 \div 21\frac{1}{3}$.

19. $(\$2025 \div 45) \times 75 + \$87.87\frac{1}{2} \times 67$.

20. $(\$843 \div 48) \times 163 + (\$1508 \div 64)$.

21. $(\$6750 \div \$93.75) \times 83\frac{3}{4} + (\$1746 \div \$18.18\frac{3}{4})$.

22. $(\$1397 \div \$21.16\frac{2}{3}) \times 98 - (\$3817 \div \$43.37\frac{1}{2})$.

FRACTIONAL PARTS OF ONE DOLLAR.

232. Operations in multiplication and division may frequently be shortened by considering the cents in the multiplier or divisor as a fractional part of one dollar. Pupils should memorize the following

TABLE.

$6\frac{1}{4}$	cents	=	$\frac{1}{16}$ of \$1.	$33\frac{1}{3}$	cents	=	$\frac{1}{3}$ of \$1.
$8\frac{1}{3}$	"	=	$\frac{1}{12}$ of \$1.	$37\frac{1}{2}$	"	=	$\frac{3}{8}$ of \$1.
$12\frac{1}{2}$	"	=	$\frac{1}{8}$ of \$1.	50	"	=	$\frac{1}{2}$ of \$1.
$14\frac{2}{7}$	"	=	$\frac{1}{7}$ of \$1.	$62\frac{1}{2}$	"	=	$\frac{5}{8}$ of \$1.
$16\frac{2}{3}$	"	=	$\frac{1}{6}$ of \$1.	$66\frac{2}{3}$	"	=	$\frac{2}{3}$ of \$1.
20	"	=	$\frac{1}{5}$ of \$1.	75	"	=	$\frac{3}{4}$ of \$1.
25	"	=	$\frac{1}{4}$ of \$1.	$87\frac{1}{2}$	"	=	$\frac{7}{8}$ of \$1.

ORAL PROBLEMS.

233. *1.* How much would 48 articles of any kind cost at \$1 each? At \$$\frac{1}{2}$ each? At \$$\frac{1}{3}$ each? At \$$\frac{1}{4}$? At $16\frac{2}{3}$ cents? At $12\frac{1}{2}$ cts.?

2. How many articles at \$1 each can be bought for \$12? At 50 cents each? At 25 cents each? At $33\frac{1}{3}$ cents? At $8\frac{1}{3}$ cents?

3. 25 cents × 16 = ? $33\frac{1}{3}$ cents × 12 = ? 20 cents × 75 = ? $8\frac{1}{3}$ cents × 36 = ? $14\frac{2}{7}$ cents × 21 = ?

4. Find the cost of 65 pounds of sugar at $8\frac{1}{3}$ cents per pound.

5. What is 24 times $\frac{1}{8}$ of \$1? 24 times $12\frac{1}{2}$ cents? 24 times $\frac{3}{8}$ of \$1? 24 times $37\frac{1}{2}$ cents? 24 times $62\frac{1}{2}$ cents? 24 times $87\frac{1}{2}$ cents?

6. $37\frac{1}{2}$ cents × 30 = ? $66\frac{2}{3}$ cents × 45 = ? $62\frac{1}{2}$ cents × 40 = ? $87\frac{1}{2}$ cents × 80 = ?

7. \$1.12$\frac{1}{2}$ × 48 = ? \$1.37$\frac{1}{2}$ × 56 = ? \$2.62$\frac{1}{2}$ × 32 = ? \$3.87$\frac{1}{2}$ × 24 = ? \$3.66$\frac{2}{3}$ × 36 = ?

8. How much will 5 dozen shovels cost at $\$.62\frac{1}{2}$ each? At $\$.66\frac{2}{3}$? At $\$.87\frac{1}{2}$?

9. What will 72 bushels of oats cost at $\$.37\frac{1}{2}$ cents? 96 bushels of rye at $\$.87\frac{1}{2}$? 48 bushels of corn at \$.75? At $\$.62\frac{1}{2}$? At $\$.66\frac{2}{3}$?

10. What will 28 lb. of cheese cost at $\$.12\frac{1}{2}$ a pound?

11. What is the cost of 54 pounds of currants at 25 cents a pound? Of 66 pounds? Of 96 pounds?

12. If \$4 was paid for rice at 3 lb. for 25 cents, how many pounds were there?

13. A man sold some butter at 25 cents per pound and received for it \$6; how many pounds were there?

14. A farmer traded 36 bushels of oats at $37\frac{1}{2}$ cents per bushel, for potatoes at 75 cents per bushel; how many bushels of potatoes did he get?

WRITTEN PROBLEMS.

234. *1.* How much will 9408 bushels of wheat cost at $\$1.33\frac{1}{3}$ per bushel? At $\$1.12\frac{1}{2}$? At $\$1.16\frac{2}{3}$?

2. Find the cost of 1960 bushels of rye at \$.875 per bushel.

3. Bought 576 quarts of cream at 25 cents per quart, and 9728 quarts of milk at $8\frac{1}{3}$ cents; what was the entire cost?

4. How many bushels of oats at $\$.37\frac{1}{2}$ can be bought for \$9300? For \$6096?

5. How long will \$140 pay my board at \$1.75 per day?

6. Find the cost of 6824 yards of silk at $\$2.12\frac{1}{2}$ per yard, and 8975 yards of silk velvet at $\$3.33\frac{1}{3}$ per yard.

7. Find the cost of 756 tons of coal at $\$4.87\frac{1}{2}$ per ton, and 924 cords of wood at $\$5.62\frac{1}{2}$ per cord.

8. At $\$1.16\frac{2}{3}$ per bushel, 8 car-loads of potatoes cost \$4375; how many bushels on an average in a car?

9. Paid \$1980 for coal at $\$4.12\frac{1}{2}$ per ton, and sold it at \$5.75 per ton; how much did I gain?

10. A speculator bought 19864 bushels of wheat at $\$1.12\frac{1}{2}$ per bushel, and sold it at $\$1.33\frac{1}{3}$ per bushel; what was his gain?

BILLS.

235. A **debtor** is one who owes money, goods or service.

236. A **creditor** is one having a claim against a debtor for money, goods, or service.

237. A **bill** is a formal statement of debits and credits, with dates, place of business, and names of parties.

When the creditor, or his agent, writes or stamps on a bill the expression *Received payment,* or *Paid,* and signs his name to the bill, it is said to be *receipted.*

238. ALBANY, N. Y., *Dec. 18, 1885.*

Mr. Henry W. Sanborn,

Bought of JOHN R. NORRIS *& Co.,*

84 yds.	*Canton Flannel*	@	*$.16*	*$13*	*44*
56 "	*Hollands*	@	*.18*	*10*	*08*
72 "	*Sheeting*	@	*.12*	*8*	*64*
48 "	*Flannel*	@	*.62*	*29*	*76*
64 "	*Duck*	@	*.25*	*16*	*00*
96 "	*Cassimere*	@	*.75*	*72*	*00*
				$149	*92*

Received payment,

John R. Norris & Co.,

Per J. S.

239. When, according to the bill, were the items mentioned above purchased? At what place? By whom? Of whom? By whom was the bill made out? Who paid the bill? By whom was it receipted? What does *per J. S.* mean? What is the amount of the extension of the first item? Of the second? Of the third? Of the fourth? Of the fifth? Of the sixth? What is the footing of the bill?

240. The following abbreviations are used in bills:

@,	At.	Dr.,	Debtor.	Net,	Without disc't
Acc't, %,	Account.	Do. or Ditto,	The same.	No. or #,	Number.
Am't,	Amount.	Doz.,	Dozen.	Pay't,	Payment.
Bal.,	Balance.	Exps.,	Expenses.	Pd.,	Paid.
Bbl.,	Barrel.	Fol.,	Folio.	Per,	By.
Bo't,	Bought.	Inst.,	This month.	Rec'd,	Received.
Co.,	Company.	Int.,	Interest.	Sunds.,	Sundries.
Cr.,	Creditor.	Mdse.,	Merchandise.	Ult.,	Last month

241. ***Copy and find the footings of the following bills:***

(1) NEW YORK, *Dec. 11, 1885.*

Mr. Henry J. Rowson,

Bought of RICHARD GLOVER *& Co.,*

25 yds.	*Velveteen*	@	*$1.85*		
18¾ "	*Satin*	"	*3.75*		
22 "	*Cashmere*	"	*1.63*		
7 "	*Brocaded Velvet*	"	*4.25*		

Received payment,
Richard Glover & Co.

(2) DETROIT, MICH., *Sept. 8, 1885.*

Mr. Mortimer Harvey,

To GEORGE *C.* SMITH, *Dr.,*

To Board for 3 adults, 6 weeks (each),	@	*$14.00*		
" " " 2 children, " "	@	*7.50*		
" Carriage, 8 times,	@	*1.50*		
" Boat, 14 "	@	*.50*		
" Laundry, 13½ doz. articles	@	*1.10*		
" Express charges		*2.75*		

Received payment,
George C. Smith.

(3)

ALBANY, N. Y., *Dec. 1, 1885.*

Mrs. Henry J. Lee,

To SCOTT & WESTON, Dr.,

1885.				
Sept. 8	*To 22½ yds. Silk Plush . . @ $3.75*			
" "	" *5 " Surah Silk . . @ 1.38*			
" *25*	" *43 " Bleached Muslin @ .09*			
Oct. 3	" *4 pieces Silk Braid . . @ 2.25*			
" "	" *2½ doz. Buttons . . . @ .75*			
" *12*	" *15 yds. Honiton Lace . @ 1.13*			
" *18*	" *30 " Spanish Lace . @ 1.88*			
" "	" *6 pairs Kid Gloves . . @ 1.75*			
Nov. 12	" *1 pair Burt's Shoes . @ 6.00*			
" "	" *21 yds. Silk . . . . @ 1.67*			

Received payment,
Scott & Weston.

(4)

PATERSON, N. J., *Nov. 11, 1885.*

Mr. John H. Colfax,

Bought of ACKERMAN & VAN GIESON,

183 lb. Ham . . . . @ 12¢			
1200 " Oat Meal . . . . " 2¢			
114 cakes Soap . . . . " 9¢			
54 gal. Molasses . . . . " 45¢			
45¾ lb. Honey . . . . " 11¢			
150 " Corn Starch, ... " 9¢			
168 " Dried Beef . . . . . . " 16¢			
16 boxes Starch . . . . " 95¢			
190 lb. Dried Peaches . . . . . " 25¢			

Received payment,
Ackerman & Van Gieson.

LEDGER COLUMNS.

242. In the addition of ledger columns it is common to write in pencil at the top of each column, in small figures, the number carried from the preceding column. Thus,

222 1
$879.89
995.96
484.93
$2360.78

The small figures in italics represent the number carried forward from the preceding column. This enables the accountant, if interrupted in the addition of a long column, to resume his work without going over the whole, and also to verify his work by columns.

243. ***Copy and find the footings of the following ledger columns:***

1.	*2.*	*3.*	*4.*
$18.25	$171.38	$236.87	$837.50
2.37	23.61	5961.00	2361.25
23.68	18.00	86.75	749.60
1.12	51.49	487.64	3000.00
148.65	638.41	8.99	568.92
23.00	83.00	2354.47	37.00
15.41	9.13	927.68	650.00
.87	832.61	516.00	1725.75
4.31	595.75	5927.84	928.38
.93	87.63	1219.58	67.44
264.85	59.47	1.65	8396.87
5.87	429.68	346.78	460.00
15.06	87.00	59.84	1739.56
1.18	5.87	2837.19	28.38
137.38	69.45	846.25	5.98
49.69	747.95	1800.00	861.29
458.86	81.00	569.43	2468.78
79.58	598.12	3769.29	367.59
5.63	.84	468.37	3875.98
75.13	1.75	85.00	625.00
12.86	33.84	17.12	83.50

244. ***Make receipted bills in correct form:***

1. On Jan. 9, 1884, Henry Cooke bought of John R. Alexander, at Philadelphia, Penn.:

13⅓ lb. sugar @ 8¢; 5¾ lb. coffee @ 32¢; 8¾ lb. cheese @ 18¢; 12⅓ lb. rice @ 9¢; 5½ qts. beans @ 12¢; 6¾ lb. starch @ 11¢.

2. Henry Esmond bo't July 7, 1861, of James M. Howard & Co., of New York:

76 yds. carpet @ $1.87; 84 yds. ingrain carpet @ $1.25; 22 yds. stair carpet @ $1.12; 57 yds. border @ $.88; 135 yds. carpet lining @ 9¢; 5 rugs @ $4.75; 41¾ yds. oil-cloth @ 58¢; 15¾ sq. yds. linoleum @ $1.60.

3. Frank W. Case, a clerk with J. H. Porter & Co., Chicago, Ill., sold the following items to Henry Sherman, and gave him a receipted bill, Aug. 7, 1885:

75 tons Lehigh coal @ $4.33; 168¾ tons Scranton coal @ $4.12; 24¾ tons cannel coal @ $18; 96⅓ tons bituminous coal @ $2.62; 48 cords hickory wood @ $6.38; 35¾ cords pine wood @ $4.75.

245. ***Supply names of persons and places, and make receipted bills with dates:***

1. 256 lb. ham @ 12½¢; 94¾ lb. bacon @ 9¢; 68 lb. dried beef @ 16¢; 325 lb. shoulder @ 10¢; 75 lb. boneless bacon @ 14¢; 32¾ lb. Bologna sausage @ 18¢.

2. 36 bbl. flour @ $8.62½; 43 bbl. new process flour @ $9.25; 875 lb. buckwheat flour @ 3¾¢; 486 lb. corn meal @ 2⅔¢; 186 lb. oatmeal @ 4¾¢; 91 bbl. rye flour @ $7.75.

3. 650 lb. mackerel @ 9½¢; 25 kits pickled herring @ $1.12½; 533 chests Young Hyson tea, each 87 lb., @ 28⅔¢; 75 sacks Maracaibo coffee, each 185 lb., @ 21⅔¢; 1680 lb. rice @ 8⅓¢; 560 lb. crackers @ 6⅔¢; 95 quarts soup beans @ 6¼¢; 32 barrels granulated sugar, each 200 lb. @ 7¾¢.

NOTE.—The pupil should be required to make original bills neatly and in proper form.

MEASURES IN COMMON USE.

LINEAR MEASURE.

246. Linear measure is used in measuring lines and distances.

TABLE.

12 inches	=	1 foot,	. . .	*ft.*
3 feet	=	1 yard,	. . .	*yd.*
$5\frac{1}{2}$ yards ($16\frac{1}{2}$ feet)	=	1 rod,	. . .	*rd.*
320 rods	=	1 mile,	. . .	*mi.*

ORAL PROBLEMS.

247. *1.* How many inches in 1 foot? In 2 feet? In 3 feet? In 4 feet? In 7 feet?

2. How many feet in 1 yard? In 2 yards? In 3 yards? In 4 yards? In 5 yards? In $5\frac{1}{2}$ yards?

3. How many inches in $1\frac{1}{2}$ feet? In $2\frac{1}{4}$ feet? In $3\frac{1}{3}$ feet? In 1 yard? In 2 yards? In $3\frac{1}{3}$ yards?

4. 2 feet equals how many inches? 3 feet? 1 yard? 2 yards? $4\frac{1}{3}$ yards?

5. How many feet in 2 rods? In 3 rods? In 4 rods? In 8 rods?

6. In 2 rods how many yards? In 3 rods? In 4 rods? In 10 rods?

7. In 72 inches how many feet? How many yards? In 108 inches how many feet? How many yards?

8. In 45 inches how many feet? Yards? In 54 inches how many feet? Yards?

9. What part of a yard is 9 inches? 12 inches? 6 inches? 18 inches? 24 inches? 27 inches? 30 inches? 32 inches?

10. How many yards in 6 feet? In 12 feet? In 30 feet? In $16\frac{1}{2}$ feet? In $4\frac{1}{2}$ feet? In 8 feet? In 11 feet?

11. How many rods in 66 feet? How many yards?

12. Find the cost of a piece of ribbon 18 in. long at 20 cents per yard.

13. How much would 27 in. of velveteen cost at \$2 per yard?

14. Find the cost of 6 ft. 9 in. of copper wire at 12¢ per foot.

15. A boy bought a chain for his dog, paying 15¢ per foot; how much did he pay for 56 inches?

16. A carpenter buys a piece of mahogany moulding 12 ft. 6 in. long at 10¢ per foot; find its cost.

17. What is the value of 90 feet of manilla rope at 12¢ per yard?

18. A lady buys a piece of silk velvet 48 in. long at \$9 per yard; how much does it cost?

WRITTEN PROBLEMS.

248. *1.* Find the cost of 28 yds. of Wilton carpet at \$5.25 per yard.

2. How much will 156 feet of piano wire cost at $12\frac{1}{2}$¢ per foot?

3. How many feet in 40 rods? In $160\frac{2}{3}$ yards? In 1 mile?

4. How many yds. in 1 mile? In $2\frac{3}{4}$ miles?

5. A man goes 36 inches at each step; how many steps must he take in going from New York to Philadelphia, a distance of 90 miles?

6. Find how many feet in 1140 in. In 63360 in.

7. Reduce 58080 feet to rods.

8. How much must a tailor pay, at \$2.87$\frac{1}{2}$ per yard, for a piece of French cassimere containing $46\frac{3}{4}$ yards?

9. In 5 miles how many rods? In 4 miles and 96 rods how many rods? How many feet?

10. Reduce 3 mi. 13 rds. to rods. To feet. To inches.

11. Find the number of rods in 5 mi. 61 rds. Of yards. Of feet.

12. Reduce 1512 in. to feet. To yards.

13. A man walked 237600 ft. in 15 hours; how many feet did he walk per hour?

14. Change 334788 in. to feet. To yards.

15. The chain used by surveyors is 66 feet long; how many lengths of it in 1 mile?

16. If carpet 27 inches wide is worth \$1.35 per yard, how mueh should be charged for carpet equally good, 1 yard wide?

17. A lady spent \$50.82 for silk at \$1.98 per yard; how many yards did she buy?

18. A farmer paid a man \$36.75 for fencing a field at $37\frac{1}{2}$¢ per rod; how many rods of fence were there?

19. A pigeon made a flight of $5\frac{3}{4}$ hours at the rate of 48 miles per hour; how far did it fly?

20. The fathom, which is equal to 6 feet, is used in measuring depths at sea. How many feet deep is the Gulf Stream where it issues from the Gulf of Mexico, if its depth is $433\frac{1}{3}$ fathoms?

21. How many feet separate two vessels at sea that are 396 cable-lengths apart, the cable-length being equal to 120 fathoms?

22. Soundings made between the Island of St. Helena and the coast of Africa give the greatest depth of the ocean as 3000 fathoms. What is the depth in feet?

23. How much more will $19\frac{3}{4}$ yds. of velvet cost at \$3.88 per yard than at \$3.67 per yard?

24. How many steps of 30 inches each does a man take in walking one mile, a mile being equal to 63360 inches?

25. I paid \$71.76 for silk at \$2.34 per yard; how many yards did I buy?

26. A tailor paid \$1389.04 for 8 equal pieces of broadcloth at \$3.88 per yard; how many yards in each piece?

27. A farmer paid \$$127\frac{1}{2}$ to have a fence built around a field the length of whose sides was 896 ft., 789 ft., 886 ft. and 795 ft. respectively; what did he pay per rod?

LIQUID MEASURE.

249. Liquid measure is used in measuring liquids.

TABLE.

4 gills (*gi.*)	=	1 pint. . . .	*pt.*
2 pints	=	1 quart. . . .	*qt.*
4 quarts	=	1 gallon. . . .	*gal.*

ORAL PROBLEMS.

250. *1.* How many gills in 1 pint? In 2 pints? In 3 pints? In $5\frac{1}{2}$ pints? In 12 pints? In $4\frac{3}{4}$ pints?

2. How many pints in 2 quarts? In 3 quarts? In $4\frac{1}{2}$ quarts? In $11\frac{1}{2}$ quarts?

3. How many gills in 1 quart? In 1 gallon? In 5 quarts? In 2 gallons? In 3 gallons? In $2\frac{1}{2}$ gallons?

4. How many gills in 1 quart and 1 gill? In 2 quarts and 1 pint? In 1 qt. 1 pt. 3 gi.? In 2 qts. $1\frac{1}{2}$ pts.?

5. How many pints in 1 gal. 3 qts. 1 pt.? In 2 gal. 2 qts. 1 pt.? 3 gal. 1 qt. 1 pt.?

6. How many pts. in 12 gi.? In 19 gi.? In 29 gi.?

7. Reduce 8 gi. to qts.; 32 gi.; 37 gi.; 43 gi.; 64 gi.

8. What is the cost of a gallon of milk at 8¢ per qt.?

9. How much will 1 gal. 3 qts. of cider cost at 9¢ per qt.?

10. Find the cost of 2 gal. 1 qt. 1 pt. of vinegar at 10¢ per pt.

11. How much will 5 pts. of brandy cost at $8 per gallon?

12. How much will 11 qts. of syrup cost at 80¢ per gallon?

13. I paid 75¢ for cider at 5 cents per qt.; how many gallons were there?

14. How many pints of cream at 80¢ per gal. can be bought for 65¢?

15. Find the cost of 2 gallons kerosene at $2\frac{1}{2}$¢ per pint.

16. A boy sold 12 quarts of berries at $6\frac{1}{4}$¢ per quart; how much did he get for them?

WRITTEN PROBLEMS.

251. *1.* How many pints in $41\frac{1}{2}$ gallons?

2. Find the cost of a hogshead of New Orleans molasses containing 63 gallons at $87\frac{1}{2}$¢ per gallon.

3. A grocer bought a barrel containing $41\frac{1}{2}$ gallons of cider at 18¢ per gallon, and sold it at 3¢ a pint; find his profit.

4. A dairyman sold 12 cans of milk, each containing $10\frac{1}{8}$ gallons, at 8¢ a quart; what did he get for it?

5. Find the gain on 5 barrels of kerosene, of $43\frac{3}{4}$ gallons each, bought at 15¢ per gallon and sold at 8¢ per quart.

6. How many times must I empty a 5-pint vessel into a barrel holding 40 gallons in order to fill the barrel?

7. If a quart of strained honey weighs $2\frac{3}{8}$ lb., how much will $19\frac{1}{2}$ gallons cost at 25¢ per pound?

8. A dairyman each day distributes among his customers 28 cans of milk, each containing $10\frac{3}{4}$ gallons, for which he charges 10¢ a quart; what should be his weekly collections?

9. A dealer paid $25 for a barrel of turpentine containing $41\frac{1}{4}$ gallons, and sold it at 3¢ a gill; find his gain.

10. A can containing $12\frac{3}{4}$ gallons of shellac, costing $1.40 per gallon, was sold at 12¢ a gill; what was the gain?

11. From a hogshead containing 63 gallons of syrup, 19 gallons 3 qts. 1 pt. were sold; how much remained?

12. How many barrels of $31\frac{1}{2}$ gallons each in 24192 gills?

13. Bought 90 gallons 3 qts. of varnish at $1.25 per gallon, and sold it at $37\frac{1}{2}$¢ a pint; find my gain.

14. How many bottles, each containing 2 gills, can be filled from a vessel containing 24 gallons 3 qts. 1 pt.?

15. One gallon of maple syrup can be made from 12 gallons 1 qt. 1 pt. of sap; how many pints of sap will make 3 gallons of syrup?

16. 5 gal. 2 qts. 1 pint of currant jelly were put into half-pint jars costing 5 cents each, and sold at $12\frac{1}{2}$¢ per jar; how much was left after paying for the jars?

DRY MEASURE.

252. Dry measure is used in measuring grain, fruit, and many other articles not liquid.

TABLE.

2 pints (*pt.*)	=	1 quart, . . .	*qt.*
8 quarts	=	1 peck, . . .	*pk.*
4 pecks	=	1 bushel, . . .	*bu.*

ORAL PROBLEMS.

253. *1.* In 2 quarts of cherries how many pints? In 3 quarts? In 4 quarts? In 12 quarts?

2. In 1 peck of crab-apples how many quarts? How many pints? In 2 pecks how many quarts? Pints?

3. How many pecks in 1 bushel? In 2 bushels? In $2\frac{1}{2}$ bushels? In $3\frac{3}{4}$ bushels?

4. How many quarts in 1 bushel? In 1 bushel and 1 peck? In $1\frac{1}{2}$ bushels? In $2\frac{1}{2}$ bushels?

5. How much will 1 bushel of chestnuts cost at 50¢ per peck? At 5¢ per quart?

6. What will 1 peck of cranberries cost at $12\frac{1}{2}$¢ per quart? 1 bushel?

7. How many pecks in 40 quarts? In 64 quarts? In 54 quarts? In 84 quarts?

8. What will 72 quarts of hazel-nuts cost at 75¢ a peck?

9. What will be the cost of 20 quarts of peaches at 50¢ a peck?

10. What part of a peck in 1 quart? What part of a peck in 2 quarts? 4 quarts? 6 quarts?

11. What part of a bushel in 2 quarts? 4 quarts? 8 quarts? 12 quarts? 16 quarts? 20 quarts? 24 quarts?

12. What part of a bushel in 1 peck? What part of a bushel in 4 quarts? 2 pecks? 2 pecks and 4 quarts? 3 pecks? 3 pecks and 4 quarts?

13. What will 3 pecks of potatoes cost at $1 per bushel? 1 bu. 1 pk. 4 qts.? 2 bu. 2 pks. 4 qts.? 3 bu. 3 pks. 4 qts.?

14. I paid $1.60 for 1 bushel of peanuts; how much did they cost per peck? How much per quart?

15. A farmer sold 2 bu. 3 pks. 4 qts. of clover-seed at $16 per bushel; how much did he get for it?

16. A man sold 1 bu. 1 pk. 3 qts. of cherries at 10¢ per quart; how much did he receive for them?

WRITTEN PROBLEMS.

254. *1.* Reduce 5 bu. 3 pks. to pecks. To quarts. To pints.

2. Change 1 bu. 1 pk. 1 qt. 1 pt. to pints.

3. How many pints in 15 bu. 1 pk. 7 qts. 1 pt.?

4. How many quarts in 864 pints? How many pecks? How many bushels?

5. Change 1565 pints to pecks, quarts and pints.

6. A grocer bought 3 bu. 3 pks. of bird seed at $3.20 per bushel and retailed it at 15¢ a quart; find his gain.

7. A man sold 3 bu. 1 pk. 7 qts. of peanuts at 5¢ a pint what did he receive for them?

8. What will be the cost of 15 bu. 3 pks. 7 qts. of cranberries at 15 cents per quart?

9. Henry gathers during the autumn 9 bu. 3 pks. 6 qts. 1 pt. of chestnuts and sells them at 17¢ per quart; with the proceeds he buys his mother 18¾ yards of silk for a dress at $1.35 per yard; how much money does he have left?

10. A farmer exchanges 5 bu. 2 pks. 4 qts. of clover-seed at $16.40 per bushel for flour at $2¼ per hundred pounds; how many pounds of flour does he receive?

11. A grocer bought 6 barrels of Baldwin apples, each containing 3 bu. 1 pk. 6 qts. at $3.75 per barrel, and retailed them at 20 cents a half-peck; how much did he gain?

12. A fruit dealer bought 2 bu. 3 pks. of Bartlett pears at $3.60 per bushel and sold them at 3¢ each; how much did he gain if there were on an average 184 pears in a bushel?

13. A grocer bought 6 bags of chestnuts, each containing 2 bu. 3 pks. 4 qts. at \$2.80 per bushel; he sold 4 bags at 14¢ a quart, and the remainder at 5¢ a pint; find his gain on the lot.

14. How many times must I empty a vessel having a capacity of 1 pk. 2 qts., into a bin holding 15 bu. 2 pks. 4 qts., in order to fill it?

15. A man bought $12\frac{3}{4}$ bushels of peaches at \$1.50 per bushel and sold them at 25¢ a half-peck; find his gain.

AVOIRDUPOIS WEIGHT.

255. Weight is the measure of the earth's attraction.

256. Avoirdupois weight is used in finding the weight of all common articles.

TABLE.

16 ounces (*oz.*)	=	1 pound, ...	*lb.*
100 pounds	=	1 hundredweight, .	*cwt.*
20 cwt., or 2000 lb.	=	1 ton, ...	*T.*

257. The *long ton* is used in weighing coal at the mines, and goods on which duty is paid by weight at the Custom House.

LONG TON TABLE.

16 ounces (*oz.*)	=	1 pound, . . .	*lb.*
28 pounds	=	1 quarter, . . .	*qr.*
4 quarters	=	1 hundredweight,	*cwt.*
20 cwt., or 2240 lb.	=	1 ton, ...	*T.*

ORAL PROBLEMS.

258. *1.* How many ounces in 1 pound? In 2 pounds?

2. What is the cost of a pound of caramels at 2¢ an ounce?

3. How many ounces in 1 lb. 5 oz.? In 1 lb. 11 oz.? In 2 lb. 5 oz.? In 3 lb. 12 oz.?

4. How much would 2 lb. 8 oz. of butter cost at 32¢ per pound?

5. In $1\frac{1}{2}$ pounds how many ounces? In $2\frac{1}{4}$ pounds? In $3\frac{1}{2}$ pounds? In $5\frac{3}{4}$ pounds?

6. How much will $1\frac{1}{2}$ pounds of almonds cost at 30¢ a pound?

7. In 1 hundredweight how many pounds? How many ounces?

8. How many pounds in 2 cwt. 58 lb.? In 5 cwt. 17 lb.? In 1 T. 1 cwt. 18 lb.? In 8 T. 5 cwt. 78 lb.?

9. How many pounds in 16 ounces? In 32 ounces? In 48 ounces? In 64 ounces? In 80 ounces? In 96 ounces?

10. What part of a pound is 2 ounces? 4 ounces? 6 ounces? 14 ounces? 10 ounces? 8 ounces? 12 ounces?

11. How many pounds in 20 ounces? In 30 ounces? In 40 ounces? In 56 ounces?

12. Six boys share 1 lb. 8 oz. of candy equally; how much does each receive?

13. What is one fourth of 2 lb. 12 oz.? Of 1 lb. 12 oz.? Of $\frac{5}{8}$ lb.? Of $\frac{3}{4}$ lb.?

14. What part of a ton is 2 cwt.? 4 cwt.? 15 cwt.? 18 cwt.? 10 cwt.? 8 cwt.? 12 cwt.?

15. How much will 15 cwt. of coal cost at \$4 per ton? 18 cwt.? 4 cwt.? 14 cwt.?

16. What is the cost of 1 T. 15 cwt. of granite at \$2 per ton? Of 2 T. 18 cwt.? Of 5 T. 16 cwt.?

17. What will 2 lb. 10 oz. of dried cherries cost at 20 cents per pound?

18. How much must I pay for 3 lb. 12 oz. butter at 32¢ per pound?

19. Four apples of equal size weigh together 3 lb. 4 oz.; what is the weight of each?

WRITTEN PROBLEMS.

259. *1.* Reduce 5 T. 17 cwt. 87 lb. to pounds.

2. Some sugar from Cuba weighed at the New York Custom House 5 T. 17 cwt. 3 qrs. 18 lb.; how many pounds were there?

3. Find the cost of 5 cwt. 75 lb. of beef at $12\frac{1}{2}$¢ per pound.

4. How much would 3 cwt. $59\frac{3}{4}$ lb. of sugar cost at 8¢ per pound?

5. A coal dealer bought at the mines at Scranton, anthracite coal weighing 56 tons; what was its weight in pounds?

6. I paid \$3.75 per ton for 175 long tons of coal, and sold it at \$3.85 per ordinary ton; what was my gain?

7. A fish dealer sold a trout weighing 8 lb. 10 oz. at 70¢ per pound; what did he receive for it?

8. A butcher bought a fat steer weighing 18 cwt. 95 lb. at $5\frac{3}{4}$¢ per pound live weight. Its weight after it was killed and dressed was $\frac{3}{5}$ of its weight when living, and he sold the meat at an average price of 13¢ per pound; what was his gain?

9. How many pounds more are there in 18 long tons than in 20 ordinary tons?

10. How many spoons weighing $1\frac{3}{4}$ ounces each can be made from 5 lb. 4 oz. of German silver?

11. Reduce 83675 ounces to higher denominations.

12. What will $3\frac{3}{4}$ cwt. of dried peaches cost at 37¢ per pound?

13. A bushel of wheat weighs 60 pounds; if a farmer receives \$20.79 for 891 pounds, how much does he receive per bushel?

14. In a canning establishment 2 lb. 12 oz. of beef were used to fill each can; how many cans were required to contain 13 cwt. 9 lb.?

15. At 4¢ per pound, how many pounds of iron can be bought for \$231.50?

REVIEW QUESTIONS.

260. Equal Parts of Units.—When two numbers are written, one above the other, with a short horizontal line between them, what do they express? What does $\frac{1}{5}$ mean? What is meant by $\frac{3}{7}$? Which is the denominator? What does it show? Which is the numerator? What does it show? How are equal parts of like units added? What is necessary to be done before adding unlike parts of like units? How are mixed numbers added? If a unit is divided in equal parts, what is the relation between the size of the parts and their number? Show how to subtract a mixed number from an integral number. How is one mixed number subtracted from another mixed number? Illustrate. What is meant by multiplying a number by a fraction? Illustrate. How is an integer multiplied by a fraction? Illustrate. How is an integer multiplied by a mixed number? A mixed number by an integer? Illustrate each. How is an integer divided by a fraction? What does the quotient show when the divisor is less than the dividend? When the divisor is greater than the dividend? How is a mixed number divided by an integer? An integer by a mixed number?

261. Decimal Parts of Units.—What is a decimal part of a unit? A decimal fraction? How are decimals written? What are decimal places? What is a mixed decimal? Why may a mixed decimal be written as a single number? How is a decimal read? How are decimals written in order to add or to subtract them? How may numbers be multiplied by 10, 100, 1000, etc.? How divided?

262. United States Money.—Give the table of United States money. Show that it is a decimal currency. Why may dollars, cents and mills be written as one number? How are cents reduced to mills? To dollars? Mills to dollars? Dollars to cents? To mills? How is United States money added? Subtracted? Multiplied? What must the multiplier be? By what may United States money be divided? What is the quotient when the divisor is abstract? When it is concrete? Give the table of the fractional parts of $1. Define: Debtor, creditor, bill. How is a bill receipted?

263. Measures in Common Use.—Give the table of linear measure. For what is linear measure used? What is a fathom? A cable-length? Give the table of liquid measure. Give the table of dry measure. What is weight? For what is avoirdupois weight used?

MISCELLANEOUS PROBLEMS.

ADDITION.

264. *1.* I pay on a certain debt at one time \$985.60 in cash; at another I give a horse and carriage worth \$465.50; again I make a cash payment of \$1731.56, after which I still owe \$2035.74; what was the original debt?

2. A young man saved \$987.50, which was \$193.80 less than he already had in a bank; how much was he worth?

3. A man sold to A $346\frac{5}{8}$ acres of land, to B $191\frac{1}{2}$ acres more than to A, to C $92\frac{3}{4}$ acres more than to A and B together, and then had $1238\frac{7}{8}$ acres left; how much had he at first?

4. A grocer buys flour for \$1375, and sells it so as to gain \$275.50; how much does he get for it?

5. Mr. Fox bought a house and lot for \$3875, had repairs made costing \$863.50, paid \$125.36 for painting, and then sold it so as to gain \$1264.89; what did he get for it?

6. Three brothers bought a farm; the first paid \$5675, the second \$8954, and the third \$6435, after which they still owed \$3655; what was the price of the farm?

7. A drover bought 4 horses at \$275 each; he sold them so as to gain on the first \$75.25, on the second \$21.50, on the third \$37.75, and on the fourth \$16.65; how much did he receive for them all?

8. A merchant deposited in bank on Monday \$375.50, on Tuesday \$585.96, on Wednesday \$736, on Thursday \$845.60, on Friday \$954, and on Saturday \$1361.85; find the amount of his deposits for the week.

9. A merchant having a certain sum of money in bank, pays by check the following bills: \$183.75, \$565.83, \$19.29, \$1165.90. He then has a balance in bank of \$8361.68; how much had he before paying the bills?

10. A ton of coal cost me \$$5\frac{4}{5}$ and a cord of wood \$$3\frac{3}{4}$; what did both cost?

11. Of a party of four miners, one found $37\frac{3}{8}$ ounces of gold, the second found $23\frac{3}{4}$ ounces, the third $46\frac{1}{2}$ ounces, and the fourth $9\frac{1}{2}$ ounces more than the first; how many ounces did all find?

12. A farmer sold $\frac{1}{3}$ of his hay to one man, $\frac{1}{4}$ to another, and $\frac{1}{6}$ to a third; what part of his hay did he sell?

13. A well is sunk through sand $17\frac{3}{4}$ feet, through clay $8\frac{2}{3}$ feet, through slate $21\frac{5}{12}$ feet, and through rock $16\frac{1}{2}$ feet; how deep is the well?

14. A manufacturer makes cloth at a cost of $87\frac{1}{2}$ cents per yard, and sells it to a jobber at a profit of $19\frac{3}{4}$ cents per yard; the jobber sells it to a retailer at a gain of $23\frac{1}{2}$ cents per yard, and the retailer sells it at a gain of $29\frac{1}{4}$ cents per yard; what does the retailer get for it?

15. From the milk of one cow a dairyman made one week $16\frac{3}{4}$ lb. of butter, the next week $14\frac{3}{8}$ lb., the third week $18\frac{1}{4}$ lb., and the fourth week $13\frac{1}{2}$ lb.; how many pounds did he make in the four weeks?

16. A lady bought ribbon for \$1.87, a pair of shoes for \$3.37, a pair of gloves for \$1.62 and a hat for \$5; how much did all cost?

17. The masons worked on a certain house $9\frac{2}{3}$ days, after which the carpenters worked $14\frac{1}{2}$ days; then came the plasterers for $5\frac{3}{4}$ days, and last of all the painters for $3\frac{1}{4}$ days; how many days were consumed by all?

18. A father went for berries with his three sons; the youngest gathered $18\frac{3}{4}$ quarts, the second $9\frac{1}{4}$ quarts more than the youngest, the oldest $5\frac{1}{2}$ more than the second, and the father $12\frac{3}{4}$ quarts more than the oldest; how many quarts did all gather?

19. A pedestrian walked $65\frac{2}{3}$ miles one day, $75\frac{5}{6}$ miles the next, and $58\frac{1}{2}$ miles the third day; how far did he walk in the three days?

20. A coal merchant sold to one customer 13 T. 15 cwt. of coal, to another $25\frac{7}{10}$ T., and to another 18 T. 12 cwt.; how much did he sell to all?

21. A farmer raised on one field 275 bu. 3 pks. 4 qts. of oats, on a second $461\frac{3}{4}$ bu., and on a third 524 bu. 2 pks. 4 qts.; how much did he raise on the three fields?

22. The four sides of a certain field are $61\frac{1}{4}$ rods, $48\frac{3}{5}$ rods, $59\frac{1}{2}$ rods, and $49\frac{3}{4}$ rods; how many rods of fence will it require to enclose the field?

SUBTRACTION.

265. *1.* Mary earned $\$8\frac{4}{5}$ and paid $\$3\frac{1}{2}$ for a hat; how much remained?

2. John is $5\frac{2}{3}$ feet high and Henry $4\frac{3}{4}$ feet; find the difference in their hight.

3. From a halibut weighing $63\frac{3}{4}$ lb. a fish dealer sold $8\frac{5}{8}$ lb., $7\frac{1}{4}$ lb., $4\frac{1}{2}$ lb. and $12\frac{7}{8}$ lb.; how much remained?

4. From a board $18\frac{1}{4}$ ft. long were cut two pieces, one $5\frac{3}{4}$ ft. long and the other $7\frac{5}{6}$ ft. long; how much remained?

5. After expending $\frac{1}{4}$ of my money and $\frac{1}{5}$ of my money, what part remains?

6. A man left an estate of \$29450, of which his wife received \$11175, his son \$975 less than his wife, and his daughter \$6350 less than his son; the remainder was left to his sister; how much did she receive?

7. A man sold his house for \$13650, his furniture for \$1875, his carpets and pictures for \$3225; if they all cost him \$21380, did he gain or lose, and how much?

8. A man paid \$775 for four horses; for the first \$185, for the second \$225, for the third \$198; how much did the fourth cost him?

9. Before paying two bills, one of \$178.38 and another of \$3725.62, I had \$7365.56 cash in bank; how much remained after I paid the bills?

10. A turkey and a chicken together weighed $27\frac{1}{4}$ lb. and the chicken weighed $8\frac{5}{8}$ lb.; what was the weight of the turkey?

11. A boy gathered $29\frac{3}{4}$ quarts of cherries and sold $12\frac{5}{8}$ quarts to one person and $10\frac{1}{2}$ quarts to another; how many quarts had he left?

12. Henry earns \$$5\frac{3}{4}$ per week, Charles \$$2\frac{9}{10}$ per week more than Henry, and James \$$3\frac{1}{2}$ per week more than Charles; how much can they together save weekly if their expenses are \$$17\frac{3}{8}$?

13. I bought a ton of anthracite coal for \$5.15, $\frac{1}{2}$ a ton of cannel coal for \$4.90, and a cord of wood for 65 cents more than the cost of the anthracite coal; how much change should I receive out of a twenty-dollar bill?

14. A lady went shopping with \$100; she spent \$35.17 for a dress, \$23.47 for a shawl, and \$5.38 for a hat; she then bought a fine rug and received \$19.21 change; how much did the rug cost?

15. The material for two suits having been cut from a piece of cloth $40\frac{2}{3}$ yds. long, $26\frac{1}{2}$ yds. remained; if one suit required $7\frac{3}{4}$ yds., how many yards were in the other suit?

16. A man owes his grocer \$$15.37\frac{1}{2}$; he pays him at one time \$10, then buys from him goods amounting to \$$7.62\frac{1}{2}$, and then gives him enough to pay the entire bill; what is the last payment?

17. A man having agreed to sink a well 40 feet, digs the first week $15\frac{5}{8}$ feet, the second $12\frac{3}{4}$ feet, and the third week finishes all but $5\frac{1}{2}$ feet; how much did he dig the third week?

18. A man bought four pieces of cloth; the first contained 65 yds. 27 in., the second 58.5 yds., the third $81\frac{3}{8}$ yds., and the fourth 75 yds. 9 in.; how many yards in the four pieces?

19. A dairyman made at one time 697 lb. 8 oz. of cheese, at another time $537\frac{5}{8}$ lb., and at a third time he made enough so that the three lots weighed in all 1843.75 lb.; how much did he make the third time?

MULTIPLICATION.

266. *1.* If flour is worth 4 cents per pound, how much must a baker pay for the flour used in making 144 loaves of bread, each loaf requiring $2\frac{5}{8}$ pounds?

2. Find the cost of $17\frac{3}{4}$ cords of wood at $5.50 per cord and 24 tons of coal at $5.25 per ton.

3. A farmer sells 8 loads of hay each of $1\frac{3}{4}$ tons at $17.85 per ton; how much does he receive?

4. A clerk earns $28.60 and spends $11.90 each week; how much can he save in 52 weeks?

5. If a mill is worth $18640, what is the value of $\frac{7}{8}$ of it?

6. A lady bought 18 yards of goods for a dress and used only $\frac{3}{4}$ of it; how many yards did she use?

7. How much would $24\frac{3}{4}$ pounds of ham cost at 18 cents a pound?

8. If 3 pounds of sugar cost 25 cents, how much would a barrel containing 216 pounds cost?

9. Find the cost of $36\frac{7}{8}$ bushels of potatoes at 95 cents per bushel.

10. A butcher sells $4\frac{3}{4}$ pounds of porterhouse steak at 24 cents per pound; how much does it bring?

11. How much must I pay for a turkey weighing $17\frac{3}{4}$ pounds at 23 cents per pound?

12. A man owes $725 and pays $\frac{2}{5}$ of the debt; how many dollars does he still owe?

13. A miller bought 996 bushels of wheat at $1.37 per bushel; he made 49 pounds of flour and 11 pounds of bran from each bushel; by selling the flour at $\$8\frac{3}{4}$ per barrel and the bran at 2 cents per pound, what did he gain, if the barrels cost 25¢ apiece, and each contained 196 pounds?

14. A drover has 875 sheep; he can sell them all to one man at $4.68 a head, or he can sell them in two lots, one lot of 495 at $4.79 each, and the remainder at $4.56; which is the better way, and what is the difference?

15. A and B bought a farm for \$8765, A paying $\frac{3}{8}$ of the price and B the remainder; how much did each pay?

16. A man bought 250 pounds of fish at $8\frac{4}{5}$ cents per pound; $\frac{3}{5}$ of it he sold at 12 cents and the rest at 16 cents per pound; find his gain.

17. A man in building a house employs 16 carpenters for 41 days at \$3.75, 9 plasterers for 13 days at \$3.25, and 8 masons for 13 days at \$4.45; he pays for plumbing \$625.50, for painting \$465.85, and for material \$3725. Find the cost of his house.

18. How much must be paid for 180 barrels of apples, each barrel containing $2\frac{1}{3}$ bushels, at $62\frac{1}{2}$ cents per bushel?

19. A farmer exchanges $18\frac{3}{4}$ bu. of potatoes at 48 cents per bushel for 258 lb. of flour at 4 cts. per pound, and pays the difference in cash; how much cash does he pay?

20. How many yards of cloth will be required to make coats for a company of soldiers consisting of 102 men, if $4\frac{3}{4}$ yards are used for each coat?

21. A man has two boarders each of whom pays him \$$5\frac{3}{4}$ per week, a third who pays \$$6\frac{1}{4}$, and a fourth who pays \$$7\frac{1}{2}$; how much will he receive from them all in $14\frac{2}{7}$ weeks?

22. A grocer buys butter in the country at $18\frac{3}{4}$ cents a pound, pays $\frac{7}{8}$ of a cent a pound freight, and sells it in New York at $32\frac{1}{2}$ cents; what is his profit on 24 tubs each containing $43\frac{3}{4}$ pounds?

23. A vessel holding $2\frac{3}{8}$ gal. is emptied 13 times into a $41\frac{1}{4}$ gallon barrel; what does the barrel lack of being full?

24. A canning company put $2\frac{3}{4}$ pounds of beef in a can, and sold it at 33 cents a can; if the meat cost $8\frac{2}{3}$ cents per pound and the cans $2\frac{1}{2}$ cents each, what was the profit on 100 dozen cans?

25. Mr. Ackerman sells 1000 pounds of cheese that cost him $12\frac{1}{2}$ cents per pound: for 371 lb. 12 oz. he receives 18 cents per pound, for 482 lb. he gets 16 cents per pound, and for the remainder 15 cents per pound; find his gain.

DIVISION.

267. *1.* The area of England is 50922 square miles and in 1880 her population was 25402461; what was the average population per square mile?

2. A merchant bought 375 tons of coal for $1406.25 and wishes to sell it so as to gain by the transaction $206.25; what must he get per ton?

3. A man who has $27850, gives his wife $7860, and establishes his son in business with half the remainder; how much does he give his son?

4. From an estate of $75380 a man leaves to his wife $23750, to his son half as much as to his wife, and the remainder in equal shares to his five daughters; how much does each daughter receive?

5. A house cost me $5375, and the grounds cost $\frac{4}{5}$ as much as the house; find the cost of both.

6. If $3\frac{3}{4}$ dozen eggs cost 75 cents, how much are they per dozen?

7. A butcher charges me 90 cents for $5\frac{5}{8}$ pounds of steak; how much is that per pound?

8. A boy worked 27 days, and received for his work a sum such that, after paying a board bill of $19.90, he had $17.90 left; how much did he receive per day?

9. My family used last year $8\frac{3}{4}$ bbl. of flour, worth $8.80 a barrel; how long must I work at $22 per week to pay for it?

10. If 161 bushels of wheat are raised on $3\frac{5}{6}$ acres, how many bushels grow on each acre?

11. How many widths of carpet $\frac{3}{4}$ of a yard wide will, if placed side by side, reach across a parlor 12 yards wide? How much will it cost at $1.25 per yard if the parlor is 45 ft. long?

12. If $12\frac{3}{4}$ acres of land cost $2040, how much at that rate must be paid for a farm of $145\frac{4}{5}$ acres?

13. How many bushels of wheat at $\$1\frac{3}{5}$ per bushel must be given for 160 barrels of flour at $\$7\frac{3}{4}$ per barrel?

14. A merchant buys a piece of cloth at $\$1\frac{3}{4}$ per yard and sells it at $\$2\frac{5}{8}$; his gain on the entire piece is $49; how many yards does it contain?

15. A drover expended $41787.25 for 168 horses at $162.50 and some cattle at $41.75 a head; how many cattle were there?

16. Two brothers earn in 29 working days $178.35 and one earns $21.75 more than the other; what is the daily wages of each?

17. Sold a quantity of wood that cost $591.25 for $997.50 and found that the gain was $3.25 per cord; how many cords were there?

18. A man exchanges his farm of 185 acres which cost him $245 per acre for a house in the city worth $8450 and 59 city lots; what did the lots cost him apiece?

19. A ten-dollar gold piece weighs 258 grains; find the value of 430 ounces of gold ready for coining, each ounce containing 480 grains.

20. A speculator bought 65 acres of ground at $538 per acre and paid on the purchase $3815. He then sold enough of it as city lots at $465 each to pay the remainder; how many lots did he sell?

21. A chicken that weighs $6\frac{3}{8}$ lb. is sold for $1.02; how much should be charged for one weighing $9\frac{3}{4}$ lb.?

22. A man going a journey of $66\frac{2}{3}$ miles travels the first day 12 hours at $2\frac{5}{8}$ miles per hour, and finishes the journey the second day at the rate of 4 miles per hour; how many hours in all does he travel?

23. A drover paid $4000 for a horse, some sheep and some hogs. The sheep at $\$2\frac{3}{5}$ each cost $650. There were $2\frac{3}{5}$ times as many hogs as sheep, bought at $\$4\frac{3}{4}$ per head. How much did he pay for the horse?

24. A merchant after selling $15\frac{3}{8}$ yards from a piece of silk, cut the remainder into 12 equal dress patterns. These he sold at $\$2\frac{2}{3}$ per yard and received for each $45. How many yards were in the piece at first?

25. With $\frac{3}{4}$ of what a man received for a ship he paid for $318\frac{3}{4}$ acres of land at $180 per acre, and for buildings costing $13125; what amount did he receive for the ship?

26. If $8\frac{3}{4}$ cords of wood cost $70, how many cords can be bought for $100?

27. How many bushels of rye at 88 cents per bushel must be given for 33 hogsheads of molasses, each containing 63 gallons, at 80 cents per gallon?

28. How long will it require 20 men to do a piece of work that 12 men can do in 15 days?

29. A piece of work is done by 15 men, each of whom receives $23.40; if 9 men had done the work, what would have been each man's share?

30. A man, out of his pay for 26 days' labor, gave his grocer $13.75 and his butcher $9.85; he then bought $2\frac{3}{5}$ tons of coal at $4.75 per ton, and $5\frac{1}{2}$ bushels of potatoes at $1.20 per bushel, after which there remained of his pay $22.45; what did he receive per day?

31. A church having 327 members owes a debt of $2356.25. Two wealthy members of the church agree to pay $\frac{2}{5}$ of the debt if the other members will contribute equally to pay the remainder. How much must be paid by each of the others in order to cancel the debt?

32. The buildings on a certain farm cost $\frac{3}{8}$ as much as the land. The farm consisted of 168 acres bought at $95.50 per acre; what did the buildings cost?

33. An excursion train requires $3\frac{7}{15}$ hours to make the run from New York to Mauch Chunk, a distance of 104 miles; what is the rate per hour?

34. A and B start from the same place at the same time to travel in opposite directions, A at the rate of $2\frac{7}{12}$ miles per hour, and B at the rate of $3\frac{2}{3}$ miles per hour; in how many hours will they be 100 miles apart?

PROPERTIES OF NUMBERS.

DEFINITIONS.

268. An **exact divisor** of a number is a divisor that is contained in it without a remainder.

Thus, 3 is an exact divisor of 6; 5 of 15; and 7 of 14.

269. A **prime number** is a number that has no exact divisors except itself and 1.

Thus, 1, 2, 3, 5, 7, 11, 13, are prime numbers.

270. A **composite number** is one that has two or more exact divisors beside itself and 1.

Thus, 6, 15, 18, 21, are composite numbers.

271. An **even number** is a number that is exactly divisible by 2.

Thus, 2, 4, 6, 8, 10, 12, 24, are even numbers.

272. An **odd number** is a number that is not exactly divisible by 2.

Thus, 1, 3, 5, 7, 9, 11, 13, 27, are odd numbers.

273. The **factors** of a number are two or more numbers which, if multiplied together, will produce it.

Thus, 7 and 9 are factors of 63; 2, 3 and 5, of 30.

274. The product obtained by using the same number two or more times as a factor is called a **power**.

Thus, $3 \times 3 \times 3$ gives 27 as a product or power.

275. A small figure written above and at the right of a number indicates how many times the number is used as a factor, and is called an **exponent**.

Thus, 2^3 means $2 \times 2 \times 2$, and the number of times 2 is used as a factor is indicated by the exponent 3.

276. The *power* obtained by using a number *twice* as a factor is called the **second power, or square**; by using a number *three* times as a factor, the **third power, or cube**; *four* times as a factor, the **fourth power,** etc.

Thus, 9 or 3^2 is the second power of 3, 27 or 3^3 the third power of 3, 81 or 3^4 the fourth power of 3, etc.

FACTORING.

1. What prime numbers multiplied together will give 6? 9? 14? 36? 42?

2. What prime numbers will exactly divide 6? 9? 14? 36? 42?

277. A **prime factor** is a prime number used as a factor.

Thus, 3, 5, 7, etc., when used as factors, are prime factors.

3. What are the prime factors of 6? Of 9? Of 14? Of 36? Of 42?

278. Principles.—I. *Every prime number that is an exact divisor of a number, is a prime factor of that number.*

II. *The prime factors of a number, or the product of two or more of them, are the only exact divisors of that number.*

III. *The continued product of all the prime factors of a number equals the number.*

279. *Give the prime factors of the following numbers:*

1. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24.
2. 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42.
3. 44, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 60, 63, 64.

280. A **composite factor** is a composite number used as a factor.

Thus, 4, 6, 8, 9, etc., when used as factors, are composite factors.

281. Two numbers, one or both of which may be composite, are said to be **prime to each other**, or **mutually prime**, when no number greater than 1 will exactly divide them both.

Thus, 16 and 35 are mutually prime.

282. The following facts, if memorized, will greatly assist in determining the factors of a number.

Any number is exactly divisible,

1. By 2, when the last figure is 0, 2, 4, 6, or 8.
2. By 3, when 3 is exactly contained in the sum of its digits.
3. By 4, when the number expressed by the two right-hand figures is exactly divisible by 4.
4. By 5, when the last figure is 0 or 5.
5. By 6, when it is divisible by both 3 and 2.
6. By 8, when the number expressed by the three right-hand figures is divisible by 8.
7. By 9, when 9 is exactly contained in the sum of its digits.
8. By 10, when the last figure is 0.

283. To find the prime factors of any composite number.

I. Required the prime factors of 420.

Process.

$$\begin{array}{r|l} 2 & 420 \\ \hline 2 & 210 \\ \hline 5 & 105 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Explanation.—The prime numbers which are exact divisors of 420 will be the prime factors. 2 is an exact divisor, because the last figure of the number is 0. The last figure of the quotient is 0 and 2 is again selected for a divisor. 5 is selected for the third divisor, because the quotient to be divided ends in 5. 21 is divisible by 3, because the sum of its digits is divisible by 3. The last quotient, 7, is prime, and is divisible only by itself.

The prime factors are, therefore, 2, 2, 5, 3, 7.

284. Rule.—*Divide the given number by any prime number which is an exact divisor. Divide the quotient by the same or another prime number, and continue the process until the quotient is a prime number. The several divisors and the last quotient will be the prime factors.*

285. ***Find the prime factors of:***

1.	75.	*6.*	150.	*11.*	256.	*16.*	1280.
2.	90.	*7.*	160.	*12.*	280.	*17.*	1872.
3.	108.	*8.*	189.	*13.*	315.	*18.*	3204.
4.	132.	*9.*	216.	*14.*	316.	*19.*	3980.
5.	144.	*10.*	240.	*15.*	484.	*20.*	7007.

CANCELLATION.

1. What is the quotient of 24 divided by 12? 3 and 8 are factors of 24, and 3 and 4 are factors of 12. What factor is common to both dividend and divisor? Rejecting this factor and dividing 8 by 4 gives what for a quotient? Has the value of the quotient been changed by rejecting a factor common to both dividend and divisor?

286. Rejecting factors common to both dividend and divisor is called **cancellation.**

287. Principle.—*Rejecting factors common to both dividend and divisor does not change the value of the quotient.*

288. To shorten the process of division when the dividend and divisor contain common factors.

I. Required to divide the product of 5, 7, and 13 by the product of 3, 5, and 7.

Process.

$$\frac{\cancel{5}\times\cancel{7}\times 13}{3\times\cancel{5}\times\cancel{7}}=\frac{13}{3}=4\tfrac{1}{3}$$

Explanation.—The division is indicated by writing the dividend above a line and the divisor below it. The common factors 5 and 7 are canceled; and 13, the remaining factor in the dividend, divided by 3, the remaining factor in the divisor, gives $4\frac{1}{3}$ for the quotient.

If the products of $5\times7\times13$ and of $3\times5\times7$ were actually found, and the first divided by the second, the result would be the same, but the process would be much longer.

II. Required to divide 35×39 by 13×14.

Explanation.—The dividend and divisor are written as in the preceding example.

Process.

$$\frac{\overset{5}{\cancel{35}} \times \overset{3}{\cancel{39}}}{\cancel{13} \times \underset{2}{\cancel{14}}} = \frac{15}{2} = 7\tfrac{1}{2}$$

The factor 13 is rejected from the divisor and from the factor 39 of the dividend, leaving the factor 3 in the dividend instead of the factor 39.

The factor 7 is rejected from the 35 of the dividend and from the 14 of the divisor, leaving the factors 5 and 2.

15, the product of the remaining factors of the dividend, divided by 2, the remaining factor of the divisor, gives $7\frac{1}{2}$ for the quotient.

289. ***Find the quotient of:***

1. $\dfrac{15 \times 14 \times 9 \times 22}{3 \times 11 \times 5 \times 7}$.

2. $\dfrac{18 \times 26 \times 33 \times 28}{11 \times 14 \times 9 \times 13}$.

3. $\dfrac{25 \times 32 \times 36 \times 42}{7 \times 5 \times 8 \times 9}$.

4. $\dfrac{72 \times 18 \times 35 \times 15}{14 \times 27 \times 12 \times 10}$.

5. $\dfrac{18 \times 80 \times 25 \times 49}{10 \times 30 \times 35 \times 24}$.

6. $\dfrac{72 \times 56 \times 54 \times 77}{33 \times 63 \times 24 \times 42}$.

7. $\dfrac{60 \times 45 \times 84 \times 55}{54 \times 35 \times 36 \times 22}$.

8. $\dfrac{88 \times 65 \times 51 \times 42}{34 \times 28 \times 33 \times 39}$.

9. $\dfrac{144 \times 121 \times 81 \times 4}{108 \times 132 \times 63 \times 33}$.

10. $\dfrac{78 \times 70 \times 68 \times 84}{15 \times 52 \times 51 \times 56}$.

11. $125 \times 45 \times 95 \times 85$ divided by $30 \times 75 \times 38 \times 51$.

12. $100 \times 95 \times 65 \times 36 \times 84$ divided by $48 \times 39 \times 76 \times 24 \times 80$.

13. $120 \times 108 \times 100 \times 121$ divided by $66 \times 32 \times 75 \times 55$.

14. $175 \times 84 \times 100 \times 117$ divided by $75 \times 125 \times 91 \times 60$.

15. $144 \times 216 \times 96 \times 143$ divided by $180 \times 117 \times 156 \times 84$.

16. $130 \times 140 \times 150 \times 160$ divided by $48 \times 28 \times 26 \times 75$.

17. $78 \times 130 \times 140 \times 250$ divided by $65 \times 125 \times 84 \times 52$.

18. $72 \times 112 \times 168 \times 144 \times 121$ divided by $88 \times 56 \times 126 \times 96 \times 99$.

19. $230 \times 250 \times 300 \times 310 \times 540$ divided by $60 \times 75 \times 125 \times 69 \times 93$.

CONCRETE PROBLEMS.

290. *1.* A merchant sold 20 hogsheads of olive oil, each containing 63 gallons, at $1.75 per gallon, and invested the money in cases of table sauce, each containing 12 bottles, at $1.25 per bottle; how many cases did he buy?

Explanation. — By the ordinary method the amount received for the olive oil sold would be found by multiplying $1.75 by 63, to find the amount received for one hogshead, and then multiplying that product by 20, the number of hogsheads, to find the entire proceeds of the sale.

Process.

$$\frac{\overset{7}{\cancel{\$1.75}} \times \overset{21}{\cancel{63}} \times \overset{4}{\cancel{20}}}{\underset{5}{\cancel{\$1.25}} \times \underset{4}{\cancel{12}}} = 147$$

The cost of one case of table sauce would then be found, and the total amount to be expended for table sauce, divided by it, would give the number of cases.

Instead of this long process, however, these multiplications and the division are simply indicated, and cancellation renders the computation very brief.

2. How many bushels of chestnuts at 15 cents per quart should be given in exchange for 24 barrels of apples, each containing 3 bushels, at $1.25 per bushel?

3. A dairyman sold 24 tubs of butter, each containing 45 lb., at $37\frac{1}{2}$¢ per lb., and with the money bought 60 pieces of muslin at $12\frac{1}{2}$¢ a yard; how many yards in each piece?

4. A father works 26 days each month for 9 months at $2.25 per day, and earns the same sum that his son earns in 13 months, working 27 days each month; how much does the son receive per day?

5. How many barrels, each containing 60 dozen of eggs, at 25 cents per dozen, are worth as much as 360 lb. of butter at $37\frac{1}{2}$ cents per pound?

6. How many crates of plums, each containing 2 bushels, will bring the same money when sold at 15 cents per quart that 16 barrels of apples, each containing 3 bushels, will bring when sold at 60 cents per peck?

7. How many men working 6 hours per day for 21 days, can do as much work as 28 men working 8 hours per day for 9 days?

8. How many boxes of crackers, containing 15 lb. each, at 12¢ a pound, will cost the same as 20 boxes of starch, each containing 8 lb., at 18¢ a pound?

9. A speculator sold 15 carloads of potatoes, of 342 bushels each, at $1.25 per bushel, and expended the money for 10 carloads of peaches at $2.25 per crate; how many crates in each carload?

GREATEST COMMON DIVISOR.

291. **A common divisor** of two or more numbers is a number that will exactly divide each of them.

Thus, 3 is a common divisor of 12 and 18.

292. The **greatest common divisor** (G. C. D.) of two or more numbers is the greatest number that will exactly divide each of them.

Thus, 6 is the greatest common divisor of 12 and 18.

The greatest common divisor is sometimes called the **greatest common measure.**

1. What are the factors of 6? Of 9? What number is a factor in both 6 and 9? What number will exactly divide both 6 and 9?

2. What are the prime factors of 12? Of 18? What prime factors are common to both numbers? What is the greatest number that will divide both 12 and 18? What are the factors of 6?

293. **Principles.**—I. *Any factor common to two or more numbers is a common divisor of those numbers.*

II. *The product of all the prime factors common to two or more numbers is the greatest common divisor of those numbers.*

294. To find the greatest common divisor of two or more numbers, when the numbers are readily factored.

I. Required the G. C. D. of 12, 18, and 30.

Explanation.—The numbers are all exactly divisible by 2, therefore 2 is a factor of the greatest common divisor. The quotients obtained by dividing the numbers by 2 are all divisible by 3, hence 3 is a factor in the greatest common divisor. There is no number that will exactly divide each of the last set of quotients, and, hence, there is no other factor common to all the numbers. Since 2 and 3 are the only exact divisors of the given numbers, their product is the greatest common divisor.

Process.

$$\begin{array}{r|rrr} 2 & 12, & 18, & 30 \\ \hline 3 & 6, & 9, & 15 \\ \hline & 2, & 3, & 5 \end{array}$$

$$2 \times 3 = 6 = \text{G.C.D.}$$

295. Rule.—*Divide the numbers by any number that is an exact divisor of all of them, divide the quotients by any exact divisor of all of them, and so continue until the quotients are not all exactly divisible by the same number. The continued product of all the exact divisors is the greatest common divisor.* (See Note.)

296. When the numbers are not readily factored, their greatest common divisor may be found as follows:

I. Required the G. C. D. of 437 and 943.

Explanation.—The G. C. D. of 437 and 943 is found by dividing 943 by 437, then dividing the first divisor 437 by the first remainder 69, and then dividing the second divisor 69 by the second remainder 23. This last division gives no remainder, and 23 is the G. C. D. required.

Process.

$$\begin{array}{l} 437\ )\ 943\ (\ 2 \\ \quad\quad\ \ \underline{874} \\ \quad\quad\quad 69\ )\ 437\ (\ 6 \\ \quad\quad\quad\quad\quad\ \underline{414} \\ \quad\quad\quad\quad\quad\ 23\ )\ 69\ (\ 3 \\ \quad\quad\quad\quad\quad\quad\quad\ \underline{69} \end{array}$$

In the case of more than two numbers, find the greatest common divisor of two of them, then of this divisor and another of the given numbers, and so on. The last divisor will be the greatest common divisor. For a full explanation of this process, see Appendix, page 321.

NOTE.—If only one such divisor can be found, that number is the *greatest common* divisor.

297. ***Find the greatest common divisor of:***

1.	18, 24, 30.	*8.*	105, 140, 245, 385.
2.	30, 45, 75.	*9.*	112, 176, 208, 240.
3.	27, 45, 63.	*10.*	117, 156, 195, 273.
4.	56, 70, 98.	*11.*	165, 225, 300, 450.
5.	63, 84, 105.	*12.*	108, 198, 234, 180.
6.	48, 72, 216.	*13.*	240, 204, 156, 276.
7.	81, 135, 216.	*14.*	144, 192, 264, 360.

15.	437, 897.	*18.*	899, 1363.	*21.*	1273, 2077, 4087.
16.	851, 1739.	*19.*	667, 1943.	*22.*	2279, 3233, 4399.
17.	1457, 1643.	*20.*	2001, 3915.	*23.*	1827, 3906, 4599.

CONCRETE PROBLEMS.

298. *1.* What is the largest whole number of dollars per head that a drover can pay for horses, and how many in all will he buy, provided he expends at one time $12549 and at another $14523, the price on both occasions being the same?

2. A man has three cisterns holding respectively 90 gallons, 138 gallons and 168 gallons; what is the capacity of the largest vessel which will exactly fill them if emptied into each an exact number of times?

3. A man has a field whose sides are 225 feet, 300 feet, 195 feet and 255 feet; what is the greatest length of rail contained an exact number of times in the length of each side?

4. A farmer has 318 bushels of wheat, 225 of rye, 450 of oats and 291 of barley; he wishes to put his grain into the largest bags of equal size that will exactly hold each kind; how many bags will he require?

5. How many boards will it take to build a fence around a piece of ground 208 feet long and 143 feet wide, the fence to be 5 boards high, and the boards of equal length and the longest possible?

LEAST COMMON MULTIPLE.

299. A multiple of a number is a number that is exactly divisible by it.

Thus, 12 is a multiple of 6; 15 of 5; 18 of 9.

300. A **common multiple** of two or more numbers is a number exactly divisible by each of them.

Thus, 12 is a common multiple of 2, 3 and 6.

301. The **least common multiple** (L. C. M.) of two or more numbers is the least number that is exactly divisible by each of them.

Thus, 18 is the least common multiple of 6 and 9.

1. What are the factors of 15? Of 20? $15 = 3 \times 5$, and $20 = 2 \times 2 \times 5$. Must a multiple of 15 contain the factors 3 and 5? Must a multiple of 20 contain the factors 2, 2, and 5? Would a number that contained all the factors of both numbers be a common multiple of the numbers? Would it be the least common multiple if it contained only the factors found in the numbers?

2. Will the product of $3 \times 2 \times 2 \times 5$ contain all the factors in 15 and 20? What is the product? Is 60 a common multiple of 15 and 20? What factor is common to both numbers? In which number does it occur the greater number of times? How many times is it used as a factor in 60? Does 60 contain any prime factor not found in 15 or 20? Is 60 the least common multiple of 15 and 20?

302. Principle.—I. *The least common multiple of two or more numbers contains the prime factors of the numbers, and no other factors.*

II. *Each prime factor occurs in the least common multiple as many times as it occurs in that one of the given numbers that contains it the greatest number of times.*

303. To find the least common multiple of two or more numbers.

I. Required the L. C. M. of 12, 30, and 45.

Process.

$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$45 = 3 \times 3 \times 5$$
$$2^2 \times 3^2 \times 5 = 180 = \text{L. C. M.}$$

Explanation. — The prime factors of 12 are 2, 2, and 3; of 30, 2, 3, and 5; and of 45, 3, 3, and 5.

The factor 2 is used twice in one of the numbers, hence it must be taken twice in the least common multiple. The same is true of the factor 3. The factor 5 occurs but once in any of the numbers and therefore is used only once in the least common multiple. The product of $2 \times 3 \times 5 \times 2 \times 3$ is 180, and since it contains all of the factors in 12, 30, and 45, it is a multiple of each of these numbers. No factor has been used that is not found in 12, 30, or 15, and therefore 180 is the least common multiple of those numbers.

304. Rule.—*Resolve the numbers into their prime factors; find the continued product of the different factors, taking each factor the greatest number of times it is found in any of the numbers; the result will be the least common multiple of the numbers.*

305. The following method of finding the least common multiple of two or more numbers is in common use.

I. Required the L. C. M. of the same numbers, 12, 30, and 45.

Process.

$$\begin{array}{r|rrr} 2 & 12 & 30 & 45 \\ \hline 3 & 6 & 15 & 45 \\ \hline 5 & 2 & 5 & 15 \\ \hline & 2 & 1 & 3 \end{array}$$

$$2^2 \times 3^2 \times 5 = 180 = \text{L.C.M.}$$

Explanation.—The numbers are written in a horizontal line, and a prime number that will divide two or more of them selected for a divisor. For the second divisor, another prime number is selected that will divide two or more of the quotients and undivided numbers. This process is continued till the quotients and undivided numbers are prime to each other. The continued product of the several divisors, the last quotients and the undivided numbers is the least common multiple.

306. ***Find the least common multiple of:***

1. 12 and 30.
2. 35 and 21.
3. 30, 42, and 70.
4. 28, 42, and 60.
5. 30, 45, and 75.
6. 36, 45, and 75.
7. 45, 72, 180.
8. 54, 90, 135.
9. 75, 125, 150.
10. 24, 32, 36, 45.
11. 36, 45, 48, 63.
12. 27, 35, 63, 81.
13. 72, 84, 108, 144.
14. 63, 105, 147, 231.

CONCRETE PROBLEMS.

307. *1.* What is the shortest piece of calico that can be cut into pieces of 8 yards, 10 yards, or 12 yards in length, and nothing remain?

2. What is the least sum of money that will exactly pay for a number of cows at $36 each, hogs at $15, or oxen at $48, and how many of either will it purchase?

3. Three steamers leave New York City the same day; the first returns every two weeks and leaves again the same day; the second returns every three weeks and leaves in like manner the same day, the third returns every 35 days. In how many days will they all be in New York together?

4. How many acres in the smallest farm that can be divided into an exact number of lots containing either 12 acres, 15 acres, 24 acres, or 32 acres each?

5. The fore wheel of a wagon is 132 inches in circumference and the hind wheel 154 inches; how far from a given point must the wagon move in order that the spokes of its wheels shall be in the same relative position as at starting?

6. What is the least debt that can be exactly discharged in payments of $75, of $125, or of $175?

7. A barrel of flour will last the first of three families 24 days, the second 40 days, and the third 64 days; they each begin a barrel of flour on the same day; how soon will the three families again begin a barrel on the same day?

COMMON FRACTIONS.

DEFINITIONS.

The lessons on Equal Parts of Units, pages 53 to 56, should be reviewed before this section is studied.

308. A **fractional unit** is one of the equal parts of an integral unit.

Thus, $\frac{1}{2}$ of a foot, $\frac{1}{8}$ of a pound, $\frac{1}{12}$, are fractional units.

309. A **fraction** is a fractional unit or a collection of fractional units.

Thus, $\frac{1}{4}$ of a dollar, $\frac{3}{4}$ of an apple, $\frac{5}{8}$, are fractions.

310. A fraction is expressed by writing below a short horizontal line the number of parts into which the unit is divided, and above the line the number of parts taken.

Thus, $\frac{3}{5}$ expresses 3 of the 5 equal parts into which the unit is divided.

311. The **denominator** of a fraction is the number below the line; the **numerator**, is the number above the line; and the denominator and numerator together are the **terms of a fraction.**

312. The **value of a fraction** depends upon the relative value of its numerator and denominator.

Thus, when the numerator and denominator are equal, as in $\frac{4}{4}$, the value of the fraction is **one**; when the numerator is less than the denominator, as in $\frac{3}{4}$, the value is **less than one**; and when the numerator is greater than the denominator, as in $\frac{5}{4}$, the value is **greater than one.**

313. A **proper fraction** is a fraction whose numerator is less than its denominator.

314. An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator.

PRINCIPLES OF FRACTIONS.

315. I. *Multiplying the numerator of a fraction by any number multiplies the value of the fraction by that number.*

Explanation.—The numerator indicates the number of equal parts taken. Multiplying it by 2 multiplies the number of parts taken by 2. Hence, the value of the fraction is multiplied by 2.

Illustration.
$\frac{3 \times 2}{12} = \frac{6}{12}$

II. *Dividing the denominator of a fraction by any number multiplies the value of the fraction by that number.*

Explanation.—The denominator of a fraction indicates the number of parts into which the unit is divided. If the denominator is divided by 2 the unit will be divided into only one half as many parts, and the parts will therefore be twice as large. Hence, the number of parts taken remaining the same, their value, or the value of the fraction, is multiplied by 2.

Illustration.
$\frac{3}{12 \div 2} = \frac{3}{6}$

III. *Dividing the numerator of a fraction by any number divides the value of the fraction by that number.*

Explanation.—The numerator indicates the number of parts taken. If the numerator is divided by 3, only one third as many parts will be taken. Dividing the numerator by 3, therefore, divides the value of the fraction by 3.

Illustration.
$\frac{9 \div 3}{12} = \frac{3}{12}$

IV. *Multiplying the denominator of a fraction by any number divides the value of the fraction by that number.*

Explanation.—The denominator indicates the number of parts into which the unit is divided. Multiplying it by 3 multiplies by 3 the number of parts into which the unit is divided, and they will therefore be only one third as large as before. Since the number of parts taken remains the same, and each part is only one third as large, multiplying the denominator by 3 divides the value of the fraction by 3.

Illustration.
$\frac{3}{4 \times 3} = \frac{3}{12}$

V. *Multiplying or dividing both terms of a fraction by the same number does not change its value.*

Explanation.—1. Multiplying the numerator by 2 multiplies the value of the fraction by 2; multiplying the denominator of the result by 2 divides its value by 2. To multiply by 2 and divide the result by 2 evidently does not change the value.

Illustration.

$\frac{3}{4}^{\times 2} = \frac{6}{4}$; $\frac{6}{4_{\times 2}} = \frac{6}{8}$

2. As before, multiplying the value of the fraction by 2, and dividing the value of the result by 2, changes the form, but not the value, of the fraction.

Illustration.

$\frac{6}{8_{\div 2}} = \frac{6}{4}$; $\frac{6}{4}^{\div 2} = \frac{3}{4}$

316. General Principle.—*The value of a fraction is multiplied by multiplying its numerator or by dividing its denominator; it is divided by dividing its numerator or multiplying its denominator; and the form but not the value of a fraction is changed by multiplying or dividing both terms by the same number.*

ORAL EXERCISES.

317. *1.* What is the effect of multiplying the numerator of $\frac{3}{4}$ by 2? Of dividing the denominator by 2?

2. In what two forms may the product of $\frac{5}{16}$ multiplied by 2 be expressed? Which is the simpler form? Why?

3. Which is the better process of multiplying $\frac{5}{18}$ by 3? By 6? By 9? By 5? By 7?

4. What is the effect of dividing the numerator of $\frac{4}{5}$ by 2? Of multiplying the denominator by 2?

5. In what two forms may the quotient of $\frac{6}{7}$ divided by 3 be expressed? Which is the simpler form? Why?

6. If the numerator of $\frac{3}{4}$ be multiplied by 3, what other operation is necessary that the value of the fraction may not be changed?

7. If the numerator of $\frac{6}{12}$ be divided by 3, what is necessary that the value of the fraction may not be changed?

REDUCTION OF FRACTIONS.

318. Reduction of fractions consists in changing their forms without changing their values.

319. A fraction is in its **simplest form** when its numerator and denominator are prime to each other.

320. To reduce fractions to their simplest form.

I. Required to reduce $\frac{48}{72}$ to an equivalent fraction having the smallest possible denominator.

Explanation.—Dividing both terms of a fraction by the same number does not alter its value. Dividing both terms of $\frac{48}{72}$ by 6 gives $\frac{8}{12}$. Dividing both terms of $\frac{8}{12}$ by 4 gives $\frac{2}{3}$. The numerator and denominator of $\frac{2}{3}$ are prime to each other, and the fraction is, therefore, in its simplest form.

Process.

$$\frac{48}{72} = \frac{8}{12} = \frac{2}{3}$$

The same result might have been reached by dividing both terms at once by 24, their *greatest common divisor.*

321. Rule.—I. *Divide the terms of the fraction by any exact divisor of both; divide the terms of the result in like manner; and so continue until a fraction is obtained whose numerator and denominator are prime to each other.* Or,

II. *Divide the terms of the fraction by their greatest common divisor.*

322. ***Reduce the following fractions to their simplest form:***

1. $\frac{36}{60}$.	*9.* $\frac{96}{240}$.	*17.* $\frac{320}{1728}$.	*25.* $\frac{917}{1441}$.
2. $\frac{45}{75}$.	*10.* $\frac{144}{156}$.	*18.* $\frac{1584}{1728}$.	*26.* $\frac{855}{1368}$.
3. $\frac{56}{84}$.	*11.* $\frac{120}{192}$.	*19.* $\frac{728}{1144}$.	*27.* $\frac{3456}{4096}$.
4. $\frac{75}{125}$.	*12.* $\frac{275}{875}$.	*20.* $\frac{385}{693}$.	*28.* $\frac{1975}{3275}$.
5. $\frac{108}{144}$.	*13.* $\frac{192}{512}$.	*21.* $\frac{204}{357}$.	*29.* $\frac{6561}{8748}$.
6. $\frac{125}{175}$.	*14.* $\frac{195}{455}$.	*22.* $\frac{475}{570}$.	*30.* $\frac{1944}{3240}$.
7. $\frac{144}{192}$.	*15.* $\frac{65}{91}$.	*23.* $\frac{345}{552}$.	*31.* $\frac{7344}{11016}$.
8. $\frac{128}{160}$.	*16.* $\frac{121}{143}$.	*24.* $\frac{2025}{3025}$.	*32.* $\frac{4104}{6156}$.

323. To reduce a mixed number to an improper fraction.

I. Required to reduce $5\frac{3}{4}$ to an improper fraction.

Explanation.—In 1 there are 4 fourths; in 5 there are 5 times 4 fourths, or 20 fourths, which is written $\frac{20}{4}$. Adding the $\frac{3}{4}$ to $\frac{20}{4}$ gives $\frac{23}{4}$, the number of fourths in $5\frac{3}{4}$.

Process.

$$5\tfrac{3}{4} = \tfrac{23}{4}$$
$$5 = \tfrac{4}{4} \times 5 = \tfrac{20}{4}$$
$$\tfrac{20}{4} + \tfrac{3}{4} = \tfrac{23}{4}$$

324. Rule.—*Multiply the integer by the denominator of the fraction, add the product to the numerator, and write the sum over the denominator.*

325. ***Reduce the following mixed numbers to improper fractions:***

1. $8\frac{2}{3}$.	*8.* $103\frac{4}{7}$.	*15.* $2345\frac{1}{2}$.	*22.* $4\frac{127}{576}$.
2. $9\frac{3}{5}$.	*9.* $463\frac{3}{4}$.	*16.* $3762\frac{3}{4}$.	*23.* $15\frac{11}{125}$.
3. $17\frac{5}{8}$.	*10.* $229\frac{7}{11}$.	*17.* $4695\frac{2}{3}$.	*24.* $73\frac{893}{1729}$.
4. $23\frac{5}{6}$.	*11.* $83\frac{16}{25}$.	*18.* $831\frac{19}{20}$.	*25.* $6\frac{173}{2025}$.
5. $32\frac{7}{9}$.	*12.* $29\frac{19}{36}$.	*19.* $783\frac{17}{25}$.	*26.* $18\frac{43}{5280}$
6. $45\frac{9}{10}$.	*13.* $87\frac{17}{28}$.	*20.* $149\frac{13}{36}$.	*27.* $357\frac{31}{100}$.
7. $21\frac{11}{12}$.	*14.* $189\frac{11}{15}$.	*21.* $98\frac{17}{45}$.	*28.* $85\frac{179}{1000}$.

326. To reduce an improper fraction to a whole or mixed number.

I. Required to reduce $\frac{1240}{16}$ to a mixed number.

Explanation.—In 1 unit there are $\frac{16}{16}$; to find the number of units in $\frac{1240}{16}$, the numerator, 1240, is divided by 16, the denominator. $\frac{8}{16}$, the fractional part of the quotient, is reduced to its simplest form, and the final result is $77\frac{1}{2}$.

Process.

$$\tfrac{1240}{16} = 77\tfrac{1}{2}$$
$$1240 \div 16 = 77\tfrac{8}{16}$$
$$77\tfrac{8}{16} = 77\tfrac{1}{2}$$

327. Rule.—*Divide the numerator of the fraction by its denominator, and reduce the fractional part of the quotient, if any, to its simplest form.*

328. ***Reduce to whole or mixed numbers the following improper fractions:***

1.	$\frac{47}{3}$.	*5.*	$\frac{864}{8}$.	*9.*	$\frac{2025}{25}$.	*13.*	$\frac{7688}{18}$.
2.	$\frac{83}{5}$.	*6.*	$\frac{202}{12}$.	*10.*	$\frac{1561}{13}$.	*14.*	$\frac{1932}{16}$.
3.	$\frac{192}{7}$.	*7.*	$\frac{1728}{15}$.	*11.*	$\frac{9328}{11}$.	*15.*	$\frac{4384}{96}$.
4.	$\frac{835}{4}$.	*8.*	$\frac{9327}{9}$.	*12.*	$\frac{8349}{24}$.	*16.*	$\frac{18365}{25}$.

329. To reduce a fraction to an equivalent fraction having a given denominator.

I. Required to reduce $\frac{2}{3}$ to an equivalent fraction whose denominator is 81.

Explanation.—Both the numerator and the denominator must be multiplied by the same number that the value of the fraction be not changed, and the number selected must, in multiplying, give 81 for the new denominator. To find this number, 81 is divided by 3, giving 27 for a quotient. Multiplying both terms of the fraction by 27 gives $\frac{54}{81}$ as the required fraction.

Process.

$$\frac{2}{3} = \frac{54}{81}$$

$$81 \div 3 = 27$$

$$\frac{2 \times 27}{3 \times 27} = \frac{54}{81}$$

330. Rule.—*Divide the required denominator by the denominator of the given fraction and multiply both terms of the fraction by the quotient.*

331. ***Perform the following reductions:***

1.	$\frac{3}{4}$ to twelfths.	*6.*	$\frac{6}{7}$ to sixty-thirds.
2.	$\frac{4}{5}$ to thirtieths.	*7.*	$\frac{5}{12}$ to eighty-fourths.
3.	$\frac{5}{8}$ to fortieths.	*8.*	$\frac{7}{4}$ to twenty-eighths.
4.	$\frac{2}{3}$ to twenty-fourths.	*9.*	$\frac{5}{3}$ to forty-eighths.
5.	$\frac{7}{9}$ to thirty-sixths.	*10.*	$\frac{11}{15}$ to 135ths.

If the quotient obtained by dividing the required denominator by the denominator of the given fraction is a mixed number, the numerator of the required fraction will also be a mixed number.

Thus, to reduce $\frac{3}{4}$ to 15ths; $15 \div 4 = 3\frac{3}{4}$; $\frac{3 \times 3\frac{3}{4}}{4 \times 3\frac{3}{4}} = \frac{11\frac{1}{4}}{15}$

11.	$\frac{2}{3}$ to twentieths.	*13.*	$\frac{3}{4}$ to tenths.
12.	$\frac{5}{8}$ to twelfths.	*14.*	$\frac{5}{6}$ to 32ds.

332. To reduce an integer to an equivalent fraction having a given denominator.

I. Required to reduce 8 to fifths.

Explanation.—In 1 there are 5 fifths, and in 8 there are 8 times 5 fifths or 40 fifths, which is written $\frac{40}{5}$.

Process.

$$8 = \frac{5}{5} \times 8 = \frac{40}{5}$$

333. Rule.—*Multiply the integer by the given denominator and write the result as the numerator of a fraction having the given denominator.*

334. To reduce a mixed number to an equivalent fraction having a given denominator.

I. Required to reduce $8\frac{3}{4}$ to twentieths.

Explanation.—Reducing $\frac{3}{4}$ to twentieths gives $\frac{15}{20}$. Reducing 8 to twentieths gives $\frac{160}{20}$. Adding the results, $\frac{15}{20}$ and $\frac{160}{20}$, gives $\frac{175}{20}$, a fraction having the required denominator.

Process.

$$20 \div 4 = 5$$

$$\begin{aligned} \tfrac{3}{4} &= \tfrac{3 \times 5}{4 \times 5} &&= \tfrac{15}{20} \\ 8 &= \tfrac{20}{20} \times 8 &&= \tfrac{160}{20} \\ \hline 8\tfrac{3}{4} & &&= \tfrac{175}{20} \end{aligned}$$

335. Rule.—*Reduce the integral and the fractional parts to fractions having the required denominator, and unite the results.*

336. ***Perform the following reductions:***

1. 15 to thirds.
2. 23 to fifths.
3. 13 to eighths.
4. 19 to sevenths.
5. 32 to fourths.
6. $9\frac{2}{3}$ to twelfths.
7. $5\frac{3}{5}$ to twentieths.
8. $8\frac{4}{7}$ to twenty-firsts.
9. $8\frac{3}{5}$ to fifteenths.
10. $7\frac{5}{8}$ to fortieths.
11. $27\frac{5}{6}$ to thirtieths.
12. $18\frac{3}{4}$ to thirty-sixths.
13. $37\frac{4}{5}$ to sixtieths.
14. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to 12ths.
15. $\frac{3}{7}$, $\frac{4}{9}$, $2\frac{4}{21}$ to 63ds.
16. $\frac{5}{8}$, $2\frac{1}{12}$, $5\frac{3}{16}$ to 48ths.
17. $\frac{4}{5}$, $\frac{9}{10}$, $\frac{11}{12}$ to 60ths.
18. $2\frac{2}{3}$, $3\frac{3}{4}$, $5\frac{9}{10}$ to 60ths.
19. $8\frac{5}{7}$, $6\frac{5}{6}$, $7\frac{11}{14}$ to 42ds.
20. $18\frac{5}{9}$, $17\frac{2}{3}$, $19\frac{3}{5}$ to 45ths.

ADDITION OF FRACTIONS.

337. Fractions in order to be added must have the same fractional unit.

338. To find the sum of two or more fractions.

I. Required to find the sum of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.

Process.

$$\frac{2}{3}+\frac{3}{4}+\frac{5}{6} = 2\frac{1}{4}$$

$$\begin{array}{r|rrr} 3 & 3 & 4 & 6 \\ \hline 2 & 1 & 4 & 2 \\ \hline & & 2 & 1 \end{array}$$

$$3 \times 2 \times 2 = 12, \text{ L. C. D.}$$

$$12 \div 3 = 4;\ \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$12 \div 4 = 3;\ \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$$12 \div 6 = 2;\ \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\frac{8}{12}+\frac{9}{12}+\frac{10}{12} = \frac{27}{12}$$

$$\frac{27}{12} = 2\frac{3}{12} = 2\frac{1}{4}$$

Explanation.—The fractions are first reduced to equivalent fractions having for a denominator the least number that is exactly divisible by the denominators of the fractions. 12 is the least common multiple of 3, 4, and 6, and is, therefore, the smallest number that can be used as a common denominator. To reduce $\frac{2}{3}$ to twelfths, 12 is divided by 3 and both terms of the fraction multiplied by the quotient, giving $\frac{8}{12}$ as a result. In the same manner, each of the other fractions is reduced to an equivalent fraction having 12 for a denominator. Adding the numerators of the equivalent fractions and writing the sum over the common denominator gives $\frac{27}{12}$ as their sum. Reducing this improper fraction to a mixed number, and the fractional part of the mixed number to its simplest form, gives $2\frac{1}{4}$, the sum required.

339. Rule.—I. *Reduce the fractions to equivalent fractions each of whose denominators is the least common multiple of the denominators of the given fractions.*

II. *Add the numerators and write the sum over the common denominator.*

III. *Reduce the result, if an improper fraction, to a whole or mixed number.*

IV. *In the case of mixed numbers, add the integers and fractional parts separately, and unite the results.*

340. ***Find the sum of:***

1. $\frac{11}{18}, \frac{5}{12}, \frac{7}{9}$.
2. $\frac{7}{15}, \frac{9}{10}, \frac{5}{6}$.
3. $\frac{17}{18}, \frac{3}{4}, \frac{7}{12}$.
4. $\frac{4}{7}, \frac{3}{4}, \frac{11}{14}$.
5. $\frac{4}{5}, \frac{9}{10}, \frac{17}{25}$.
6. $\frac{11}{16}, \frac{19}{24}, \frac{5}{12}$.
7. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.
8. $\frac{11}{12}, \frac{13}{18}, \frac{19}{20}$.
9. $\frac{19}{28}, \frac{4}{7}, \frac{7}{12}$.
10. $\frac{23}{36}, \frac{17}{18}, \frac{5}{27}$.
11. $\frac{8}{9}, \frac{6}{7}, \frac{13}{21}$.
12. $\frac{11}{18}, \frac{4}{5}, \frac{23}{30}$.
13. $\frac{6}{7}, \frac{7}{12}, \frac{9}{16}$.
14. $\frac{4}{5}, \frac{5}{9}, \frac{19}{21}$.
15. $2\frac{3}{4}, 3\frac{4}{5}, 7\frac{9}{10}, 9\frac{1}{2}$.
16. $4\frac{5}{6}, 7\frac{3}{8}, 4\frac{5}{12}, 6\frac{5}{18}$.
17. $2\frac{31}{36}, 3\frac{17}{48}, 9\frac{61}{72}, 8\frac{15}{16}$.
18. $\frac{17}{36}, 2\frac{13}{24}, \frac{11}{12}, 16\frac{8}{9}$.
19. $832\frac{11}{14}, 625\frac{7}{12}, 42\frac{19}{28}$.
20. $461\frac{5}{14}, 396\frac{29}{35}, 82\frac{9}{10}$.
21. $19\frac{35}{64}, 13\frac{4}{5}, 6\frac{31}{48}, 8\frac{13}{40}$.

22. $203\frac{5}{8}$ miles, $87\frac{11}{16}$ miles, $123\frac{7}{10}$ miles, and $197\frac{23}{32}$ miles.

23. \$$835\frac{3}{4}$, \$$1765\frac{4}{5}$, \$$623\frac{1}{2}$, \$$465\frac{7}{10}$, and \$$738\frac{11}{20}$.

24. $83\frac{9}{16}$ acres, $138\frac{9}{10}$ acres, $875\frac{5}{8}$ acres, $236\frac{23}{40}$ acres, $147\frac{17}{30}$ acres, and $98\frac{7}{16}$ acres.

CONCRETE ORAL PROBLEMS.

341. *1.* Henry paid $\frac{1}{2}$ of a dollar for a ball, $\frac{1}{4}$ of a dollar for a bat, and $\frac{1}{10}$ of a dollar for a top; how much did they all cost?

2. A lady bought a hat for \$$1\frac{1}{8}$ and some ribbon for \$$1\frac{3}{4}$; how much did she pay for both?

3. A farmer sold $\frac{7}{8}$ of a ton of hay to one man, $\frac{3}{4}$ to another, and $\frac{5}{8}$ to another; how much did he sell to all?

4. One demijohn contains $2\frac{3}{4}$ gal. of wine, another $3\frac{1}{8}$ gal., and a third $2\frac{5}{8}$ gal.; how much do they all contain?

5. A fisherman caught three trout; the first weighed $4\frac{3}{4}$ lb., the second $3\frac{5}{8}$ lb., and the third $5\frac{1}{2}$ lb.; find their united weight.

6. A family eat $2\frac{3}{8}$ lb. of bread for breakfast, $1\frac{3}{4}$ lb. for dinner, and $3\frac{1}{2}$ lb. for supper; how much do they consume daily?

7. Mary picked $8\frac{3}{4}$ qts. of berries, James $9\frac{5}{8}$ qts., Henry $12\frac{1}{2}$ qts., and Anna $7\frac{1}{2}$ qts.; how many did all pick?

8. A man sold $4\frac{5}{8}$ acres of land to one man, and $11\frac{7}{10}$ acres to another, and had $20\frac{3}{4}$ acres left; how many acres had he at first?

9. John sold from a basket of berries $4\frac{1}{4}$ quarts, spilled $3\frac{3}{4}$ quarts, and had $8\frac{1}{2}$ quarts left; how many quarts were in the basket at first?

10. A lady having a certain sum of money bought a dress for $\$3\frac{3}{5}$, some muslin for $\$2\frac{3}{4}$, a pair of gloves for $\$1\frac{1}{2}$, and had $\$1\frac{4}{5}$ remaining; how much had she at first?

11. During the month of December a man consumed in his kitchen range $1\frac{7}{10}$ tons of coal, in his parlor stove $\frac{3}{4}$ of a ton, in his dining-room $\frac{7}{8}$ of a ton, and in his study $\frac{4}{5}$ of a ton; how much was consumed in all?

12. Mr. Howard receives $\$3\frac{4}{5}$ wages per day, his son Henry $\$2\frac{3}{4}$, and his son Charles $\$1\frac{7}{20}$; what is the sum of their daily wages?

13. A certain room is $4\frac{4}{5}$ yards wide and $6\frac{5}{8}$ yards long what is the distance around the room?

14. A wood-cutter chopped and split three trees in one day; the first contained $1\frac{5}{16}$ cords, the second $\frac{7}{8}$ of a cord, and the third $1\frac{3}{4}$ cords; how many cords in the three trees?

15. A man paid $\$5\frac{4}{5}$ for a pair of boots for himself, $\$3\frac{7}{16}$ for a pair of shoes for his wife, and $\$3\frac{3}{4}$ for a pair of shoes for his daughter; find the cost of all.

16. From a certain tree a farmer sold $4\frac{5}{8}$ bushels of apples to one person and $3\frac{3}{4}$ bushels to another, and retained the remainder, $10\frac{11}{16}$ bushels; how many bushels did the tree bear?

CONCRETE WRITTEN PROBLEMS.

342. *1.* A sold $\frac{2}{5}$ of his cattle to one man, $\frac{3}{8}$ of them to another, and $\frac{1}{6}$ of them to a third; what part of his cattle did he sell?

2. A man bought three farms; the first contained $158\frac{9}{16}$ acres, the second $231\frac{3}{4}$ acres, and the third $374\frac{7}{8}$ acres; how many acres did he buy in all?

3. From A to B is $168\frac{4}{5}$ miles, from B to C $96\frac{7}{10}$ miles, from C to D $293\frac{11}{16}$ miles, and from D to E $184\frac{7}{12}$ miles; how far is it from A to E through B, C and D?

4. A steamer from New York to Havana made $231\frac{5}{8}$ miles the first day, $307\frac{11}{12}$ miles the second day, and $351\frac{5}{6}$ miles the third day, and was then within $348\frac{5}{8}$ miles of Havana; what is the distance from New York to Havana?

5. A train ran from New York to Philadelphia in $4\frac{11}{16}$ hours, from Philadelphia to Harrisburg in $4\frac{11}{16}$ hours, from Harrisburg to Altoona in $5\frac{7}{10}$ hours, and from Altoona to Pittsburg in $5\frac{5}{12}$ hours; what was the time from New York to Pittsburg?

6. James gathered $9\frac{11}{16}$ bushels of walnuts, Henry $2\frac{3}{4}$ bushels more than James, and Louis $5\frac{3}{8}$ bushels more than Henry; how many bushels did they all gather?

7. A dealer bought three car-loads of apples; the first contained $368\frac{3}{4}$ bushels, the second $84\frac{1}{2}$ bushels more than the first, and the third $49\frac{7}{8}$ bushels more than the first; how many bushels in all?

8. An author's annual income from the sale of his books was as follows: from the sale of a grammar \$$168\frac{3}{8}$, of a speller \$$199\frac{19}{20}$, and of an arithmetic \$$783\frac{9}{10}$; what were his total receipts?

9. A man expended of his monthly wages \$$23\frac{13}{20}$ for a suit of clothes, \$$28\frac{7}{10}$ for his board, \$$12\frac{3}{4}$ for incidentals, and had \$$34\frac{9}{10}$ left; find his wages by the month.

10. A lady went shopping with a certain sum of money; she bought some velvet for \$$36\frac{4}{5}$, some silk for \$$43\frac{11}{20}$, some ribbons for \$$2\frac{3}{4}$, and had \$$68\frac{5}{8}$ left; how much had she at first?

11. Mr. Dean gave from his farm $85\frac{11}{16}$ acres to the first of his three sons, $18\frac{7}{10}$ acres more to the second than to the first, and to the third $5\frac{3}{4}$ acres more than to the second, after which he had $397\frac{1}{2}$ acres left; how many acres were in the farm at first?

12. From a hogshead of molasses, a grocer sold $5\frac{1}{2}$ gallons, $15\frac{5}{8}$ gallons and $7\frac{1}{4}$ gallons, after which there remained $35\frac{3}{8}$ gallons; how many gallons did the hogshead contain at first?

SUBTRACTION OF FRACTIONS.

343. Before the difference between two unlike fractions can be found they must be made to express like fractional units.

344. To find the difference between two fractions.

I. Required to find the difference between $\frac{11}{12}$ and $\frac{5}{8}$.

Explanation. — The least common multiple of the denominators is 24; hence, both fractions may be reduced to twenty-fourths. $\frac{11}{12}$ is equal to $\frac{22}{24}$, and $\frac{5}{8}$ is equal to $\frac{15}{24}$. Subtracting $\frac{15}{24}$ from $\frac{22}{24}$ gives $\frac{7}{24}$, the difference between the fractions.

Process.

$$\frac{11}{12}-\frac{5}{8}=\frac{7}{24}$$
$$24\div 12=2;\quad \frac{11\times 2}{12\times 2}=\frac{22}{24}$$
$$24\div 8=3;\quad \frac{5\times 3}{8\times 3}=\frac{15}{24}$$
$$\frac{22}{24}-\frac{15}{24}=\frac{7}{24}$$

345. Rule.—I. *Reduce the fractions to equivalent fractions having the least common denominator.*

II. *Subtract the numerator of the subtrahend from that of the minuend, and write the difference over the least common denominator.*

III. *In the case of mixed numbers, treat the integral and the fractional parts separately.*

346. ***Find the value of:***

1. $\frac{8}{9}-\frac{5}{6}$.	*5.* $\frac{19}{24}-\frac{7}{16}$.	*9.* $\frac{19}{28}-\frac{5}{12}$.	*13.* $\frac{23}{39}-\frac{11}{52}$.
2. $\frac{7}{8}-\frac{5}{6}$.	*6.* $\frac{19}{20}-\frac{11}{15}$.	*10.* $\frac{15}{16}-\frac{5}{6}$.	*14.* $\frac{31}{34}-\frac{13}{51}$.
3. $\frac{7}{9}-\frac{5}{12}$.	*7.* $\frac{23}{24}-\frac{19}{36}$.	*11.* $\frac{31}{36}-\frac{14}{27}$.	*15.* $\frac{29}{38}-\frac{17}{57}$.
4. $\frac{11}{12}-\frac{7}{18}$.	*8.* $\frac{23}{30}-\frac{7}{12}$.	*12.* $\frac{37}{48}-\frac{19}{36}$.	*16.* $\frac{43}{51}-\frac{19}{34}$.

17. $\frac{3}{4}+\frac{4}{5}-\frac{5}{6}+\frac{7}{12}-\frac{4}{15}$.	*23.* $27\frac{11}{12}-4\frac{4}{9}+3\frac{7}{16}$.
18. $\frac{5}{6}+\frac{7}{8}-\frac{5}{9}+\frac{5}{24}-\frac{7}{36}$.	*24.* $34\frac{1}{5}-9\frac{3}{4}+12\frac{9}{10}-5\frac{5}{6}$.
19. $\frac{2}{3}-\frac{2}{5}+\frac{7}{10}+\frac{5}{6}-\frac{7}{12}$.	*25.* $83\frac{1}{4}-69\frac{2}{3}+17\frac{5}{8}-18\frac{5}{6}$.
20. $\frac{8}{9}-\frac{5}{6}+\frac{1}{2}-\frac{1}{4}+\frac{11}{12}$.	*26.* $78\frac{5}{12}-19\frac{4}{9}-23\frac{3}{4}+\frac{23}{36}$.
21. $4\frac{4}{5}-2\frac{2}{3}+7\frac{7}{10}-6\frac{5}{6}$.	*27.* $123\frac{4}{9}-19\frac{13}{15}+75\frac{17}{45}$.
22. $8\frac{9}{10}-3\frac{7}{12}+10\frac{4}{5}-8\frac{11}{15}$.	*28.* $18\frac{19}{36}-7\frac{23}{48}+15\frac{23}{54}$.

CONCRETE ORAL PROBLEMS.

347. *1.* A boy having an apple gave $\frac{1}{3}$ of it to one playmate and $\frac{3}{5}$ of it to another; how much was left?

2. A lady bought a pair of gloves for $\$1\frac{3}{4}$ and some worsted for $\frac{3}{5}$ of a dollar; how much change should she receive from a five-dollar bill?

3. A grocer had a barrel of flour and sold to one customer $\frac{1}{6}$ of it, to a second $\frac{3}{8}$ of it and to a third he sold all that remained; what part of a barrel did he sell to the third?

4. William earned $\$1\frac{3}{4}$, Frank $\$1\frac{1}{5}$, and Russell enough to make the sum of their earnings $\$5$; how much did Russell earn?

5. Three boys together caught 25 pounds of fish. The first boy caught $4\frac{5}{16}$ pounds, and the second boy caught $5\frac{3}{4}$ pounds more than the first; how many pounds of fish did the third boy catch?

6. Two men start from the same place at noon and travel in the same direction along the same road, the first $2\frac{3}{4}$ miles per hour, and the second $4\frac{3}{5}$ miles per hour; how far apart are they at 2 o'clock?

7. A man owing $\$7\frac{17}{20}$ paid at one time $\$2\frac{3}{4}$, and at another $\$3\frac{2}{5}$; how much does he still owe?

8. Mary can make a dress in $2\frac{7}{12}$ hours less than Jane, who requires $9\frac{1}{3}$ hours to make it; how long will it take Mary to make the dress?

9. From a board $13\frac{5}{8}$ feet long, there were cut $5\frac{3}{4}$ feet; how many feet remained?

10. A boy having $\$2\frac{3}{4}$, expended $\$\frac{19}{20}$, and afterward earned $\$1\frac{1}{5}$; how much had he then?

11. From a piece of ground containing $3\frac{3}{4}$ acres, there were sold to one man $1\frac{1}{5}$ acres, and to another $\frac{7}{10}$ of an acre; how much remained?

12. A grocer sold 3 pounds 12 ounces of butter from a roll weighing $8\frac{5}{8}$ pounds; how much remained?

CONCRETE WRITTEN PROBLEMS.

348. *1.* A student employs $9\frac{13}{16}$ hours daily at his books, $2\frac{7}{12}$ hours at his meals, $4\frac{9}{10}$ hours in physical exercise, and the remainder in sleep; how many hours does he sleep daily?

2. From a cheese weighing $86\frac{7}{8}$ lb., there was sold at one time $15\frac{15}{16}$ lb., at another $23\frac{3}{4}$ lb., and at a third enough to leave unsold $19\frac{1}{2}$ lb.; how many pounds were sold the third time?

3. From a bin containing $568\frac{1}{2}$ bushels of wheat there were sold at different times $187\frac{1}{4}$ bushels, $83\frac{1}{2}$ bushels, and $148\frac{3}{4}$ bushels; how many bushels remained?

4. From New York to Chicago is 952 miles; a passenger train leaves each of these places by the same road, at the same time, to run to the other place; at the end of 12 hours the train from Chicago has gone $347\frac{5}{12}$ miles, and the other is $371\frac{7}{10}$ miles from New York; how far apart are the trains?

5. Two men undertake to save \$1000 apiece; when one of them lacks $\$635\frac{4}{5}$ of having \$1000, they both together have $\$987\frac{3}{4}$; how much has each?

6. A drover sold some sheep for $\$8375\frac{3}{4}$, and by so doing gained $\$1986\frac{4}{5}$; how much did they cost?

7. A man bought a store for $\$18765\frac{1}{2}$, fixtures for $\$1246\frac{3}{4}$, and goods for $\$27932\frac{4}{5}$. After selling goods to the amount of $\$15938\frac{17}{20}$ he sold what remained, together with the store and fixtures, for $\$40985\frac{1}{4}$; find his gain.

8. The proprietor of a seminary received during a certain year $\$15831\frac{19}{20}$ for tuition, $\$7587\frac{3}{5}$ for room-rent, fuel and gas, and $\$12692\frac{3}{4}$ for board; his expenses during the year were $\$8875\frac{4}{5}$ for salaries of teachers, $\$9438\frac{1}{2}$ for provisions, and $\$5429\frac{9}{10}$ for incidentals; what was his profit?

9. The exact circumference of the earth at the equator is $24899\frac{11}{500}$ miles; what is the error if it is given as 25000 miles?

10. Of a farm containing $697\frac{17}{32}$ acres, $256\frac{5}{8}$ acres were in wheat, $297\frac{3}{4}$ acres in grass, and the remainder in corn; how much was in corn?

MULTIPLICATION OF FRACTIONS.

349. To multiply a fraction by an integer or an integer by a fraction.

I. Required to multiply $\frac{5}{24} \times 12$.

Explanation.—A fraction is multiplied by a whole number by multiplying its numerator or by dividing its denominator. By the first process, the numerator of the fraction is multiplied by 12, giving for the product $\frac{60}{24} = 2\frac{12}{24} = 2\frac{1}{2}$. By the second process, the denominator of the fraction is divided by 12, giving for the product $\frac{5}{2} = 2\frac{1}{2}$.

First Process.

$$\frac{5}{24} \times 12 = \frac{5 \times 12}{24} = \frac{60}{24}$$

$$\frac{60}{24} = 2\frac{12}{24} = 2\frac{1}{2}$$

Second Process.

$$\frac{5}{24} \times 12 = \frac{5}{24 \div 12} = \frac{5}{2} = 2\frac{1}{2}$$

The second process is the shorter of the two, but is practicable only when the denominator of the fraction is exactly divisible by the multiplier.

II. Required to multiply 24 by $\frac{5}{6}$.

Explanation.—24 multiplied by $\frac{5}{6}$ means $\frac{5}{6}$ of 24. $\frac{1}{6}$ of 24 is 4; $\frac{5}{6}$ of 24 are 5 times 4, or 20.

Or, since the result will be the same, the whole number may be multiplied by the numerator and the product divided by the denominator.

First Process.

$$24 \times \frac{5}{6} = 20$$
$$24 \div 6 = 4$$
$$4 \times 5 = 20$$

Second Process.

$$24 \times \frac{5}{6} = 20$$
$$24 \times 5 = 120$$
$$120 \div 6 = 20$$

350. Rule.—I. *Divide the integer by the denominator of the fraction and multiply the quotient by the numerator.* Or,

II. *Multiply the integer by the numerator of the fraction and divide the product by the denominator.*

NOTE.—The former is the better process when the integer is exactly divisible by the denominator of the fraction.

351. ***Perform the operations indicated:***

1. $18 \times \frac{2}{9}$.	*6.* $1575 \times \frac{19}{25}$.	*11.* $\frac{11}{25} \times 65$.
2. $21 \times \frac{6}{7}$.	*7.* $1728 \times \frac{43}{48}$.	*12.* $168 \times \frac{31}{48}$.
3. $54 \times \frac{3}{4}$.	*8.* $2025 \times \frac{17}{25}$.	*13.* $83 \times \frac{11}{31}$.
4. $\frac{13}{20} \times 150$.	*9.* $\frac{8}{9} \times 195$.	*14.* $\frac{7}{9} \times 117$.
5. $196 \times \frac{2}{3}$.	*10.* $\frac{18}{19} \times 78$.	*15.* $\frac{13}{36} \times 864$.

352. To find the product of two or more fractions or mixed numbers.

I. Required to find the product of $\frac{8}{9}$ and $\frac{2}{3}$.

Process.

$$\frac{8}{9} \times \frac{2}{3} = \frac{16}{27}$$

$$\frac{8}{9} \times 2 = \frac{16}{9};\ \frac{16}{9} \div 3 = \frac{16}{27}$$

Explanation.—Multiplying $\frac{8}{9}$ by 2, the numerator of the multiplier, gives $\frac{16}{9}$. But 2 is 3 times as great as the multiplier $\frac{2}{3}$; hence, $\frac{16}{9}$ is 3 times as great as it should be. Dividing $\frac{16}{9}$ by 3, which is done by multiplying the denominator by 3, gives $\frac{16}{27}$. Hence, $\frac{8}{9} \times \frac{2}{3} = \frac{16}{27}$. This result has for its numerator the product of the numerators of the fractions, and for its denominator the product of their denominators.

II. Required the product of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

Process.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{60} = \frac{2}{5}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12};\ \frac{6}{12} \times \frac{4}{5} = \frac{24}{60}$$

$$2 \times 3 \times 4 = 24$$

$$3 \times 4 \times 5 = 60$$

$$\frac{24}{60} = \frac{2}{5}$$

By Cancellation.

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{2}{5}$$

Explanation.—The continued product of several fractions is only an extension of the above process. Thus, the product of $\frac{2}{3} \times \frac{3}{4}$ is $\frac{6}{12}$; and the product of $\frac{6}{12} \times \frac{4}{5} = \frac{24}{60}$. The same result is obtained by taking the continued product of the numerators for the numerator of the product and the continued product of the denominators for the denominator of the product.

In practice, equal factors are rejected from numerators and denominators before the products are found.

III. Required the product of $4\frac{3}{8}$, $5\frac{5}{7}$, $\frac{14}{15}$.

Process.

$$4\frac{3}{8} \times 5\frac{5}{7} \times \frac{14}{15} = 23\frac{1}{3}$$

$$\frac{\overset{5}{\cancel{35}}}{\cancel{8}} \times \frac{\overset{5}{\cancel{40}}}{\cancel{7}} \times \frac{14}{\underset{3}{\cancel{15}}} = \frac{70}{3} = 23\frac{1}{3}$$

Explanation.—The mixed numbers are first reduced to improper fractions and the multiplication then performed as above.

353. Rule.—I. *Reduce mixed numbers to improper fractions and indicate the multiplication.*

II. *Reject factors common to the numerators and denominators.*

III. *Find the product of the numerators for the numerator of the result, and the product of the denominators for the denominator of the result, and reduce the result, if an improper fraction, to a whole or a mixed number.*

NOTE.—The word *of* written between fractions denotes multiplication

354. ***Find the product of the following:***

1. $\frac{3}{4}$, $\frac{11}{12}$, and $\frac{7}{11}$.
2. $8\frac{2}{3}$, $\frac{5}{13}$, and $\frac{6}{7}$.
3. $\frac{7}{8}$, $4\frac{4}{5}$, and $\frac{5}{14}$.
4. $\frac{4}{9}$, $\frac{3}{7}$, and $8\frac{2}{3}$.
5. $4\frac{2}{3}$, $\frac{11}{14}$, and $4\frac{1}{11}$.
6. $8\frac{3}{4}$, $3\frac{5}{7}$, and $\frac{9}{13}$.
7. $\frac{3}{4}$, $\frac{5}{9}$, $\frac{12}{13}$, and $\frac{3}{5}$.
8. $\frac{6}{7}$, $\frac{8}{9}$, $\frac{21}{32}$, and $\frac{2}{3}$.
9. $\frac{31}{48}$, $\frac{24}{35}$, $\frac{70}{93}$, and $\frac{4}{7}$.
10. $4\frac{3}{4}$, $6\frac{2}{5}$, $\frac{5}{8}$, and $5\frac{2}{3}$.
11. $5\frac{4}{5}$, $4\frac{2}{7}$, $8\frac{1}{6}$, $4\frac{3}{8}$.
12. $\frac{2}{3}$, $6\frac{3}{5}$, $\frac{7}{11}$, and $8\frac{4}{7}$.
13. $16\frac{4}{5}$, $\frac{11}{28}$, $11\frac{2}{3}$, and $\frac{17}{22}$.
14. $9\frac{2}{11}$, $7\frac{6}{7}$, $23\frac{1}{3}$, and $\frac{39}{56}$.
15. $\frac{2}{3}$ of $4\frac{1}{2} \times \frac{3}{4}$ of $2\frac{2}{3}$.
16. $\frac{7}{8}$ of $4\frac{4}{5} \times \frac{3}{4}$ of $\frac{8}{9}$.
17. $\frac{5}{8}$ of $21\frac{1}{3} \times \frac{4}{5}$ of $5\frac{5}{7}$.
18. $(17\frac{3}{4} - 13\frac{4}{5}) \times (8\frac{2}{3} + \frac{4}{5})$
19. $(8\frac{5}{8} + 7\frac{2}{3}) \times (\frac{4}{5} + \frac{9}{10})$.
20. $(24\frac{4}{7} - 8\frac{5}{6}) \times 9\frac{7}{12}$.

CONCRETE ORAL PROBLEMS.

355. *1.* At $\$\frac{5}{8}$ per yard, what will 9 yards of cloth cost ?

2. What will $\frac{5}{8}$ of a cord of wood cost at $7 a cord ?

3. How much will $\frac{3}{4}$ of a gallon of molasses cost at $\frac{4}{5}$ of a dollar per gallon ?

4. A man can walk $3\frac{3}{4}$ miles per hour ; how far can he walk in $\frac{4}{5}$ of an hour ?

5. At $\$3\frac{3}{4}$ per day, how much can a man earn in $2\frac{2}{3}$ days ?

6. Selling apples at 5 for 3 cents, how much do I receive for 10 apples ?

7. What will $3\frac{4}{5}$ yards of velvet cost at $\$3\frac{3}{4}$ per yard?

8. A boy gave $9\frac{3}{4}$ qts. of berries, at 3 qts. for 25 cts., as part payment for a hat worth \$1.00; how much is still due?

9. How much more will $8\frac{3}{4}$ dozen eggs cost at 6 dozen for \$1, than $2\frac{5}{8}$ pounds of butter at 40¢ a pound?

10. A grocer bought $5\frac{7}{8}$ lb. of butter at 25¢ a pound, and sold it at 35 cts. a pound; how much did he gain?

11. A man owning $\frac{2}{3}$ of a mill sold $\frac{3}{4}$ of his share for \$3000; at this rate, what was the value of the mill?

12. If it takes $\frac{3}{8}$ of a yard of cloth to make a vest, how much will it take to make 12 vests?

13. A man who earns \$60 per month pays $\frac{5}{8}$ of a month's wages for a watch; what does the watch cost?

14. A grocer sold $8\frac{3}{4}$ lb. of brown sugar at $5\frac{3}{8}$¢ per pound; how much change should he give out of a half-dollar?

15. If $\frac{5}{6}$ of $\$8\frac{3}{4}$ will pay a certain bill, how much is the bill?

CONCRETE WRITTEN PROBLEMS.

356. *1.* Three brothers raised $568\frac{3}{4}$ bushels of wheat on a field, and agreed to divide equally what they received for it. They sold it at $\$1\frac{3}{5}$ per bushel; what was each one's share?

2. Find the cost of $68\frac{5}{8}$ yards of carpet at \$1.17 per yard.

3. A tailor cut 8 suits of $7\frac{4}{5}$ yds. each from a piece of cloth and $11\frac{17}{18}$ yds. remained; what was in the piece at first?

4. I owned a house that cost me \$9500, and insured it for $\frac{3}{4}$ of its cost. It was destroyed by fire, and I received only $\frac{7}{8}$ of the insurance; how much was my loss?

5. A lady had $\$16\frac{1}{5}$ in her purse and spent $\frac{2}{3}$ of it; how much remained?

6. The grain raised on a certain field was taken to market in loads of $27\frac{3}{4}$ bushels each; after 9 loads were taken, there remained $\frac{3}{4}$ of a load; how much was there in all?

7. Find the cost of $17\frac{4}{5}$ tons of coal at $\$5\frac{4}{5}$ a ton.

8. A certain sum of money was shared equally among 9 men, each receiving $\$19\frac{4}{5}$; what was the sum divided?

9. A man owing me $\$3785\frac{3}{4}$ failed, and could pay me only $\frac{4}{5}$ of my claim; how much did I lose?

10. A railroad train ran for $19\frac{3}{5}$ hours at the average rate of $24\frac{5}{12}$ miles per hour; find the distance passed over.

11. Mr. Chester bought a farm of $374\frac{1}{2}$ acres at $\$98\frac{1}{2}$ per acre; what did he pay for the farm?

12. A drover bought 18 sheep at $\$4\frac{17}{20}$ each, and one hog; they all cost \$100; find the cost of the hog.

13. A cubic foot of water weighs $62\frac{1}{2}$ pounds, and gold is $19\frac{1}{2}$ times as heavy as water; what is the weight of a cubic foot of gold?

14. A grocer bought 36 dozen eggs at the rate of 16 eggs for 25 cents, and sold them at the rate of 9 for 25 cents· what was his gain?

15. A commission merchant received a consignment of 48 tubs of butter, each containing $41\frac{7}{8}$ lb., for which he was charged $31\frac{1}{4}$¢ per lb.; he sold the entire lot at $43\frac{3}{4}$¢ a pound; find his gain.

16. A clock gains $5\frac{3}{4}$ minutes a day; if it is right at noon on Jan. 1, what time will the clock indicate at noon on Jan. 30?

17. A drover bought 700 hogs at $\$12\frac{19}{20}$ a head; he sold $\frac{3}{5}$ of them at $\$16\frac{2}{5}$, $\frac{3}{4}$ of the remainder at $\$10\frac{1}{2}$ a head, and what still remained at $\$14\frac{3}{5}$ a head; what was his gain?

18. Find the cost of $159\frac{2}{5}$ acres of land at $\$108\frac{1}{2}$ per acre.

19. A and B set out from the same place, at the same time, in opposite directions. A travels at the rate of $4\frac{3}{8}$ miles per hour, and B at the rate of $3\frac{4}{5}$ miles per hour; how far apart will they be after traveling $6\frac{7}{12}$ hours?

20. The buildings on Mr. Stewart's farm are worth $\frac{3}{7}$ as much as the farm; what are the buildings worth if the farm consists of $71\frac{2}{5}$ acres, worth $\$98\frac{3}{4}$ per acre?

DIVISION OF FRACTIONS.

357. To divide a fraction by an integer.

I. Required to divide $\frac{6}{7}$ by 3.

Explanation.—The value of a fraction is divided by dividing its numerator or by multiplying its denominator. Dividing the numerator 6 by 3 and writing the quotient 2 over the denominator of the fraction gives $\frac{2}{7}$ as the result.

First Process.

$$\frac{6}{7} \div 3 = \frac{6 \div 3}{7} = \frac{2}{7}$$

Or, multiplying the denominator by 3 gives $\frac{6}{21}$, which in its lowest terms is $\frac{2}{7}$, the same result.

Second Process.

$$\frac{6}{7} \div 3 = \frac{6}{7 \times 3} = \frac{6}{21} = \frac{2}{7}$$

The same result is obtained by writing the divisor 3 in the form of a fraction, $\frac{3}{1}$, inverting it, and proceeding as in multiplication.

358. Rule.—I. *Divide the numerator or multiply the denominator of the fraction by the integer.* Or,

II. *Write the divisor as a fraction, invert it, and proceed as in multiplication.*

359. To divide an integer by a fraction.

I. Required to divide 4 by $\frac{6}{7}$.

Explanation.—Dividing 4 by 6 gives $\frac{2}{3}$ for a quotient. But the divisor is not 6, but $\frac{6}{7}$, or $\frac{1}{7}$ of 6. Hence, the true quotient will be 7 times $\frac{2}{3}$, or $\frac{14}{3}$, equal to $4\frac{2}{3}$.

Process.

$$4 \div \frac{6}{7} = 4 \times \frac{7}{6} = \frac{28}{6} = 4\frac{2}{3}.$$

Analysis.

$$4 \div 6 = \frac{4}{6} = \frac{2}{3};$$

$$\frac{2}{3} \times 7 = \frac{14}{3} = 4\frac{2}{3}.$$

The same result is obtained by inverting the divisor and proceeding as in multiplication.

360. Rule.—I. *Multiply the integer by the denominator of the fraction and divide the product by the numerator.* Or,

II. *Invert the divisor and proceed as in multiplication.*

EXAMPLES.

361. ***Find the value of:***

1. $9 \div \frac{2}{3}$.	*9.* $\frac{11}{12} \div 20$.	*17.* $(\frac{3}{4} \times \frac{2}{3}) \div 5$.
2. $12 \div \frac{3}{4}$.	*10.* $\frac{6}{7} \div 12$.	*18.* $(\frac{7}{8} \times \frac{4}{5}) \div 14$.
3. $20 \div \frac{4}{5}$.	*11.* $\frac{12}{13} \div 24$.	*19.* $21 \div (\frac{4}{9} - \frac{1}{3})$.
4. $21 \div \frac{7}{9}$.	*12.* $\frac{15}{32} \div 45$.	*20.* $35 \div (\frac{5}{8} + \frac{1}{4})$.
5. $38 \div \frac{5}{7}$.	*13.* $(\frac{3}{4} - \frac{1}{6}) \div 24$.	*21.* $51 \div (\frac{4}{5} - \frac{3}{8})$.
6. $63 \div \frac{14}{15}$.	*14.* $(\frac{1}{3} + \frac{1}{4}) \div 36$.	*22.* $75 \div (\frac{5}{6}$ of $\frac{3}{5})$.
7. $\frac{8}{9} \div 4$.	*15.* $(\frac{2}{5} + \frac{3}{8}) \div 31$.	*23.* $50 \div (\frac{7}{8}$ of $\frac{2}{14})$.
8. $\frac{5}{8} \div 3$.	*16.* $(\frac{9}{10} - \frac{1}{2}) \div 8$.	*24.* $100 \div (5\frac{1}{4} \times \frac{1}{6})$.

362. To divide a fraction or a mixed number by a fraction or a mixed number.

I. Required to divide $\frac{3}{4}$ by $\frac{2}{3}$.

Explanation. — Dividing $\frac{3}{4}$ by 2 gives $\frac{3}{8}$ for a quotient. But the divisor is not 2, but $\frac{2}{3}$, or $\frac{1}{3}$ of 2. Hence, the true quotient will be 3 times $\frac{3}{8}$, or $\frac{9}{8}$, equal to $1\frac{1}{8}$.

Process.

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}.$$

Analysis.

$$\frac{3}{4} \div 2 = \frac{3}{8}; \quad \frac{3}{8} \times 3 = \frac{9}{8} = 1\frac{1}{8}.$$

The same result is obtained by multiplying the dividend $\frac{3}{4}$ by the divisor inverted.

II. Required to divide $2\frac{1}{4}$ by $3\frac{2}{3}$.

Explanation.—Reducing $2\frac{1}{4}$ and $3\frac{2}{3}$ to improper fractions gives $\frac{9}{4}$ and $\frac{11}{3}$. Dividing $\frac{9}{4}$ by $\frac{11}{3}$ gives $\frac{27}{44}$.

Process.

$$2\frac{1}{4} \div 3\frac{2}{3} = \frac{9}{4} \div \frac{11}{3};$$
$$\frac{9}{4} \div \frac{11}{3} = \frac{9}{4} \times \frac{3}{11} = \frac{27}{44}.$$

NOTE.—When either the dividend or the divisor is a mixed number, reduce it to an improper fraction, and proceed as above.

363. Rule.—I. *Reduce mixed numbers to improper fractions.*

II. *Invert the divisor and proceed as in multiplication.*

364. ***Perform the operations indicated:***

1. $\frac{11}{12} \div \frac{4}{5}$.	*9.* $\frac{21}{40} \div \frac{35}{48}$.	*17.* $4\frac{3}{4} \div 3\frac{4}{5}$.
2. $\frac{5}{9} \div \frac{11}{12}$.	*10.* $\frac{30}{49} \div \frac{25}{63}$.	*18.* $5\frac{8}{9} \div 10\frac{3}{5}$.
3. $\frac{7}{8} \div \frac{3}{4}$.	*11.* $9 \div \frac{3}{4}$.	*19.* $5\frac{3}{5} \div 4\frac{2}{3}$.
4. $\frac{14}{15} \div \frac{2}{3}$.	*12.* $21 \div \frac{7}{8}$.	*20.* $9\frac{4}{9} \div 13\frac{3}{5}$.
5. $\frac{19}{20} \div \frac{14}{15}$.	*13.* $45 \div \frac{6}{7}$.	*21.* $3\frac{3}{7} \div 9\frac{3}{5}$.
6. $\frac{23}{48} \div \frac{13}{24}$.	*14.* $56 \div \frac{14}{15}$.	*22.* $8\frac{2}{9} \div 12\frac{1}{3}$.
7. $\frac{27}{31} \div \frac{45}{62}$.	*15.* $156 \div \frac{12}{25}$.	*23.* $\frac{4}{5} \div 1\frac{5}{7}$.
8. $\frac{18}{35} \div \frac{27}{40}$.	*16.* $225 \div \frac{45}{68}$.	*24.* $\frac{8}{9} \div 4\frac{4}{5}$.

365. Multiplication and division of fractions may generally be shortened by cancellation.

I. Required to divide the continued product of $\frac{8}{9}$, $5\frac{4}{5}$, and $7\frac{7}{11}$ by the continued product of $4\frac{4}{5}$, $8\frac{2}{7}$, and $\frac{28}{33}$.

Process.

$$\frac{\frac{8}{9} \times 5\frac{4}{5} \times 7\frac{7}{11}}{4\frac{4}{5} \times 8\frac{2}{7} \times \frac{28}{33}} = \frac{\frac{8}{9} \times \frac{29}{5} \times \frac{84}{11}}{\frac{24}{5} \times \frac{58}{7} \times \frac{28}{33}} =$$

$$\frac{\cancel{8} \times \cancel{29} \times \overset{3}{\cancel{84}} \times \cancel{5} \times 7 \times \overset{3}{\cancel{33}}}{\underset{3}{\cancel{9}} \times \cancel{5} \times \cancel{11} \times \underset{3}{\cancel{24}} \times \underset{2}{\cancel{58}} \times \cancel{28}} = \frac{7}{6} = 1\frac{1}{6}$$

Explanation.—Reducing mixed numbers to improper fractions, the numerators of the dividend are written above a horizontal line and the denominators of the dividend below the line. Then the denominators of the divisor are written above the line and the numerators below it. In other words, the factors of the divisor are all inverted. Canceling, multiplying together the uncanceled factors on each side of the line, and reducing the result to a mixed number gives $1\frac{1}{6}$ as the required quotient.

366. ***Perform the operations indicated:***

1. $\frac{12}{13} \div 9\frac{1}{3}$.	*5.* $18\frac{3}{4} \div \frac{15}{16}$.	*9.* $(6\frac{1}{2} \times \frac{1}{9}) \div \frac{5}{27}$.
2. $\frac{24}{25} \div 10\frac{2}{3}$.	*6.* $65\frac{5}{9} \div 7\frac{3}{8}$.	*10.* $(5\frac{4}{9} \times \frac{13}{14}) \div \frac{2}{3}$.
3. $\frac{16}{25} \div 8\frac{2}{3}$.	*7.* $(\frac{4}{7} \text{ of } \frac{14}{15}) \div \frac{24}{35}$.	*11.* $(9\frac{2}{7} \div \frac{4}{11}) \div \frac{7}{8}$.
4. $\frac{15}{17} \div 3\frac{5}{6}$.	*8.* $(\frac{5}{8} \text{ of } \frac{24}{35}) \div \frac{9}{14}$.	*12.* $(\frac{7}{11} - \frac{8}{33}) \div \frac{5}{9}$.

13. $(\frac{8}{9} - \frac{5}{6}) \div (\frac{3}{4} - \frac{2}{3})$.	*17.* $(10\frac{4}{5} \div 2\frac{4}{7}) \div 7\frac{7}{8}$.
14. $(\frac{2}{3} \times \frac{5}{9}) \div (\frac{4}{5} \times \frac{10}{13})$.	*18.* $(8\frac{5}{8} \div 18\frac{2}{5}) \div 9\frac{6}{11}$.
15. $(\frac{2}{3} \div \frac{8}{9}) \div (\frac{2}{3} \times \frac{8}{9})$.	*19.* $(6\frac{3}{7} \times 1\frac{13}{15}) \div \frac{2}{5}$.
16. $(\frac{4}{5} \div \frac{5}{6}) \div (\frac{4}{5} \times \frac{5}{6})$.	*20.* $(13\frac{3}{5} \div 1\frac{6}{11}) \div 8\frac{3}{4}$.

367. Division of fractions may be indicated by writing the dividend above the divisor with a line between them, as a simple fraction is written. Expressions so formed are called **complex fractions.**

Thus, $\dfrac{5}{\frac{3}{4}}$, $\dfrac{\frac{2}{3}}{\frac{3}{4}}$, $\dfrac{\frac{5}{6}}{4\frac{1}{2}}$, $\dfrac{4\frac{2}{3}}{5\frac{5}{6}}$, $\dfrac{(\frac{2}{3}+\frac{3}{4})\times\frac{5}{6}}{(\frac{2}{3}\text{ of }5\frac{1}{2})\div\frac{7}{8}}$, etc., are complex fractions.

368. ***Perform the operations indicated:***

1. $\dfrac{4\frac{7}{8}}{8\frac{4}{5}}$.

2. $\dfrac{5\frac{8}{9}}{\frac{13}{27}}$.

3. $\dfrac{\frac{5}{6}}{\frac{10}{13}}$.

4. $\dfrac{\frac{13}{20}}{7\frac{4}{5}}$.

5. $\dfrac{\frac{2}{3}+\frac{3}{4}}{3\frac{1}{3}\times\frac{4}{5}}$.

6. $\dfrac{\frac{4}{5}\text{ of }\frac{5}{8}\text{ of }3\frac{1}{2}}{\frac{2}{3}\times(\frac{8}{9}-\frac{5}{6})}$.

7. $\dfrac{4\frac{2}{3}-(6\frac{2}{3}\times\frac{3}{5})}{2\frac{3}{4}+(8\frac{3}{4}\div 13\frac{1}{8})}$.

8. $\dfrac{11\frac{3}{7}\div(8\frac{1}{3}\div 2\frac{1}{2})}{2\times 1\frac{1}{2}\times 1\frac{1}{3}}$.

9. $\dfrac{4\frac{2}{3}\times 5\frac{1}{5}\times 8\frac{3}{4}}{8\frac{2}{5}\times 4\frac{7}{8}\times 4\frac{1}{6}}$.

10. $\dfrac{15\frac{3}{4}\times 6\frac{2}{7}\times 10\frac{2}{3}}{8\frac{2}{5}\times 4\frac{7}{12}\times\frac{24}{35}}$.

11. $\dfrac{8\frac{5}{6}\times 10\frac{1}{5}\times 17\frac{1}{3}}{18\frac{2}{5}\times 14\frac{1}{6}\times 3\frac{9}{10}}$.

12. $\dfrac{12\frac{4}{5}\times 7\frac{7}{11}\times 32\frac{2}{3}}{8\frac{5}{9}\times 28\frac{4}{5}\times 38\frac{1}{4}}$.

CONCRETE ORAL PROBLEMS.

369. *1.* A pigeon can fly 20 miles farther in an hour than in $\frac{3}{5}$ of an hour; how far can it fly in one hour?

2. A man earns \$$6\frac{2}{3}$ less in $2\frac{3}{4}$ days than in $5\frac{3}{4}$ days; what does he earn per day?

3. A farmer sold $3\frac{3}{8}$ lb. of butter to one customer, and $2\frac{3}{4}$ lb. to another, and received \$.25 more from the first than from the second; what did he receive per pound for his butter?

4. If $\frac{3}{5}$ of a barrel of flour cost \$$3\frac{3}{5}$, what was the cost per barrel?

5. If $\frac{5}{8}$ of a pound of cheese costs the same as $\frac{3}{4}$ of a pound of ham, what does cheese cost per pound when ham costs 10 cents per pound?

6. Find the cost of $3\frac{5}{8}$ pounds of caramels, if 2 pounds cost 40 cents.

7. A man exchanged $1\frac{1}{2}$ dozen of eggs for 3 pounds of sugar; what was sugar worth a pound, if eggs were worth 21¢ per dozen?

CONCRETE WRITTEN PROBLEMS.

370. *1.* If $9\frac{1}{3}$ barrels of flour are divided equally among 7 families of 5 persons each, how much will be given to each family, and how much to each person?

2. In a jail containing 2137 prisoners, each receives $\frac{7}{8}$ of a pound of meat daily; how long will 29918 pounds last them?

3. How long will it require a man to save $5\frac{3}{4}$ days' wages, who spends each day $\frac{5}{8}$ of what he earns?

4. A farmer sold $235\frac{3}{4}$ bushels of wheat; he afterward sold what was left at $\$1.12\frac{1}{2}$ per bushel, and received for it $\$246\frac{3}{8}$; how much had he at first?

5. A man bequeathed $\frac{2}{3}$ of his money to his wife, $\frac{4}{5}$ of the remainder to his son, and what still remained to his daughter; the wife's share was $\$9875\frac{7}{10}$ more than the son's share; how much did each receive?

6. A lady expended $\frac{2}{5}$ of her money for a silk dress, and then found that to pay \$36 for a shawl would require $\frac{3}{8}$ of what she had left; how much had she at first?

7. To cover the floor of a certain room with carpet 1 yard wide at $\$1.33\frac{1}{3}$ per yard cost $\$38\frac{2}{5}$; how much would it cost if the carpet were $\$1\frac{4}{5}$ per yard?

8. A boy bought $\frac{3}{4}$ of a bushel of chestnuts and sold $\frac{2}{3}$ of them for what he paid for all, and the remainder at cost; he gained by the transaction $\$1\frac{1}{4}$; at what price per bushel did he buy them?

9. A can cut $2\frac{3}{4}$ cords of wood in $8\frac{1}{4}$ hours, and B can cut $2\frac{2}{3}$ cords in $6\frac{2}{3}$ hours; how much can both cut in 12 hours?

10. I spent $\frac{2}{5}$ of my money for an overcoat, and $\frac{4}{5}$ of the remainder for a suit of clothes; how much had I at first if the overcoat cost $\$7\frac{1}{2}$ less than the suit of clothes?

11. Anna can make a dress in $8\frac{2}{3}$ hours, and Susan in $9\frac{3}{4}$ hours; how long would it take them to make the dress working together?

12. A man paid \$$15\frac{3}{8}$ for a coat, \$$5\frac{1}{2}$ for a hat, and \$$6\frac{3}{4}$ for a pair of boots; he then found that of the money in his purse at first only $\frac{5}{8}$ remained; how much had he at first?

13. A gentleman paid \$184.25 board for himself and daughter for $9\frac{4}{7}$ weeks; for himself he paid \$$10\frac{1}{8}$ per week; how much did he pay per week for his daughter?

14. I owe my butcher and my grocer together \$36.40, and $\frac{3}{4}$ of what I owe the former equals what I owe the latter; how much do I owe each?

15. A farmer raised the first of two years $462\frac{1}{2}$ bushels of wheat; $\frac{3}{5}$ of what he raised the first year was 160 bushels less than he raised the second year; how much did he raise the second year?

16. It takes Mr. Howard $5\frac{1}{2}$ hours longer to drive to a town $82\frac{1}{3}$ miles distant, than to go by rail; how far does he drive per hour if the time by rail is $3\frac{1}{6}$ hours?

17. A tailor cut from a piece of cloth material for two suits, using for the first suit $7\frac{3}{4}$ yds. and for the second $\frac{3}{8}$ of a yard less than for the first; he had then used $\frac{11}{32}$ of the entire piece; how many yards were in the piece at first?

18. Three men buy a farm. The first pays \$5385; the second pays \$1275 more than the first, and the first and second together pay $\frac{3}{5}$ of the price of the farm; how much does the third pay?

19. If $4\frac{3}{4}$ tons of coal cost \$$21\frac{3}{8}$, what cost $3\frac{3}{10}$ tons?

20. A man lost $\frac{2}{5}$ of his money, and after gaining $\frac{3}{4}$ as much as he lost, had \$8154; how much had he at first?

21. The buildings on a certain farm are worth $\frac{3}{7}$ as much as the land, and the land is worth \$4784 more than the buildings; find their united value.

22. A man traded a farm of $143\frac{5}{8}$ acres, worth \$$123\frac{1}{5}$ an acre, for another farm containing $229\frac{4}{5}$ acres; how much did the second farm cost an acre?

23. A's work is worth $\frac{3}{4}$ of B's, and together they earn \$2450 in a year; what does each earn?

24. In how many days can a man earn as much as a boy earns in 24 days, if the man earns as much in 2 days as the boy in 3 days?

25. If 40 bushels of potatoes are produced on $\frac{5}{8}$ of an acre, how many acres will it take to yield 248 bushels?

26. I bought a house and paid $\frac{1}{4}$ of the price down, and one year afterward I paid $\frac{2}{3}$ of the remainder; what was the price of the house if the two payments amounted to \$9600?

27. A man's savings when he lays by $\frac{2}{5}$ of his salary yearly are \$60 more than when he lays by $\frac{3}{8}$ of his salary; what is his salary?

28. An employer increased the wages of his men $\frac{1}{8}$, and it required \$4842.45 weekly to pay them; what did they receive each week before the increase?

29. If $\frac{3}{4}$ of a farm is worth \$690 more than $\frac{3}{5}$ of it, how much is the whole farm worth?

30. A can walk a certain distance in $14\frac{4}{5}$ hours, and B whose rate per hour is $\frac{2}{3}$ that of A, walks $2\frac{1}{2}$ miles per hour what is the distance?

MISCELLANEOUS PROBLEMS IN FRACTIONS.

371. *1.* A man owing a debt paid $\frac{3}{5}$ of it, and afterward paid $\frac{3}{4}$ of the remainder. To make the second payment required \$195.60 less than to make the first payment. What was the debt at first?

2. A grocer bought $480\frac{3}{4}$ bushels of potatoes at $\$.87\frac{1}{2}$ per bushel; he sold $\frac{2}{3}$ of them at $\$1.12\frac{1}{2}$ per bushel, and the remainder at \$.75. What was his gain?

3. A speculator bought 297 head of cattle and sold them so that $\frac{3}{4}$ of what he received for them was equal to their cost; if his gain on all was \$6806.25, what did he pay for them per head?

4. What should a farmer get for a crop of 950 bushels of potatoes, sold at the rate of $\$7\frac{3}{4}$ for $9\frac{1}{2}$ bushels?

5. At \$1.87½ per yard, what will it cost to carpet a house requiring $168\frac{5}{8}$ yards?

6. Two men own a steamer worth \$60000; what is the value of each share if one is $\frac{7}{8}$ of the other?

7. How many bushels of oats at \$.37½ per bushel will pay for 36 barrels of flour at \$$8\frac{7}{8}$ per barrel?

8. A man in business lost $\frac{4}{7}$ of his invested money, and then gained $\frac{3}{5}$ as much as remained, after which he had \$24000; how much had he at first?

9. At \$2.87½ per yard, how many yards of cloth can be paid for with the money received from the sale of 92 bushels of corn at \$$\frac{7}{8}$ per bushel?

10. What is that number whose $\frac{5}{7}$ exceeds its $\frac{3}{5}$ by 160?

11. What will be the cost of 14 dozen of eggs at the rate of 7 eggs for \$¼?

12. A boy bought apples at the rate of 5 for 3 cents, and sold them at the rate of 4 for 5 cents; if his gain was \$1.69 how many did he sell?

13. $\frac{2}{3}$ of what I paid for a horse was what I paid for a sleigh, and together they cost \$340; what was the cost of each?

14. A farmer gave his two sons 286 acres of land; $\frac{5}{8}$ of the share of the one was equal to the share of the other; how many acres did each receive?

15. In how many days will a man earn \$100 by cutting wood at \$$\frac{2}{3}$ per cord, if he can cut 3¾ cords per day?

16. A grocer bought a barrel of syrup containing 42¾ gallons, at \$.75 per gallon; after losing $\frac{1}{9}$ of it by leakage he sold the remainder at \$1.15 per gallon; what was his gain?

17. How many barrels of apples, bought at \$$2\frac{7}{8}$ and sold at \$$3\frac{4}{5}$, will realize a gain of \$111?

18. After paying $\frac{2}{5}$ of a debt, and then ¾ of the remainder, I owe \$$431\frac{3}{8}$ less than at first; what was the debt originally?

19. I pay 16 cents per pound for a turkey weighing $23\frac{9}{16}$ lb.; what change should I receive out of a 10-dollar bill?

20. To dig a certain cellar took 5 men $14\frac{2}{5}$ days; how many men could have done the work in 12 days?

21. On 12 barrels of flour a grocer made a profit of $\$4\frac{1}{2}$ by selling it all for $\$106\frac{1}{2}$; what did the flour cost him per barrel?

22. If \$9520 is divided between A and B so that A receives \$3 as often as B \$4, how much does each receive?

23. A can do a piece of work in $8\frac{3}{4}$ days; what part of the work can he do in $5\frac{3}{4}$ days?

24. A alone can dig a cellar in $15\frac{3}{4}$ days, and B alone can dig it in $16\frac{4}{5}$ days; how long will it take them together to dig the cellar?

25. If a man earns $\$15\frac{3}{4}$ per week when he works 10 hours per day, how much should he receive per week when he works 8 hours per day?

26. A man failing in business paid 65 cents on each dollar of his debts; how much did he owe a creditor who received in settlement \$2665?

27. When A alone plows a field it takes him 8 days, but when assisted by B, it takes 5 days to plow it; in what time could B alone do it?

28. A farm hand engages to work $5\frac{1}{2}$ months for \$121, but works only $3\frac{3}{5}$ months; how much at that rate should he receive?

29. A father earns as much in 3 days as his son earns in 5 days; in a certain time they together earned \$103; how much of it was earned by each?

30. In a school $\frac{13}{20}$ of all the pupils belong to the primary department, and the remainder, 588, belong to the grammar department; how many pupils in the entire school?

31. A butcher sold a roast weighing 8 lb. 12 oz. for \$1.75; what did he charge per pound?

32. If I own $\frac{4}{7}$ of a mill, and $\frac{3}{5}$ of my share is worth $8400, what is the entire mill worth?

33. A lady paid $\frac{2}{5}$ of her money for a silk dress and $\frac{3}{5}$ of the remainder for a shawl; if the dress cost $5.25 more than the shawl, how much money did she have at first?

34. A workman who can make a suit of clothes in $2\frac{3}{4}$ days receives $2.80 per day; how much should be paid per day to a workman who takes $3\frac{3}{5}$ days to make a similar suit?

35. If 5 horses can eat a ton of hay in 8 days, how long will $3\frac{3}{8}$ tons last 9 horses?

36. A man fails owing $20250; of this amount he can pay only $13500; how much should a creditor receive whose claim is $1569?

37. What number must be multiplied by $\frac{5}{8}$ of $3\frac{3}{4}$ to give a product of $25\frac{4}{5}$?

38. Divide 100 into two parts such that $\frac{2}{3}$ of the first part shall be equal to $\frac{3}{4}$ of the second part.

39. I paid $\frac{2}{5}$ of the cost of a house when I purchased it, $\frac{2}{25}$ of its cost at the next payment, and $\frac{1}{5}$ of its cost at the third payment; if I still owed $960, what was the price of the house?

40. What should be paid for a piece of cloth $42\frac{2}{3}$ yards long and $1\frac{1}{4}$ yards wide, if $28.50 is paid for a piece of the same quality $12\frac{2}{3}$ yards long and 27 inches wide?

41. Mary can make a dress in $18\frac{2}{3}$ hours, but with the assistance of Kate the work requires only $11\frac{1}{5}$ hours; in how many hours can Kate alone make it?

42. A man bought a quantity of eggs at 15 cents a dozen and sold them at the rate of 8 for 25 cents; if his gain by the transaction was $13.50, how many eggs were there?

43. If 4 men can do as much work as 7 boys, how much should a boy receive per day if a man is paid $2.80?

44. How many miles per hour must a man travel to make a journey of $52\frac{1}{3}$ miles in 15 hours and 40 minutes?

45. What will be the cost of a hogshead of molasses

containing $126\frac{3}{4}$ gallons, if a barrel of molasses of the same quality containing $43\frac{3}{4}$ gallons cost \$$36\frac{3}{4}$?

46. If $\frac{1}{7}$ of $\frac{2}{3}$ of $1\frac{4}{5}$ times a number is $28\frac{4}{5}$, what is the number?

47. A farmer after selling $\frac{13}{16}$ of his wheat had 192 bushels left; how much wheat had he at first?

48. A speculator bought a quantity of grain, $\frac{1}{3}$ of which was wheat at \$1.45 per bushel, $\frac{1}{3}$ rye at \$1.08 per bushel, and the remainder corn at \$.83 per bushel. How many bushels were there if the entire cost of the grain was \$6073.20?

49. A man earning \$$23\frac{3}{4}$ per week expended \$$18\frac{4}{5}$ per week; in what time could he save enough to pay a debt of \$495?

50. If $9\frac{3}{4}$ tons of hay cost \$$121\frac{7}{8}$, how many tons can be bought for \$175?

51. A horse and a buggy cost \$570; if the horse cost $1\frac{3}{8}$ times as much as the buggy, what was the cost of each?

52. A drover put $\frac{2}{5}$ of his sheep in one field, $\frac{5}{7}$ of the remainder in a second field, and what still remained, which was 180, in a third field; how many sheep had he in all?

53. A fruit dealer bought 96 crates of plums, each containing $1\frac{3}{4}$ bushels, at \$$2\frac{3}{4}$ per bushel, and sold them at 45 cents a half-peck; what was his gain?

54. A man bought 125 acres of land at \$$275\frac{3}{4}$ per acre, and divided it into city lots, of which there were $17\frac{53}{125}$ in an acre. He sold the lots at \$45 each; how much did he gain?

55. A butcher dressed 15 sheep weighing $1687\frac{1}{2}$ pounds; what was their average cost at $12\frac{1}{2}$ cents per pound?

56. A and B together can do a certain work in 40 hours, and B can do it alone in 60 hours; in how many hours can A alone do it?

57. A farm was bought at \$$42\frac{3}{4}$ per acre, $\frac{3}{8}$ of it was sold at \$$54\frac{1}{2}$ per acre, $\frac{2}{3}$ of the remainder at \$$65\frac{3}{8}$ per acre, and the remainder, $60\frac{4}{5}$ acres, at \$$75\frac{3}{4}$ per acre; what was the gain?

DECIMALS.

The lessons on Decimal Parts of Units should be reviewed before this subject is studied.

372. A **decimal fraction** or **decimal** is a fraction whose denominator is 10 or some power of 10.

The denominator of a decimal fraction may be indicated by writing only the numerator and prefixing the decimal point or the decimal point and one or more decimal ciphers. Thus,

$\frac{1}{10}$	is	written	**.1**	$\frac{5}{1000}$	is	written	**.005**
$\frac{5}{10}$	"	"	**.5**	$\frac{55}{1000}$	"	"	**.055**
$\frac{5}{100}$	"	"	**.05**	$\frac{555}{1000}$	"	"	**.555**
$\frac{55}{100}$	"	"	**.55**	$\frac{5}{10000}$	"	"	**.0005**

When the denominator is 100 it is the second power of 10, or 10×10; when 1000, it is the third power of 10, or $10 \times 10 \times 10$; when 10000, it is the fourth power of 10, or 10 taken four times as a factor. In every case it consists of 1 with one or more ciphers annexed, the number of ciphers indicating the number of times it contains 10 as a factor.

The denominator is indicated by so placing the decimal point that the number of places after it in the numerator shall indicate the number of ciphers in the denominator, or the number of times the denominator contains 10 as a factor. Thus,

.05	indicates	**2**	ciphers	in	the	denominator,	or	$\frac{5}{100}$
.005	"	**3**	"	"	"	"	"	$\frac{5}{1000}$
.00055	"	**5**	"	"	"	"	"	$\frac{55}{100000}$
.055555	"	**6**	"	"	"	"	"	$\frac{55555}{1000000}$

373. A **complex decimal** is a decimal and a common fraction written together.

Thus, $.7\frac{1}{2}$, $.33\frac{1}{3}$, $.014\frac{2}{7}$, are complex decimals.

374. Mixed decimals are integers and decimals written together. For brevity they are usually called *decimals.*

Thus, 15.5, 125.25, 75.005 are mixed decimals.

TABLE.

Names of Orders.	Millions.	Hund.-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund.-thousandths.	Millionths.	Ten-millionths.	Hund.-millionths.	Billionths.
Number.	3	4	6	0	9	8	5	.	6	4	3	0	7	5	2	9	5
Orders.	7th. .	6th. .	5th. .	4th. .	3d. . .	2d. . .	1st. . .		1st. . .	2d. . .	3d. . .	4th. .	5th. .	6th. .	7th. .	8th. .	9th. .
	Integral Orders.								Decimal Orders.								

PRINCIPLES OF DECIMALS.

375. I. *Annexing or rejecting a cipher at the right of a decimal does not affect its value.*

Explanation.—Writing .5 as a common fraction and multiplying both terms by 10 gives $\frac{50}{100}$, or .50. Conversely, writing .50 as a common fraction and dividing both terms by 10 gives .5.

Illustration.

$$.5 = \tfrac{5}{10};\ \tfrac{5 \times 10}{10 \times 10} = \tfrac{50}{100} = .50$$

II. *Inserting a cipher between a decimal and the point divides the decimal by 10.*

Explanation.—Writing .5 as a common fraction, and dividing the fraction by 10 gives $\frac{5}{100}$, or .05.

Illustration.

$$.5 = \tfrac{5}{10};\ \tfrac{5}{10} \div 10 = \tfrac{5}{100} = .05$$

III. *Rejecting a cipher from the left of a decimal multiplies the decimal by 10.*

Explanation.—Writing .05 as a common fraction and multiplying the fraction by 10 gives $\frac{5}{10}$, or .5.

Illustration.

$$.05 = \tfrac{5}{100};\ \tfrac{5}{100} \times 10 = \tfrac{5}{10} = .5$$

376. ***Write from dictation and read:***

1.	.985.	*7.*	2.0002.	*13.*	.327806594.
2.	.075.	*8.*	.98756.	*14.*	25.0000056.
3.	3.03.	*9.*	.10101.	*15.*	50000.005.
4.	.005.	*10.*	1.0101.	*16.*	3.03030303.
5.	.501.	*11.*	9.1001.	*17.*	500.5005.
6.	90.9.	*12.*	.04256.	*18.*	3890.00755.

377. ***Express decimally and read:***

1.	$\frac{125}{1000}$.	*6.*	$\frac{84}{1000000}$.	*11.*	$\frac{3270065}{1000000000}$.
2.	$\frac{75}{10000}$.	*7.*	$\frac{10501}{10000000}$.	*12.*	$\frac{202002}{100000000}$.
3.	$3\frac{33}{1000}$.	*8.*	$\frac{7001}{1000000}$.	*13.*	$\frac{516}{100000000}$.
4.	$35\frac{35}{100}$.	*9.*	$53\frac{56}{100000}$.	*14.*	$\frac{1001}{100000000}$.
5.	$175\frac{15}{10000}$.	*10.*	$101\frac{101}{1000}$.	*15.*	$1001\frac{10002}{10000000}$.

378. ***Express in figures as decimals:***

1. Seventy-five hundred and seventy-five hundredths.

2. Nine hundred ninety and nine thousandths.

3. Five thousand five and five hundred-thousandths.

4. Five thousand and fifteen hundred-thousandths.

5. Five thousand five hundred-thousandths.

6. Eight million eighty thousand and eighty-five millionths.

7. Eight million eighty thousand eighty and five millionths.

8. Seven billion fifty million fifty and one thousand one billionths.

9. Three billion one hundred thousand three and three thousand five hundred one hundred-millionths.

10. Five hundred ten million ten thousand ten and ten thousand one hundred five ten-millionths.

11. 8 billion 801 million 725 thousand 540 and 125 thousand 305 hundred-billionths.

12. 5 trillion 320 billion 70 thousand 839 and 561 million 46 thousand 346 trillionths.

REDUCTION OF DECIMALS.

379. To reduce a common fraction to a decimal.

I. Required to reduce $\frac{3}{4}$ to a decimal.

Explanation.—Ciphers are annexed to the numerator 3, the resulting number divided by the denominator 4, and as many decimal places pointed off in the quotient as there were ciphers annexed.

Process.

$$4\,\underline{)\,3.00}$$
$$.75$$

Analysis.

$$\frac{3}{4} = \frac{\frac{30}{10}}{4} = \frac{\frac{300}{100}}{4} = \frac{75}{100} = .75$$

In the analysis, the numerator 3 is both multiplied and divided by 10, giving $\frac{30}{10}$, a result which, if divided by the denominator 4, gives $.7\frac{1}{2}$, a complex decimal.

Multiplying both terms of $\frac{30}{10}$ by 10 gives $\frac{300}{100}$; dividing $\frac{300}{100}$ by the denominator 4 gives $\frac{75}{100}$, or .75, a simple decimal.

380. Rule.—I. *Annex ciphers to the numerator and divide the resulting number by the denominator.*

II. *Point off in the quotient as many decimal places as there were ciphers annexed, prefixing decimal ciphers if necessary.*

NOTE.—If the denominator of a fraction contains prime factors other than 2 or 5, the fraction cannot be reduced to a simple decimal. In such case, the fraction may be changed to a complex decimal, or the division may be carried to several places and the remainder disregarded. In this case, the sign + is used to show that the value is *approximate.*

Thus, $\frac{2}{7} = .28\frac{4}{7}$, or $.2857+$; $\frac{1}{3} = .3\frac{1}{3} = .333+$.

381. ***Reduce the following to equivalent decimals:***

1. $\frac{1}{2}$, $\frac{4}{5}$.	*6.* $\frac{19}{32}$, $\frac{17}{20}$.	*11.* $\frac{119}{250}$, $\frac{193}{256}$.
2. $\frac{3}{5}$, $\frac{5}{8}$.	*7.* $\frac{31}{40}$, $\frac{59}{64}$.	*12.* $\frac{3}{5}$ of $1\frac{3}{8}$.
3. $\frac{7}{8}$, $\frac{11}{16}$.	*8.* $\frac{23}{64}$, $\frac{67}{80}$.	*13.* $\frac{17}{32}$ of $\frac{5}{8}$.
4. $\frac{15}{16}$, $\frac{9}{25}$.	*9.* $\frac{43}{80}$, $\frac{117}{125}$.	*14.* $\frac{7}{8}$ of $10\frac{4}{5}$.
5. $\frac{24}{25}$, $\frac{7}{32}$.	*10.* $\frac{93}{125}$, $\frac{113}{128}$.	*15.* $\frac{4}{5}$ of $12\frac{4}{5}$.

382. ***Change to complex decimals, or to approximate decimals of four places:***

1. $\frac{5}{7}$, $\frac{5}{6}$.	*4.* $\frac{9}{13}$, $\frac{11}{14}$.	*7.* $\frac{2}{3}$ of $8\frac{4}{5}$.
2. $\frac{8}{9}$, $\frac{2}{3}$.	*5.* $\frac{14}{15}$, $\frac{16}{17}$.	*8.* $\frac{9}{13}$ of $17\frac{3}{5}$.
3. $\frac{10}{11}$, $\frac{11}{12}$.	*6.* $\frac{5}{18}$, $\frac{17}{19}$.	*9.* $\frac{5}{6}$ of $25\frac{4}{7}$.

383. To reduce a decimal fraction to a common fraction.

I. Required to reduce .125 to a common fraction in its simplest form.

Process.

$$.125 = \frac{125}{1000}$$

$$\frac{125}{1000} = \frac{25}{200} = \frac{1}{8}$$

Explanation.—The decimal .125 is written as a common fraction $\frac{125}{1000}$. Both terms of $\frac{125}{1000}$ are divided by any convenient common factor, as 5, and both terms of the resulting fraction are divided in like manner by 25. The terms of $\frac{1}{8}$ being prime to each other, the fraction is in its simplest form.

384. Rule.—*Express the decimal as a common fraction and reduce it to its simplest form.*

385. ***Change the following decimals to equivalent fractions in their simplest form:***

1. .16.	*5.* \$.325.	*9.* .035.	*13.* .0425.
2. .75.	*6.* \$.68.	*10.* .0225.	*14.* .00528.
3. .96.	*7.* .675.	*11.* .0084.	*15.* .01875.
4. .64.	*8.* .925.	*12.* .6125.	*16.* .04096.

NOTE.—Complex decimals and mixed decimals are reduced in the same manner as simple decimals.

Thus, $.14\frac{2}{7} = \dfrac{14\frac{2}{7}}{100} = \dfrac{\frac{100}{7}}{100} = \frac{1}{7}$.

386. ***Reduce to common fractions in their simplest form:***

1. $.3\frac{1}{2}$.	*4.* \$$.33\frac{1}{3}$.	*7.* \$$3.62\frac{1}{2}$.	*10.* $2.87\frac{1}{2}$.
2. $.04\frac{1}{2}$.	*5.* \$$.28\frac{4}{7}$.	*8.* \$$1.31\frac{1}{4}$.	*11.* $3.43\frac{3}{4}$.
3. $.16\frac{2}{3}$.	*6.* \$$1.12\frac{1}{2}$.	*9.* \$$.37\frac{1}{2}$.	*12.* $.166\frac{2}{3}$.

ADDITION AND SUBTRACTION OF DECIMALS.

387. To find the sum of two or more decimals.

I. Required to find the sum of 125.5, 174.13, and 85.017.

Explanation.—The decimals are written so that both integral and decimal units of the same order stand in the same column, and the addition is then performed as in whole numbers.

Process.

```
 125.5
 174.13
  85.017
 -------
 384.647
```

388. To subtract one decimal from another.

II. Required to subtract 175.025 from 621.01.

Explanation.—The decimals are written so that both integral and decimal units of the same order stand in the same column, and the subtraction is then performed as in whole numbers.

Process.

```
 621.01
 175.025
 -------
 445.985
```

NOTE.—A vacant decimal order, as thousandths in the minuend above, is treated as if filled with a cipher, since .01 = .010.

389. Rule.—*Write the numbers so that units of the same order stand in the same column, and then perform the addition or subtraction as in whole numbers.*

390. *Write from dictation and find the sum of:*

1.	*2.*	*3.*	*4.*
867.0251	1.0117	.018025	3.01515
65.0028	35.9099	.60061	5.11188
910.1707	64.7203	.170017	.20002
808.001	114.0114	.0012	.11997

5.	*6.*	*7.*	*8.*
95.199	220.0025	.193003	5.22022
32.6505	63.0808	.070707	16.01301
16.002	118.099	.18961	14.29099
198.6209	60.9098	.50055	9.80008

391. ***Find the value of:***

1. \$16.023 + \$9.0075 + \$229.01 + \$305.3005 + \$1192.
2. 200.002 + 19.809 + .6205 + 3.001 + 18.0088.
3. .61025 + 195.1 + 245.0542 + 618 + 7.0027.
4. 35.10001 + 351.0808 + 3.51005 + .026189.
5. .8046 + .70007 + .069 + .320032 + .010109.
6. \$17.002 + \$.2509 + \$8156.303 + \$212 + \$.212.

392. ***Perform the subtractions indicated:***

1. \$99.062 — \$17.915.
2. 506.982 — 505.899.
3. 61.0001 — 15.7089.
4. 1256 — 125.6636.
5. \$14.009 — \$12.0875.
6. 156 — .209654.
7. 50.9286 — 49.8097.
8. 100.1 — 99.87654.
9. 50.005 — 49.0069.
10. 17 — 15.1725.
11. .69212 — .34987.
12. 123.001 — 45.5258.

CONCRETE WRITTEN PROBLEMS.

393. *1.* A man who owned 4 plots of ground had them surveyed. The first contained 87.875 acres, the second 59.3125 acres, the third 125.125 acres, and the fourth 71.6875 acres; how much land was there in the four plots?

2. The mercury in a barometer stood at 30.15 inches, and at the approach of a storm fell suddenly .8 of an inch; at what height did it then stand?

3. A ship on a voyage made 329.625 miles the first day, 53.75 miles more the second day than the first, and 299.125 less the third day than the first and second together; she was then 196.7815 miles from her destination; find the number of miles in the voyage.

4. A party of surveyors in running the boundary line of a certain state measured 18.175 miles the first day, 15.989 miles the second day, and 19.365 miles the third day. On the fourth day they reached the end of the line after measuring 9.35 miles; what was the length of the boundary?

5. During a certain year a rain gauge at San Francisco registered a total rain-fall of 27.935 inches, and one at Salt Lake City registered 4.3125 inches; how much greater was the rainfall at the former place than at the latter?

6. The tonnage of a certain ship was 3628.8765, and that of another 4521.35; find the difference.

7. Mr. Brown having a farm of 800 acres gives to his oldest son 265.325 acres, to the second son 37.615 acres less, to the third son 25.128 acres less than to the second, and to the fourth son the remainder; how many acres does the last receive?

MULTIPLICATION OF DECIMALS.

394. To find the product of two or more decimals.

I. Required to multiply .027 by .05.

Process.

$$\begin{array}{r} .027 \\ .05 \\ \hline .00135 \end{array}$$

Analysis.

$$.027 = \tfrac{27}{1000}$$

$$.05 = \tfrac{5}{100}$$

$$\tfrac{27}{1000} \times \tfrac{5}{100} = \tfrac{135}{100000}$$

Explanation.—The decimals are so written that the multiplication may be performed the most readily. 27 is multiplied by 5 as if both were integers, and decimal ciphers are prefixed to 135 until the product contains as many decimal places as the multiplicand and multiplier together.

The analysis shows why the product is thus pointed off. The decimals are written as common fractions, and the multiplication is performed. The number of ciphers in the denominator of the product is equal to the number of ciphers in the denominators of the multiplicand and the multiplier together. It follows, therefore, that 027 × .05 = .00135.

395. Rule.—*Perform the multiplication as in integers, and point off as many decimal places in the product as there are in the multiplicand and the multiplier together.*

396. ***Find the value of:***

1. $.0875 \times 875.$	*5.* $.0098 \times .005.$	*9.* $\$19.66\tfrac{2}{3} \times .1525.$
2. $9.125 \times 62.5.$	*6.* $892.5 \times .009.$	*10.* $\$75.025 \times 198.5$
3. $75.05 \times 6.25.$	*7.* $\$8.625 \times .0756.$	*11.* $\$112.25 \times 496.5$
4. $12.56 \times .075.$	*8.* $\$93.875 \times .065.$	*12.* $\$13.08\tfrac{1}{3} \times 92.35.$

13. $(\$1.875 + \$1.0625) \times 75.$

14. $(187.5 - 75.65 + .05) \times 8.75.$

15. $(\$75.56 - \$18.33\tfrac{1}{3}) \times 12.5 - (\$87.75 - \$79.625) \times 1.75.$

16. $(3275 - 32.75) \times .0565 - (4965 - 496.5) \times .0075.$

17. $(87 - .09 + 9.5) \times .125 - (9.8 + 96 - .005) \times .005.$

CONCRETE WRITTEN PROBLEMS.

397. *1.* How far can a man drive in 15.75 hours at the rate of 9.875 miles per hour?

2. What will 59.625 cwt. of beef cost at $11.625 per cwt.?

3. A man paid $75.875 per acre for a farm containing 968.375 acres; what did it cost him?

4. Find the freight on 5865.875 tons of coal at $1.125 per ton.

5. Find the cost of carrying a cargo of 975 bales of cotton from New Orleans to Liverpool at $3.625 per bale.

6. In a perch of stone there are 24.75 cubic feet; how many cubic feet in a wall containing 56.625 perches, and what will it cost at $3.375 a perch?

7. A farmer raised on a field of 39.925 acres an average of 35.75 bushels of wheat to the acre, and sold it all at $1.375 per bushel; what did he get for it?

8. The circumference of the earth is 3.14159 times its diameter; what is the circumference, the diameter at the equator being 7925.604 miles?

9. A locomotive weighing 45.896 tons drew a train of 39 cars across a bridge; if the cars weighed on an average 9.875 tons, and the entire train, including the locomotive, was on the bridge at once, what was the weight on the bridge?

10. At \$.0625 per ton, what toll must a vessel of 3685.925 tons pay for the privilege of passing through the Suez Canal?

11. In a city of 57840 inhabitants .125 of the population are children attending the public schools; find how many children attend the public schools.

DIVISION OF DECIMALS.

398. To find the quotient of one decimal divided by another.

I. Required to divide .625 by .5.

Explanation. — The division is performed as in whole numbers, and as many decimal places pointed off in the quotient as the decimal places in the dividend exceed in number those of the divisor.

The analysis shows the reason for thus pointing off the quotient. By the inversion of the divisor each cipher it contains cancels a cipher in the denominator of the dividend. The denominator of the product, therefore, contains as many ciphers less than the denominator of the dividend as there are ciphers in the denominator of the divisor Hence, it follows that $.625 \div .5 = 1.25$.

Process.

$$.5\,)\,\underline{.625}$$
$$1.25$$

Analysis.

$$.625 \div .5 = \frac{625}{1000} \div \frac{5}{10} = \frac{\overset{125}{\cancel{625}}}{\underset{100}{\cancel{1000}}} \times \frac{\cancel{10}}{\cancel{5}} = \frac{125}{100} = 1.25$$

II. Required to divide 1.1 by .08.

Explanation.—One cipher is annexed to 1.1, so that the number of decimal places in the dividend may be equal to the number of decimal places in the divisor. If 1.10 were exactly divisible by .08, the quotient would be a whole number, since an integer multiplied by .08 would give a product with two decimal places. There being a remainder, however, additional ciphers are annexed to the dividend until the division is exact, or until the result is sufficiently accurate for practical purposes.

Process.

$$.08\,)\,\underline{1.10\,00}$$
$$13.75$$

III. Required to divide .0025 by .5.

Explanation. — The division is performed as in whole numbers. Decimal ciphers are prefixed to the significant figure of the quotient until the quotient contains as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.

Process.

$$.5\,\overline{)\,.0025}$$
$$.005$$

Analysis.

$$.0025 \div .5 = \frac{25}{10000} \div \frac{5}{10} = \frac{\overset{5}{\cancel{25}}}{\cancel{10000}} \times \frac{\cancel{10}}{\cancel{5}} = \frac{5}{1000} = .005$$

399. Rule.—*Annex ciphers to the dividend, if necessary; divide as in whole numbers, and point off the quotient so that the decimal places of the divisor and quotient shall together be equal to the decimal places of the dividend.*

400. ***Find the value of:***

1. 31.5÷.126.	*5.* 3.706÷.017.	*9.* 177.12÷.1025.
2. 5.2÷.005.	*6.* 3217÷.0625.	*10.* 467.46÷.0504.
3. 12.6÷.0012.	*7.* 1.69÷.0013.	*11.* 25075÷1.003.
4. .66÷.015.	*8.* 884÷.0026.	*12.* .7967÷.00514.

401. ***Find quotients to four decimal places:***

1. 23.5÷9.3.	*5.* .2555÷.16.	*9.* 1.019÷.036.
2. 7.013÷.0163.	*6.* 20.2÷.0006.	*10.* $50.05÷1.13.
3. 1.09÷1.0097.	*7.* $1304÷$141.	*11.* $25.25÷$.17.
4. $123÷$609.	*8.* $92÷$3.33.	*12.* .0719÷27.53.

402. ***Perform operations, and give results in decimal form:***

1. $1.005+10.0023\div.0015.$	*4.* $(2.655\div11\frac{4}{5})\div.00225.$
2. $(9.05\times1.08)\div.00027.$	*5.* $(7\frac{1}{2}\times\frac{1}{5}+\frac{17}{25})\div.0005.$
3. $12\frac{3}{8}+1.275+\frac{19}{10000}+6.$	*6.* $(\frac{5}{16}\times.028)\div.00035.$

CONCRETE WRITTEN PROBLEMS.

403. *1.* If 175.75 bushels of potatoes cost $147.63, what will 56.25 bushels cost ?

2. If the freight on 395.52 cwt. of beef from Texas to New York is $247.20, what is the freight per cwt.?

3. At what average rate per hour must a train run in order to go from New York to Chicago in $33\frac{1}{3}$ hours, the distance being 952 miles?

4. Lead weighs 11.445 times as much as an equal bulk of water, and a cubic foot of water weighs 62.5 lb. With how many cubic feet of lead must a yacht be ballasted in order that the ballast shall weigh 91.56 tons?

5. A man walked 12.575 miles on Monday, 12.09 on Tuesday, 15.125 on Wednesday, and by Saturday evening had completed 100 miles; at what rate did he travel the last three days?

6. A charitable man divided 95.625 tons of coal equally among 125 poor people; if the coal was worth $4.60 a ton, what was the value of each share?

7. A farm of 150 acres is sold in plots of 1.25 acres each, the price for each plot being $80.25; what is received for the farm?

8. A man paid .18 of his salary for rent, .22 for food, .15 for clothing and incidental expenses, and put the remainder, amounting to $900, in the bank; what was his salary?

9. If 227.25 acres of land were divided into 18 equal fields, how many acres would there be in each field?

10. To pay a company of soldiers consisting of 108 men, each receiving the same, required $2565; how much was paid to each?

11. The circumference of the earth at the equator is 24899.022 miles; find to 4 decimal places the length of one degree of longitude at the equator, each degree being $\frac{1}{360}$ of the circumference.

12. The sovereign, a gold coin of England, is worth $4.8665; how many sovereigns are worth $15397.606?

13. Of an army .22 were disabled in battle, .0125 deserted, and .035 were discharged. There remained 15236 men; how many were in the army at first?

REVIEW QUESTIONS.

404. Properties of Numbers.—Define and illustrate exact divisor, prime number, composite number, even number, odd number. What are the factors of a number? What is a power? An exponent? Define and illustrate prime factor, composite factor. Give the principles of factoring. When are numbers prime to each other? What numbers are divisible by 2? By 3? By 4? By 5? By 6? By 8? By 9? By 10? Give the rule for finding the prime factors of a number. Give the principles governing cancellation. Define and illustrate common divisor, and greatest common divisor. Give the principles. Give the rule. What is a multiple of a number? A common multiple of two or more numbers? The least common multiple of two or more numbers? Give the principles. The rule.

405. Common Fractions.—Define and illustrate fractional unit, fraction, denominator, numerator. Upon what does the value of a fraction depend? What is a proper fraction? An improper fraction? Give the principles of fractions and illustrate each. What is reduction of fractions? When is a fraction in its simplest form? Give the rule for reducing a fraction to its simplest form. For reducing a mixed number to an improper fraction. Give the rule for reducing to an equivalent fraction having a given denominator:—a fraction; an integer; a mixed number. What is necessary that fractions may be added? Subtracted? Give the rule for addition of fractions. For subtraction. What is the rule for multiplying an integer by a fraction or a fraction by an integer? For finding the product of two or more fractions or mixed numbers? For dividing a fraction by an integer? An integer by a fraction? What is the rule for dividing a fraction by a mixed number or a mixed number by a fraction? A fraction by a fraction?

406. Decimals.—What is a decimal? How may the denominator of a decimal be indicated? What is a complex decimal? A mixed decimal? Illustrate each. State the principles of decimals and illustrate. Give the rule for reducing a common fraction to a decimal. What is an approximate decimal? What is the rule for reducing a decimal fraction to a common fraction? For adding and subtracting decimals? For multiplying decimals? For dividing decimals? How is the product pointed off in multiplication of decimals? How is the quotient pointed off in division of decimals?

DENOMINATE NUMBERS.

407. A **denominate number** is a concrete unit or a collection of concrete units.

Thus, $5, 6 pounds, 9 miles and 17 rods, are denominate numbers.

408. A **simple denominate number** expresses units of but one denomination.

Thus, 5 pounds, 9 feet, 8 rods, are simple denominate numbers.

409. A **compound denominate number** expresses two or more denominations of the same nature.

Thus, 5 gallons 3 quarts 1 pint is a compound denominate number.

The scales of compound denominate numbers are generally variable.

Thus, for a compound denominate number consisting of *pints*, *quarts*, *pecks* and *bushels*, the scale is 2, 8, 4, while for United States money the scale is 10.

410. A **denominate fraction** is a fraction expressing one or more of the equal parts of a denominate unit.

Thus, $\frac{1}{8}$ of a yard, $\frac{5}{8}$ of a mile, .95 of a ton, are denominate fractions.

REDUCTION OF DENOMINATE NUMBERS.

411. The **reduction** of a denominate number is the process of changing its denomination without changing its value.

Thus, changing *miles* to *inches*, or *inches* to *miles*, is reduction.

412. **Reduction descending** is the process of changing a denominate number to an equivalent denominate number of lower denomination.

Thus, to change *bushels* to *quarts* is reduction descending.

413. Reduction ascending is the process of changing a denominate number to an equivalent denominate number of higher denomination.

Thus, to change *minutes* to *years* is reduction ascending.

414. To reduce a compound denominate number to lower denominations.

I. Required to reduce 15 bu. 3 pks. 5 qts. to quarts.

Explanation.—The 15 bushels are reduced to pecks by multiplying by 4, since there are 4 pecks in one bushel, and the 3 pecks are added, giving 63 pecks. The 63 pecks are reduced to quarts by multiplying by 8, since there are 8 quarts in one peck, and the 5 quarts are added, giving 509 quarts, the denomination required.

Process.

15 bu. 3 pks. 5 qts.
4
63 pks.
8
509 qts.

Ans. 15 bu. 3 pks. 5 qts. = 509 qts.

Note.—See Appendix, page 329.

ORAL EXERCISES.

415. *1.* How many ounces in 2 lb.? In 3 lb.? In 5 lb.? In 10 lb.?

2. How many feet in 6 yds.? In 12 yds.? In 2 rds.?

3. Reduce 5 bu. 3 pks. to pecks. To quarts.

4. Change 5 lb. 10 oz. to ounces. 5 cwt. 75 lb. to pounds.

5. Reduce 3 gal. 3 qts. 1 pt. 1 gi. to gills.

6. What will 2 bu. 3 pks. 5 qts. of chestnuts cost at 10¢ a quart?

7. What will be the cost of 7 gal. 2 qts. 1 pt. of milk at 5¢ a pint?

8. Find the cost of 5 cwt. 83 lb. of maple sugar at 10¢ a pound.

9. What will 5 T. 15 cwt. of ice cost at 20 cents a cwt.?

10. What cost 10 bu. 3 pks. of potatoes at 40¢ a peck?

11. What will it cost, at 10 cents a yard, to enclose a square field 10 rods on a side with a stone fence?

WRITTEN PROBLEMS.

416. *1.* Reduce 95 T. 83 lb. 12 oz. to ounces.

2. Change 1175 mi. 175 rds. 4 yds. 2 ft. 11 in. to inches.

3. A grocer sold 75 bu. 3 pks. 7 qts. of cranberries at 15¢ a quart; what did he get for them?

4. In 75 long tons 3 qrs. 95 lb. of coal, how many pounds?

5. A butcher bought 9 T. 17 cwt. 93 lb. of beef at 8$\frac{3}{4}$¢ a pound, and sold it at 14¢ a pound; what did he gain?

6. An ocean steamer during a voyage of 3825 miles burns coal at the rate of 3 T. 5 cwt. 95 lb. every 25 miles; how many pounds of coal does she burn in the voyage?

7. How many steps of 30 inches each must a man take in walking 13 mi. 224 rds. 3 yds.?

8. What will be the cost of grading 31 mi. 175 rds. 4 yds. 2 ft. of road at 56¢ a foot?

9. How many panels of fence 12 feet long will it take to enclose a pasture 60 rods long and 48 rods wide?

10. How many times will a wheel 22 feet in circumference turn in going 11 mi. 132 rds.?

417. To reduce a compound denominate number to higher denominations.

I. Required to reduce 937865 inches to miles.

Explanation. — Dividing 937865 inches by 12, the number of inches in one foot, gives 78155 feet, and 5 inches. Dividing 78155 feet by 3, the number of feet in one yard, gives 26051 yards, and 2 feet. Dividing 26051 yards by 5½, the number of yards in one rod, gives 4736 rods, and 6 half-yards, equal to 3 yards. Dividing 4736 rods by 320, the number of rods in one mile, gives 14 miles and 256 rods.

Process.

12	937865 in.
3	78155 ft. + 5 in.
5½	26051 yds. + 2 ft.
2	2
11	52102 half-yards.
320	4736 rds. + 6 half-yds.
	14 mi. + 256 rds.

Ans. 14 mi. 256 rds. 3 yds. 2 ft. 5 in.

ORAL EXERCISES.

418. *1.* How many gallons in 96 gills? In 160 gills?

2. Reduce 35 pints to higher denominations. 75 gills.

3. How many T., cwt., and lb. in 3875 pounds?

4. Change 160 inches to higher denominations.

5. In 320 quarts how many pecks? How many bushels?

6. What will 144 inches of ribbon cost at 30¢ a yard?

7. What will 96 pints of cider cost at 40¢ a gallon?

8. Find the cost of 640 pints of chestnuts at $4 a bushel.

WRITTEN PROBLEMS.

419. *1.* How many tons in 8756945 ounces?

2. Change 185675 gills to gallons.

3. How many bushels in 83760 pints?

4. Reduce 9596875 inches to miles.

5. How many long tons in 18698375 ounces?

6. At $1.25 a rod, what will be the cost of building 16170 feet of fence?

7. A man sold nuts at 5¢ a half-pint and received for them $76.80; how many bushels did he sell?

8. A contractor was paid $9180 for making a road; if he received $.50 a yard, how many miles were there?

420. To reduce a denominate fraction to integers of lower denominations.

I. Required to reduce $\frac{5}{6}$ of a bushel to integers of lower denominations.

Process.

$$\tfrac{5}{6} \times 4 = \tfrac{20}{6} = 3\tfrac{1}{3} \text{ pks.}$$
$$\tfrac{1}{3} \times 8 = \tfrac{8}{3} = 2\tfrac{2}{3} \text{ qts.}$$
$$\tfrac{2}{3} \times 2 = \tfrac{4}{3} = 1\tfrac{1}{3} \text{ pts.}$$

Explanation.—Multiplying $\frac{5}{6}$ of a bushel by 4, the number of pecks in a bushel, gives $\frac{20}{6}$ pecks, equal to $3\frac{1}{3}$ pecks. Multiplying $\frac{1}{3}$ of a peck by 8, the number of quarts in a peck, gives $\frac{8}{3}$ quarts, equal to $2\frac{2}{3}$ quarts. Multiplying $\frac{2}{3}$ of a quart by 2, the number of pints in a quart, gives $\frac{4}{3}$ pints, equal to $1\frac{1}{3}$ pints.

Ans. 3 pks. 2 qts. $1\frac{1}{3}$ pts.

II. Required to reduce .975 of a gallon to integers of lower denominations.

Explanation	Process.
Explanation.—Multiplying .975 of a gallon by 4, the number of quarts in a gallon, gives 3.9 quarts. Multiplying .9 of a quart by 2, the number of pints in a quart, gives 1.8 pints. Multiplying .8 of a pint by 4, the number of gills in a pint, gives 3.2 gills.	.975 gal.
	4
	3.900 qts.
	2
	1.800 pts.
	4
	3.200 gills.

Ans. 3 qts. 1 pt. 3.2 gi.

ORAL EXERCISES.

421. *1.* How many pecks in $\frac{4}{5}$ bu.? In $\frac{9}{10}$ bu.?

2. How many yards in .8 of a rod? In .9 of a rod?

3. Change .75 of a gallon to quarts. .65 of a gallon to lower denominations.

4. In $\frac{3}{8}$ of a bushel how many pecks and quarts? In .375 of a bushel?

5. How much will $\frac{11}{16}$ of a gallon of machine oil cost at 5 cents a gill?

6. What will .625 of a pound of caramels cost at 4 cents an ounce?

WRITTEN PROBLEMS.

422. *1.* Reduce .3125 of a mile to lower denominations.

2. In $\frac{15}{16}$ of a mile how many rods? Yards? Feet?

3. How many quarts in 5.875 bushels? In 17.75 bushels how many pints?

4. Reduce .8375 of a ton to integers of lower denomination.

5. In 7.75 long tons how many pounds? Ounces?

6. What will be the cost of 13.4375 miles of insulated copper wire at $3\frac{1}{2}$ cents a foot?

7. A manufacturer made .46875 of a ton of German silver into spoons, each weighing one ounce. Allowing nothing for waste, how much are they worth at $62\frac{1}{2}$ cents a dozen?

423. To reduce a denominate number or a denominate fraction to a fraction of higher denomination.

I. Required to reduce 3 qts. 1 pt. to the fraction of a bushel.

Explanation.—3 quarts 1 pint reduced to pints gives 7 pints, and 1 bushel reduced to pints gives 64 pints. 1 pint is $\frac{1}{64}$ of a bushel, and 7 pints are $\frac{7}{64}$ of a bushel.

Process.

$$\begin{aligned} 3 \text{ qts. } 1 \text{ pt.} &= 7 \text{ pts.}; \\ 1 \text{ bushel} &= 64 \text{ pts.}; \\ 3 \text{ qts. } 1 \text{ pt.} &= \tfrac{7}{64} \text{ bu.} \end{aligned}$$

II. Change .75 of a pint to the decimal of a gallon.

Explanation.—Dividing .75 of a pint by 2, the number of pints in a quart, gives .375 of a quart. Dividing .375 of a quart by 4, the number of quarts in a gallon, gives .09375 of a gallon.

Ans. .09375 gal.

Process.

$$\begin{array}{r|l} 2 & .75 \text{ pt.} \\ 4 & .375 \text{ qt.} \\ & .09375 \text{ gal.} \end{array}$$

III. Change $\frac{5}{8}$ of a foot to the fraction of a mile.

Explanation.—Dividing $\frac{5}{8}$ of a foot by $16\frac{1}{2}$ reduces it to the fraction of a rod. Dividing that result by 320 reduces it to the fraction of a mile, the result being $\frac{1}{8448}$ of a mile.

Process.

$$\tfrac{5}{8} \div 16\tfrac{1}{2} \div 320 =$$
$$\tfrac{5}{8} \times \tfrac{2}{33} \times \tfrac{1}{320} = \tfrac{1}{8448}$$

ORAL EXERCISES.

424. *1.* What part of a bushel in 1 peck? In 12 quarts?

2. What part of a bu. in 1 pk. 5 qts.? In 2 pks. 3 qts.?

3. What part of a gallon in $\frac{3}{4}$ of a quart? In $\frac{1}{2}$ of a pint?

4. What will 3 qts. of syrup cost at 80 cts. a gallon?

5. What will 2 pks. 3 qts. of nuts cost at 96 cts. a bushel?

6. In 1.5 cwt. what part of a ton?

WRITTEN PROBLEMS.

425. *1.* 16.8 hundredweight is what part of a ton?

2. If $14.85 is paid for 7 bu. 2 pks. 7 qts. 1 pt. of chestnuts, what would a bushel cost at that rate?

3. How much will 17 gal. 1 qt. 1 pt. 1 gi. of varnish bring, if retailed at 96 cents a gallon?

4. What will 11 cwt. 75 lb. of coal cost at $4.80 a ton?

5. When $15.24 is paid for 2 barrels of cranberries containing together 5 bu. 3 pks. 6 qts. 1 pt., what is the price per bushel?

6. A contractor was paid $4860 per mile for grading the roadbed of a railroad; how much should he get for grading 536 rods?

7. What will it cost to make 25.6 rods of wire fence, at the rate of $62.75 a mile?

ADDITION AND SUBTRACTION OF COMPOUND DENOMINATE NUMBERS.

Compound denominate numbers are added and subtracted the same as abstract numbers, except that with the latter the *scale* is 10, while in denominate numbers the *scale* usually varies.

426. To find the sum of two or more compound denominate numbers.

I. Required the sum of 57 mi. 95 rds. 4 yds. 2 ft. 9 in., 83 mi. 225 rds. 3 yds. 1 ft. 8 in., 71 mi. 171 rds. 2 ft. 5 in.

Explanation. — The sum of the column of inches is 22 inches, equal to 1 foot 10 inches. The 10 is written under the column of inches, and the 1 is added in with the column of feet, giving 6 feet, equal to 2 yds. 0 ft. The 0 is written under the column of feet, and the 2 added in with the column of yards. In this manner the operation proceeds from column to column, until all have been added.

Process.

mi.	rds.	yds.	ft.	in.
57	95	4	2	9
83	225	3	1	8
71	171	0	2	5
212	172	$3\frac{1}{2}$	0	10
		$\frac{1}{2}$	=1	6
212	172	3	2	4

If fractions occur, as under the column of yards in this example, they are reduced to integers of lower denomination, and added to the like lower denominations.

WRITTEN PROBLEMS.

427. ***Find the sum of the following:***

1. 175 mi. 300 rds. 4 yds. 2 ft. 9 in., 836 mi. 267 rds. 3 yds 1 ft. 5 in., and 327 mi. 183 rds. 1 yd. 2 ft. 10 in.

2. 156 bu. 3 pks. 5 qts. 1 pt., 397 bu. 1 pk. 7 qts., 549 bu 6 qts. 1 pt., and 95 bu. 2 pks. 6 qts.

3. 164 gal. 3 qts. 1 pt. 3 gi., 195 gal. 2 qts. 2 gi., 117 gal. 1 qt. 1 pt., 143 gal. 3 qts. 1 pt. 2 gi., and 227 gal. 3 qts. 1 pt. 3 gi.

4. 15 T. 17 cwt. 87 lb. 15 oz., 35 T. 15 cwt. 93 lb. 12 oz., 43 T. 12 cwt. 54 lb. 13 oz., and 79 T. 19 cwt. 47 lb. 14 oz.

5. 75.15625 mi., $129\frac{7}{12}$ mi., $169\frac{5}{16}$ mi., and $228\frac{11}{30}$ mi.

428. To find the difference between two compound denominate numbers.

I. Required to find the difference between 183 bu. 2 pks. 2 qts. and 98 bu. 3 pks. 5 qts. 1 pt.

Process.

bu.	pks.	qts.	pts.
183	2	2	0
98	3	5	1
84	2	4	1

Explanation.—The denominate numbers are written so that units of the same denomination stand in the same column. Changing 1 of the 2 quarts to pints and subtracting 1 pint gives 1 pint. Changing 1 of the 2 pecks to quarts, adding it to the 1 quart remaining, and from the sum subtracting the 5 quarts of the subtrahend, gives 4 quarts. Like treatment of the remaining columns gives 2 pecks in the column of pecks and 84 bushels in the column of bushels.

WRITTEN PROBLEMS.

429. *1.* Find the difference between 497 bu. 1 pk. 5 qts. and 189 bu. 3 pks. 7 qts. 1 pt.

2. From 91 mi. 183 rds. 4 yds. 1 ft. 5 in. subtract 49 mi. 184 rds. 5 yds. 2 ft. 9 in.

3. From 193 T. 17 cwt. 19 lb. 5 oz. take 139 T. 19 cwt. 89 lb. 15 oz.

4. From a hogshead of molasses containing 126 gallons there were drawn 79 gal. 1 qt. 1 pt.; how much remained in the hogshead?

5. What is the difference between $\frac{23}{44}$ of a mile and 59 rds. 3 yds. 2 ft. 9 in.?

6. From the sum of 75 bu. 3 pks. 5 qts. 1 pt. and 93 bu. 2 pks. 7 qts. take 49 bu. 1 pk. 6 qts. 1 pt.

MULTIPLICATION OF COMPOUND DENOMINATE NUMBERS.

430. To multiply a compound denominate number by an abstract number.

I. Required to multiply 5 mi. 178 rds. 4 yds. 2 ft. 10 in. by 8.

Process.

mi.	rds.	yds.	ft.	in.
5	178	4	2	10
				8
44	151	$\frac{1}{2}$	1	8
		$\frac{1}{2}$ =	1	6
44	151	1	0	2

Explanation.—The product of 10 inches by 8 is 80 inches, equal to 6 feet 8 inches. Adding the 6 feet to the product of 2 feet by 8 gives 22 feet, equal to 7 yards 1 foot. Adding the 7 yards to the product of 4 yards by 8 gives 39 yards, equal to 7 rods $\frac{1}{2}$ yard. Adding the 7 rods to the product of 178 rods by 8 gives 1431 rods, equal to 4 miles 151 rods. Adding the 4 miles to the product of 5 miles by 8 gives 44 miles. Finally, the $\frac{1}{2}$ yard reduced to integers of lower denominations and added gives 44 mi. 151 rds. 1 yd. 0 ft. 2 in. for the answer.

WRITTEN PROBLEMS.

431. *1.* Multiply 33 gal. 3 qts. 1 pt. 3 gi. by 12.

2. Multiply 15 mi. 218 rds. 3 yds. 1 ft. 9 in. by 35.

3. An average of 37 bu. 3 pks. 6 qts. 1 pt. of wheat to the acre was produced on a field of 23 acres; how much wheat was produced on the entire field?

4. What is the total capacity of 45 casks, the capacity of each being 35 gal. 3 qts. 1 pt. 2 gi.?

5. A train of 39 cars is loaded with pig iron; the average load of a car being 9 T. 17 cwt. 93 lb., what was the weight of pig iron on the entire train?

6. A pedestrian traveled for 17 days at the average rate of 23 mi. 189 rds. 5 yds. a day; how far did he travel?

7. A party of linemen put up in one day 23 coils of telegraph wire, each coil containing 50 rds. 4 yds. 2 ft. of wire; how much did they put up in all?

8. A regiment of soldiers ate 1 T. 3 cwt. 85 lb. of meat in one day; how much would last them a month of 30 days?

9. A letter-carrier traveled on an average 13 mi. 195 rds. 2 yds. a day; how far would he travel in 26 days?

DIVISION OF COMPOUND DENOMINATE NUMBERS.

432. To divide a compound denominate number by an abstract number.

I. Required to divide 215 bu. 1 pk. 1 qt. 1 pt. by 9.

Process.

	bu.	pk.	qt.	pt.
9)	215	1	1	1
	23	3	5	1

Explanation.—Dividing 215 bushels by 9 gives 23 bu. and a remainder of 8 bushels, equal to 32 pecks. The sum of 32 pecks and 1 peck divided by 9 gives 3 pecks and a remainder of 6 pecks, equal to 48 quarts. The sum of 48 quarts and 1 quart divided by 9 gives 5 quarts and a remainder of 4 quarts, equal to 8 pints. The sum of 8 pints and 1 pint divided by 9 gives 1 pint.

Ans. 23 bu. 3 pks. 5 qts. 1 pt.

WRITTEN PROBLEMS.

433. *1.* Divide 44 mi. 125 rds. 4 yds. 0 ft. 10 in. by 8.

2. Divide 337 bu. 1 pk. 4 qts. 1 pt. by 13.

3. Divide 1768 T. 2 cwt. 8 lb. 2 oz. by 61.

4. A farmer delivered for shipment 2110 bu. 2 pks. of grain in 18 wagon-loads; what average amount did he deliver at a load?

5. A steamer that crossed the ocean from New York to Havre in 8 days, burned during the voyage 789 T. 11 cwt. of coal. Reckoning this as long tons, what average amount was burned daily?

6. A milkman in a month of 30 days served his customers with 5068 gal. 1 pt. of milk; what was the average daily delivery?

7. A farmer delivered to a speculator 3432 bu. $3\frac{3}{4}$ pks. of potatoes in 17 days; what was the average daily delivery?

8. A certain hogshead contained exactly 149 gal. 1 pt. 3 gi. of molasses. If the molasses would just fill 25 kegs of equal size, how much would each keg hold?

MEASURES OF LENGTH AND DISTANCE

For table of ordinary Linear Measure, see page 89.

SURVEYORS' LINEAR MEASURE.

434. Surveyors' Linear Measure is used by surveyors in measuring distances.

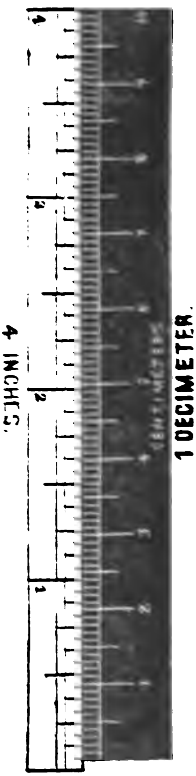
7.92	inches	=	1 link, . . .	*l.*
25	links	=	1 rod, . . .	*rd.*
100	links or 4 rds.	=	1 chain, . . .	*ch.*
80	chains	=	1 mile, . . .	*mi.*

NOTE.—The chain used by surveyors is called Gunter's chain and consists of 100 links. Its length is 792 inches, equal to 66 feet, or 4 rods. Its divisions are decimal, so that chains and links may be written as one number in the same manner as dollars and cents.

Thus, since 35 links equal .35 of a chain, 25 chains 35 links is written *25.35 chains.*

METRIC LINEAR MEASURES.

10 millimeters (*mm.*)	=	1 centimeter, . *cm.*
10 centimeters	=	1 decimeter, . *dm.*
10 decimeters	=	1 meter. . . *m.*
10 meters	=	1 dekameter, . *Dm.*
10 dekameters	=	1 hektometer. *Hm.*
10 hektometers	=	1 kilometer. . *Km.*
10 kilometers	=	1 myriameter, *Mm.*



NOTE 1.—The Metric System of weights and measures has been legalized in the United States. Its fundamental unit is the meter, from which all the other units of the system are derived. The meter is equal to 39.37 + inches.

NOTE 2.—The meter is defined as the ten-millionth part of the distance from the Equator to the North Pole, measured on the meridian passing through Paris. Later measurements have shown that the meter does not exactly correspond with the length required by the definition.

NOTE 3.—The principal point of superiority of the metric measures is in their decimal scale. As in the case of United States money, several denominations may be written together as one number. Thus, 9 Mm. 7 Km. 5 Hm. 3 Dm. 8 m. 4 dm. 6 cm. 5 mm. may be written as one denomination: 97538465 mm., or 9753846.5 cm., or 97538.465 m., or 9.7538465 Mm., etc. Reduction ascending and reduction descending thus become a mere matter of moving the decimal point.

ORAL EXERCISES.

435. *1.* How many rods in 1 chain? In 100 links? In 25 links? In 75 links?

2. What part of 4 rods in 10 links? What part of 1 rod? What part of 4 rods in 50 links? In 75 links? In 25 links?

4. A farmer delivered for shipment 2110 bu. 2 pks. of grain in 18 wagon-loads; what average amount did he deliver at a load?

5. A steamer that crossed the ocean from New York to Havre in 8 days, burned during the voyage 789 T. 11 cwt. of coal. Reckoning this as long tons, what average amount was burned daily?

6. A milkman in a month of 30 days served his customers with 5068 gal. 1 pt. of milk; what was the average daily delivery?

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25	links	=	1 rod, . . .	*rd.*
100	links or 4 rds.	=	1 chain, . . .	*ch.*
80	chains	=	1 mile, . . .	*mi.*

NOTE.—The chain used by surveyors is called Gunter's chain and consists of 100 links. Its length is 792 inches, equal to 66 feet, or 4 rods. Its divisions are decimal, so that chains and links may be written as one number in the same manner as dollars and cents.

Thus, since 35 links equal .35 of a chain, 25 chains 35 links is written *25.35 chains.*

METRIC LINEAR MEASURES.

10 millimeters (*mm.*)	=	1 centimeter,	*cm.*
10 centimeters	=	1 decimeter,	*dm.*
10 decimeters	=	1 meter,	*m.*
10 meters	=	1 dekameter,	*Dm.*
10 dekameters	=	1 hektometer,	*Hm.*
10 hektometers	=	1 kilometer,	*Km.*
10 kilometers	=	1 myriameter,	*Mm.*

4 INCHES.

1 DECIMETER.

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435. *1.* How many rods in 1 chain? In 100 links? In 25 links? In 75 links?

2. What part of 4 rods in 10 links? What part of 1 rod? What part of 4 rods in 50 links? In 75 links? In 25 links?

3. How many links in 1 rod? In 2 rods? In 8 rods?

4. How many chains in 1 mile? How many rods?

5. How many inches in 1 link? In 10 links? In 100 links? In 1000 links? In 100000 links?

6. How many feet in 100 links? In 10 links? In 1000 links? In 100000 links?

7. How many links in 66 feet? In 6.6 feet? In .66 feet? In 660 feet? In 6600 feet?

8. How many mm. in 8 cm.? In 3 dm.? In 2 m.? In 5 Dm.?

9. How many mm. in 9.5 cm.? In 1.85 dm.? In 56.875 m.?

10. How many m. in 87500 mm.? In 18750 dm.?

WRITTEN PROBLEMS.

436. *1.* How many links in 79 ch. 95 l.? In 198 ch. 5 l.? In 370 ch. 7 l.?

2. Reduce 18725 links to chains. 9872.5 l. to chains.

3. How many chains in 25 miles? In 18.75 miles?

4. Change 518 mi. 96 rds. to chains. To links.

5. Reduce .625 mi. to chains. To links.

6. How many miles in 189750 links? In 500000 links?

7. How many links in 633600 inches? In 52800 ft.?

8. How many links in .78525 of a mile?

9. How many chains in 975.75 rods? In 70400 yds.?

10. How many miles in 3565.85 chains?

11. Reduce $\frac{5}{8}$ of a mile to links. To chains.

12. How many links in 5 mi. 17 rds. 5 yds.?

13. Change 8 Km. 5 Dm. 5 m. 3 dm. to cm. To mm.

14. Find the sum of 18 Mm. 6 Km. 5 Hm. 3 m., 23 Mm. 7 Km. 1 Hm. 5 Dm., and 17 Mm. 5 Dm.

NOTE.—In the Metric System numbers ma[illegible] in one denomination, and the operation performed as in t[illegible]ple numbers.

15. Find the difference between [illegible] 5 Dm. 7 m. 2 dm. 5 cm.

3. How many links in 1 rod? In 2 rods? In 8 rods?

4. How many chains in 1 mile? How many rods?

5. How many inches in 1 link? In 10 links? In 100 links? In 1000 links? In 100000 links?

6. How many feet in 100 links? In 10 links? In 1000 links? In 100000 links?

7. How many links in 66 feet? In 6.6 feet? In .66 feet? In 660 feet? In 6600 feet?

8. How many mm. in 8 cm.? In 3 dm.? In 2 m.? In 5 Dm.?

9. How many mm. in 9.5 cm.? In 1.85 dm.? In 56.875 m.?

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12. How many links in 5 mi. 17 rds. 5 yds.?

13. Change 8 Km. 5 Dm. 5 m. 3 dm. to cm. To mm.

14. Find the sum of 18 Mm. 6 Km. 5 Hm. 3 m., 23 Mm. 7 Km. 1 Hm. 5 Dm., and 17 Mm. 5 Dm.

NOTE.—In the Metric System numbers may be written in one denomination, and the operation performed as in the addition of whole numbers.

15. Find the difference between 2356.85 m. and 3 Hm. 5 Dm. 7 m. 2 dm. 5 cm.

MEASURES OF SURFACE.

SQUARE MEASURE.

144	square inches (*sq. in.*)	=	1 square foot, . .	*sq. ft.*
9	square feet	=	1 square yard, . .	*sq. yd.*
$30\frac{1}{4}$	square yards	=	1 square rod, . .	*sq. rd.*
160	square rods	=	1 acre, ...	*A.*
640	acres	=	1 square mile, . .	*sq. mi.*

SURVEYORS' SQUARE MEASURE

437. Surveyors' Square Measure is used by surveyors in computing the area of land.

625	square links (*sq. l.*)	=	1 square rod, . . .	*sq. rd.*
16	square rods	=	1 square chain, . . .	*sq. ch.*
10	square chains	=	1 acre, ...	*A.*

METRIC SQUARE MEASURE.

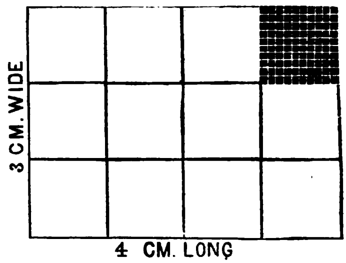
100 sq. millimeters (*sq. mm.*)	=	1 sq. centimeter,	*sq. cm.*
100 sq. centimeters	=	1 sq. decimeter, .	*sq. dm.*
100 sq. decimeters	=	1 sq. meter, . .	*sq. m.*
100 sq. meters	=	1 sq. dekameter,	*sq. Dm.*
100 sq. dekameters	=	1 sq. hektometer,	*sq. Hm.*
100 sq. hektometers	=	1 sq. kilometer, .	*sq. Km.*

NOTE.—The principal unit is the square meter, equal to 1.196 sq. yds

METRIC LAND MEASURE.

100 centares (*ca.*)	=	1 are, ...	*a.*
100 ares	=	1 hectare, . . .	*Ha.*

NOTE.—The *are*, equal to a square dekameter, or 100 square meters is 119.6 square yards, very nearly. The *hektare* is equal to 2.471 acres.



438. The amount or extent of surface enclosed within any given lines is the **area** of that surface.

439. In the measurement of surfaces, some other surface, usually of square form, and smaller, is used as a unit of measure. The number of times the surface to be measured contains the measuring surface determines the area.

Thus, if the unit of measure is a square 1 inch on each side, and the surface to be measured contains this measure 20 times, the area is 20 square inches.

440. Plane or flat surfaces having four straight sides and right angles for corners are called **rectangles.**

Thus, floors, walls, and ceilings are generally rectangular.

441. A rectangle whose sides are equal is called a **square.**

442. *The area of a rectangle is found by multiplying its length by its breadth.*

Thus, the area of the rectangle shown above, which is exactly 4 cm. long and 3 cm. wide, is equal to 12 square centimeters.

For a fuller explanation of the above rule, see Appendix, page 330.

NOTE.—A square 1 foot on a side may be spoken of either as *a foot square*, or *a square foot;* a square 2 feet on a side is 2 *feet square* and contains 4 square feet. In like manner, 5 *rods square* equals 25 *square rods*, and 2 *miles square* equals 4 *square miles.*

ORAL EXERCISES.

443. *1.* How many sq. in. in 2 sq. ft.? In 10 sq. ft.?

2. What is the area of a rectangle 12 in. by 10 in.?

3. How wide is a rectangle whose length is 12 in. and its area 108 sq. in.?

4. In 45 sq. ft. how many sq. yds.? In 81 sq. ft.?

5. How many sq. ft. in 6 sq. yds.? In $8\frac{1}{3}$ sq. yds.?

6. How many sq. yds. in 48 sq. ft.? In 69 sq. ft.?

7. How many sq. yds. in a floor 5 yds. long and 4 yds wide? How many sq. ft.?

8. At 50 cents per sq. yd., what will it cost to cover with oilcloth a floor 6 yds. long and 5 yds. wide?

9. How many sq. yds. of plastering will cover a ceiling 20 ft. long and 18 ft. wide?

10. How many acres in 160 sq. rds.? In 1600 sq. rds.?

11. How many farms of 160 A. each in 1 sq. mi.? If the farms are square, what is the length of each side?

12. How many sq. rds. in $\frac{5}{8}$ of a sq. mile? In $\frac{3}{8}$ of a sq mile? In .75 of a sq. mile? In .5 of a sq. mile?

13. How many sq. l. in 1 sq. ch.? In 2 chains square?

14. How many square chains in 9 chains square?

15. How many square chains in 1 acre?

16. How many acres in 500000 sq. l.? In 650000 sq. l.?

17. How many sq. l. in a square 1 chain on each side? How many in 2 square chains? In 2 chains square?

18. What is the area of a rectangle 1000 links long and 100 links wide? 10 chains long and 1 chain wide?

19. How many sq. cm. in 500 sq. mm.? In 1500 sq. mm.?

20. In 5 sq. m. how many sq. dm.? Sq. cm.? Sq. mm.?

21. What is the area of a ceiling 8 m. long and 5 m. wide?

22. A 5-cent nickel is exactly 2 cm. in diameter; how many could be placed on a line one meter long?

WRITTEN PROBLEMS.

444. *1.* Reduce 1 acre to square feet. To square inches.

2. How many square feet in a square mile? How many square yards?

3. Reduce 860800 square feet to integers of higher denominations.

4. How many square rods in 160.875 acres?

5. Reduce 57 A. 96 sq. rds. to the decimal of a square mile.

6. Change .3215 of a sq. mi. to lower denominations.

7. A room 30 feet long by 24 feet wide was covered with linoleum at $1.15 per square yard; what was the cost?

8. A man bought a rectangular piece of land 32.5 rds. long and 20.8 rds. wide; what did it cost him at $90 per acre?

9. At 30 cents per square yard, what is the cost of papering the sides of a room 24 ft. long and 18 ft. wide, the ceiling being 12 ft. high, making no allowance for doors and windows?

NOTE.—The cost of lumber is estimated at so much per square foot inch thick. For greater thicknesses the cost is $\frac{1}{4}$ more for each $\frac{1}{4}$ of an inch added to the thickness.

Thus, if a board 1 inch thick is sold at 4¢ per sq. ft., 5¢ will be charged for a sq. ft. $1\frac{1}{4}$ in. thick, 6¢ when it is $1\frac{1}{2}$ in. thick, etc. When the thickness is less than 1 inch, the price is usually the same as for inch lumber.

10. At 5¢ per sq. ft., what will be the cost of a plank 2 in. thick, 16 ft. long and $1\frac{1}{4}$ ft. wide?

11. Find the cost, at 15¢ per foot, of a mahogany plank $1\frac{1}{2}$ in. thick, 18 in. wide and 20 feet long.

12. The usual size of a city lot is 25 feet by 100 feet; how many such lots in 1 acre?

13. Find the length of a rectangular farm 18.5 rods wide containing 7.4 A.

14. How much must be paid at $3\frac{1}{2}$¢ per foot for inch boards to lay a floor 20 ft. by 30 ft.?

15. A field is 28.8 rods long and 20.25 rods wide; how many acres does it contain?

16. What is the length of a rectangular piece of ground 4 rods wide containing 1 A.?

17. Reduce 1912500 square links to acres.

18. A rectangular field was 10 ch. 75 l. long and 5 ch. 96 l. wide; what was its area in acres?

19. At $95 per acre, what cost a rectangular piece of ground 15 chains long and 12 chains wide?

20. What part of a sq. mile is a square 5 ch. on a side?

21. How many square links in $23\frac{5}{8}$ acres?

22. How many acres in a field 10 chains square?

23. A rectangular farm containing $275\frac{5}{8}$ A. is 62.5 chains long; how many chains wide is it?

24. A man bought 4 acres of land at $275 per acre, laid it out in town lots 1 chain wide and 2 chains deep, and sold them at $190 each; what was his gain by the transaction?

25. How many sq. m. in a floor 15.375 m. by 8.8 m.?

26. How many sq. m. in a ceiling 10.5 m. by 6.6 m.?

27. How many hectares in a rectangular field 837.5 m. long and 108 m. wide? How many acres?

NOTE.—For additional examples and illustrations, see Measurements, Appendix, pages 360–368.

MEASURES OF VOLUME.

CUBIC MEASURE.

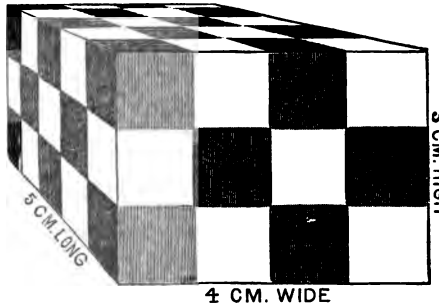
1728 cubic inches (*cu. in.*)	=	1 cubic foot, . .	*cu. ft.*
27 cubic feet	=	1 cubic yard, . .	*cu. yd.*
16 cubic feet	=	1 cord foot, . . .	*cd. ft.*
128 cubic feet	=	1 cord, ...	*cd.*

NOTE.—A pile of wood 8 feet long, 4 feet wide, and 4 feet high contains a *cord*. One foot of the length of such a pile is a *cord foot*. A perch of stone or masonry is 24.75 cu. ft.

METRIC CUBIC MEASURE.

1000 cubic millimeters (*cu. mm.*)	=	1 cubic centimeter,	*cu. cm.*
1000 cubic centimeters	=	1 cubic decimeter,	*cu. dm.*
1000 cubic decimeters	=	1 cubic meter, .	*cu. m.*

NOTE.—The *cubic meter* is equal to 35.3166 cubic feet. When used to measure *wood* or *stone*, it is called a *stere* (pronounced *stair*). Ten *steres* called a *dekastere*, are equal to 2.759 cords.



445. The **volume** or **contents** of a solid is the space it occupies. It is expressed by the number showing how many times it contains some other volume taken as a unit of measure. The unit of measure is generally a cube, and smaller than the volume to be measured.

The above figure represents 3 layers, one above another, each being 1 centimeter in depth. Each layer has 4 rows with 5 cubic centimeters in a row, making 4 times 5 cu. cm., or 20 cu. cm. in each layer. In the solid, therefore, there are 3 times 20 cu. cm., or 60 cu. cm. That is, the volume of the solid is equal to 3 times 4 times 5 cu. cm. Hence,

The volume of a solid is equal to the continued product of its length, breadth and thickness.

NOTE.—See Appendix, p. 330

A **cube** is a solid whose surface consists of six equal squares or faces.

ORAL EXERCISES.

446. *1.* How many cubic inches in a cube 2 inches each way? 3 inches? 4 inches? 5 inches? 10 inches?

2. If a 2-inch cube weighs a pound, what will be the weight of a 3-inch cube of similar material?

3. At one dollar a cubic yard, how much will it cost to dig a cellar 5 yards square, 1 yard deep?

4. At $6 a cord, what will 6 cord feet of wood cost?

5. How many times 3 cubic inches in a 3-inch cube?

6. How many cubic feet in $1\frac{2}{3}$ cubic yards?

7. How many blocks of marble, each containing one cubic foot, are equal to a block 4 ft. long, 3 ft. wide, 2 ft. thick?

8. What part of a cord in 100 cubic feet?

9. How many cu. mm. in an ivory cube whose faces are 2 cm. square?

10. How many steres in a block of granite 5 m. long, 4 m. wide, and 2 m. high?

11. How many dekasteres of wood in a pile 10 m. long, 1 m. wide, and 2 m. high?

12. At 50 cents a cu. m., how much should a man get for digging a cellar 1 m. deep, 10 m. long, and 8 m. wide?

13. How many cu. mm. in 8 cu. cm.? In 2 cu. dm.?

WRITTEN PROBLEMS.

447. *1.* How many cu. ft. in a granite block 8 ft. square and 6 ft. deep, and what is it worth at 75¢ per cu. ft.?

2. What will it cost at 25 cts. a cubic yard to dig a cellar 35 ft. long, 18 ft. wide, 6 ft. deep?

3. What is the volume of a beam $1\frac{1}{2}$ ft. wide, $1\frac{1}{3}$ ft. thick and 40 ft. long?

4. What is the value of a pile of wood 100 ft. long, 4 ft. wide, 6 ft. high, at $3.75 per cord?

5. A shed 90 ft. long, 32 ft. wide, 14 ft. high, is entirely filled with wood; what is it worth at $5 per cord?

6. How many cubic ft. of air in a school-room 30 ft. 6 in. long, 24 ft. wide, 12 ft. high?

7. A man undertakes to deliver 75 cords of wood; how many loads will it require of 6.25 cord feet each?

8. The pedestal of a monument has a base 7.5 m. square and 5.6 m. high; how many steres does it contain? How many dekasteres? How many cubic feet?

9. How many steres of wood in a pile 18.6 m. long, 2 m. high, and 1 m. wide? How many cords?

10. What will it cost to excavate a cellar 9.5 m. long, 8.3 m. wide, and 2.6 m. deep, at $1.25 a cubic meter?

11. What is the weight of a block of marble 2.5 m. long, 2.4 m. wide, and 1.6 m. thick, the weight of a cubic foot being $168\frac{3}{4}$ lb.?

MEASURES OF CAPACITY.

For tables of Liquid Measure and Dry Measure, see pages 92 and 94.

APOTHECARIES' LIQUID MEASURE.

448. Apothecaries' Liquid Measure is used in compounding *liquid* medicines.

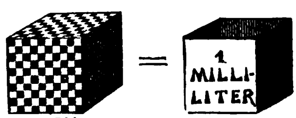
60 minims (♏)	=	1 dram, . . .	*f* ʒ.
8 drams	=	1 ounce, . . .	*f* ℥.
16 ounces	=	1 pint, . . .	*O.*
8 pints	=	1 gallon, . . .	*Cong.*

NOTE.—*Cong.* is an abbreviation of *congius*, a *gallon;* and *O.* an abbreviation of *octavius*, an *eighth* (of a gallon, *i. e.*, a pint). A *minim* of most liquids is about a drop. A pint of pure water weighs a little more than a pound.

METRIC CAPACITY MEASURE.

10 milliliters (*ml.*)	=	1 centiliter, . . .	*cl.*
10 centiliters	=	1 deciliter, . . .	*dl.*
10 deciliters	=	1 liter, ...	*l.*
10 liters	=	1 dekaliter, . . .	*Dl.*
10 dekaliters	=	1 hektoliter, . . .	*Hl.*
10 hektoliters	=	1 kiloliter, . . .	*Kl.*

NOTE 1.—The *liter*, equal to a cubic decimeter, is 61.028 cubic inches, 2.1135 pints, or 33.816 fluid ounces. The unit of measure for small quantities of liquids, as in mixing medicines and in philosophical experiments, is the milliliter, equal to 16.23 minims.



1 CU. CM.

NOTE 2.—A gallon is equal to 3.785 l., and 2.8375 bush. make a Hl.

ORAL EXERCISES.

449. *1.* How many minims in an ounce? In a dram?

2. How many ounces in a pint? In a quart?

3. An invalid takes 20 minims of a certain medicine at a dose, 3 times daily; how long will 2 ounces last?

4. What is a quart of paregoric worth at 5 cents an ounce?

WRITTEN PROBLEMS.

450. *1.* Reduce Cong. 5 O. 7 f℥ 13 fʒ 3 to drams.

2. How many liters in a hogshead containing 128 gallons?

3. Reduce 58.7695 Kl. to ml.

4. How much should a druggist get for 12.75 liters of tincture of opium retailed at 25 cents a deciliter?

5. How many bushels will a bin hold that is 8 ft. 6 in. long, 4 ft. 8 in. wide, and 2 ft. 6 in. deep?

NOTE.—A bushel contains 2150.42 cubic inches, and a gallon 231 cubic inches.

6. A cubical box 1 yd. each way is full of wheat; what is it worth at $1.25 a bushel?

NOTE.—For additional examples, see pages 360–368.

MEASURES OF WEIGHT.

TROY WEIGHT.

451. Troy weight is used in weighing gold, silver and jewels.

24 grains (*gr.*)	=	1 pennyweight,	*dwt.*
20 pennyweights	=	1 ounce, . . .	*oz.*
12 ounces	=	1 pound, . . .	*lb.* or ℔

NOTE 1.—A weight called the *carat*, equal to 3⅕ Troy grains, nearly, is used in weighing diamonds.

NOTE 2.—The term *carat* is employed in speaking of the fineness of gold. Thus, pure gold is 24 carats fine, 18 carat gold contains ¾ gold and ¼ alloy, and 16 carat gold contains ⅔ gold and ⅓ alloy.

APOTHECARIES' WEIGHT.

452. Apothecaries' weight is used by druggists in weighing drugs not liquid.

Drugs are bought and sold at wholesale by avoirdupois weight.

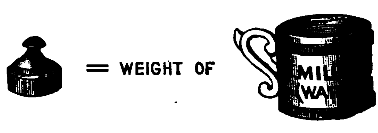
20 grains (*gr.*)	=	1 scruple, . .	℈.
3 scruples	=	1 dram, . . .	ʒ.
8 drams	=	1 ounce, . . .	℥.
12 ounces	=	1 pound, . . .	℔.

NOTE.—The Troy ounce and the apothecaries' ounce contain each 480 grains, while the avoirdupois ounce contains 437.5 grains.

METRIC WEIGHT.

10 milligrams (*mg.*)	=	1 centigram, . . .	*cg.*
10 centigrams	=	1 decigram, . . .	*dg.*
10 decigrams	=	1 gram, ...	*g.*
10 grams	=	1 dekagram, . . .	*Dg.*
10 dekagrams	=	1 hektogram, . .	*Hg*
10 hektograms	=	1 kilogram, or kilo,	*Kg.*
1000 kilos	=	1 tonne, ...	*T.*

NOTE 1.—The *gram* is equal to 15.4323 grains, the *kilo* to 2.2046 lb. avoirdupois, and the *tonne* to 2204.6 lb. avoirdupois. *Tonne* is rarely used, the number of *kilos* being preferable.



NOTE 2.—The weight of one *cubic centimeter* of pure water at its greatest density (39.2° Fahrenheit) is a *gram*; the weight of a *cubic decimeter*, or a *liter*, of water at the same temperature is a *kilogram*, or a *kilo*, and the weight of a cubic meter of water is a *tonne*.

NOTE 3.—The original 5-cent nickel weighs 5 grams, and is 2 centimeters in diameter.

ORAL EXERCISES.

453. *1.* How many ounces in 2 lb. Troy? In 3 lb.?

2. How many oz. in 1.25 lb.? In 2.75 lb.? In $4\frac{3}{4}$ lb.?

3. How many pounds in 18 oz.? In 30 oz.? In 40 oz.?

4. In 2 oz. how many dwt.? In $1\frac{1}{2}$ oz.? In $2\frac{3}{4}$ oz.?

5. In 40 dwt. how many ounces? In 100 dwt.?

6. In 1 lb. how many dwt.? In .25 lb.? In $\frac{7}{8}$ lb.?

7. How many dwt. in 24 gr.? In 48 gr.? In 480 gr.?

8. How many ounces in 480 gr.? In 4800 gr.?

9. How many ounces in 2 ℔? In $2\frac{1}{2}$ ℔? In 2.25 ℔?

10. What part of a ℔ in ℥ 2? In ℥ 3? In ℥ 9?

11. How many doses of quinine of 5 grains each in 1 dram of quinine? In $\frac{1}{2}$ dram? In $2\frac{1}{2}$ drams?

12. What is the value of 1 dram of quinine at 1 ct. per grain? Of 1 oz.? Of 2 oz.?

13. What will 2.5 Dg. of calomel cost at 4¢ a gram?

14. How many grams in a kilogram? In a hektogram?

15. Which is greater, 4 grams or one dram?

16. How many pounds in a kilo? In 10 kilos? In 100?

17. How much greater is the French *tonne* than the ordinary ton?

18. How many 5-cent nickels would weigh a kilogram?

WRITTEN PROBLEMS.

454. *1.* Reduce 19 lb. 10 oz. 18 dwt. 15 gr. to grains.

2. Change 576000 grains to pounds.

3. In a bar of 18-carat gold weighing 11 lb. 5 oz. 13 dwt. 8 gr., how many grains of pure gold?

4. Change 19875 gr. to higher denomination.

5. What decimal part of a pound is 4032 grains?

6. Reduce 10 oz. 10 dwt. to the decimal of a pound.

7. In .0625 lb. how many grains?

8. How many rings, each weighing 7 dwt. 15 gr., can be made from a bar of gold weighing 1 lb. 6 oz. 6 dwt.?

9. A mass of gold at the mint ready for coining weighs 44 lb. 9 oz. 10 dwt.; how many 10-dollar gold pieces, each weighing 258 grains, can be made of it?

10. What is the value of a bar of 16-carat gold, weighing 3 lb. 9 oz. 15 dwt. 18 gr., at 60¢ a pennyweight?

11. In one pound avoirdupois there are 7000 grains; how many Troy pounds in 144 lb. avoirdupois?

12. Reduce 25 lb. 12 oz. avoirdupois to lb., oz., etc., Troy.

13. Reduce ℔ 5 ℥ 9 ʒ 5 ℈ 2 gr. 15 to grains.

14. Change 87575 grains to higher denomination.

15. How many ℔ in 4896 scruples?

16. In 144 lb. avoirdupois how many pounds apothecaries' weight?

17. A druggist bought an avoirdupois pound of Turkey opium for $6.50 and retailed it at 15 cents a dram; what was his gain?

18. Reduce ℥ 3 ʒ 7 ℈ 0 gr. 15 to the decimal of a pound.

19. Change .875 of a ℔ to integers of lower denomination.

20. A retail druggist bought a barrel of powdered borax containing 198 lb. avoirdupois at 9½¢ a lb., and sold it for 5¢ an oz. apothecaries' weight; required his profit.

21. A pound avoirdupois of quinine, bought for $50, is sold at 25 cents a scruple; what is the gain?

22. A druggist bought a barrel of refined gum camphor containing 240 lb. for $35; for how much must he retail it by the ℔ to double his money?

23. How many kilos of water will a tank hold that is 2.35 m. long, 1.95 m. wide, and .66 m. deep?

24. How many kilos in a rectangular mass of lead 1.5 dm. long, 1.24 dm. wide, and .95 dm. thick, the weight of lead being 11.4 times that of an equal bulk of water?

25. Find the cost of 12.875 kg. of quinine at 14 cents per gram.

26. What will be the cost of a tonne of beet-root sugar at 8 cents a kilo? At 5 cents a pound?

MISCELLANEOUS MEASURES.

ENGLISH MONEY.

4 farthings (*far.*)	=	1 penny, . .	*d.*	=	\$.02027+
12 pence	=	1 shilling, .	*s.*	=	.2433+
2 shillings	=	1 florin, . .	*fl.*	=	.48665+
5 shillings	=	1 crown, . .	*cr.*	=	1.2166
20 shillings	=	1 sovereign, .	*sov.*		
		1 pound, . .	*£*		

NOTE.—The pound is worth \$4.8665 in gold.

WRITTEN PROBLEMS.

455. *1.* Reduce £20 17s. 8d. to farthings.

2. Reduce £5 14s. 6d. 1 far. to farthings.

3. Change 19728 farthings to higher denominations.

4. What part of a pound is 720 farthings? 384 farthings?

5. What part of a pound is 11s. 3d.? 3s. 4d.?

6. Reduce £$\frac{5}{6}$ to shillings and pence; £$\frac{3}{8}$; £$\frac{11}{16}$.

7. Change 15s. 10d. 2 far. to the fraction of a pound.

8. Reduce £17.925 to pounds, shillings, etc.; £15.6275.

9. Reduce 9110$\frac{5}{6}$ shillings to farthings; 22$\frac{1}{2}$ crowns to pence.

10. How many pence in £75 18s. 9d.? How many farthings?

11. Reduce 12s. 6d. to the decimal of a pound.

12. Reduce 10s. 7d. 2 far. to the decimal of a pound.

PAPER MEASURE.

24 sheets	=	1 quire.
20 quires	=	1 ream.
2 reams	=	1 bundle.
5 bundles	=	1 bale.

ORAL EXERCISES.

456. *1.* How many sheets in $\frac{1}{2}$ of a quire? In $\frac{1}{4}$ of a quire? In 3 quires? In $1\frac{1}{2}$ quires? In $\frac{3}{4}$ of a quire?

2. How many reams in 100 quires? In 200 quires?

3. In 2 bales how many bundles? How many reams? How many quires?

4. A stationer bought a ream of note-paper for 2 dollars, and sold it at 15 cts. a quire; what was his profit?

5. What is the gain on 5 quires of paper bought at 12 cents a quire and sold at a cent a sheet?

TIME MEASURE.

60 seconds (*sec.*)	=	1 minute,	. . . .	*min.*
60 minutes	=	1 hour,	. . . . .	*hr.*
24 hours	=	1 day,	. . . . .	*da.*
7 days	=	1 week,	. . . . .	*wk.*
365 days, or 12 calendar months	=	1 common year,	. .	*yr.*
366 days	=	1 leap year.		
100 years	=	1 century.		

NOTE 1.—Any year the number of which is exactly divisible by 4, but not by 100, is a leap year. When the number is divisible exactly by 400 it is also a leap year.

NOTE 2.—The solar or tropical year has a length of 365 da. 5 hr. 48 min. 49.7 sec.

ORAL EXERCISES.

457. *1.* Which of the following are leap years: 1886? 1870? 1872? 1900? 2000? 2004?

2. What part of a day in 4 hr? In 6 hr.? 8 hr.? 9 hr.? 10 hr.? 12 hr.? 15 hr.? 16 hr.? 20 hr.?

3. What part of one rotation on its axis does the earth perform in 15 hours?

4. How many hours in a week? In 10 weeks? In 5 weeks?

5. How many weeks in a common year?

6. What is the greatest possible number of Sundays in a month? The least number?

7. How many times 4 days in 4 weeks? In 12 weeks?

8. What part of 3 weeks is 6 days? 3 days?

9. How much less than $\frac{1}{12}$ of a year is 30 days?

10. What part of a week is 3 da. 12 hrs.? 3 da. 8 hrs.?

11. How many hours in $\frac{5}{8}$ of a week? In .75 of a week?

WRITTEN PROBLEMS.

458. *1.* How many seconds in a tropical year?

2. How many seconds in 1 day? How many minutes?

3. Reduce 2160000 seconds to days.

4. How many ticks will a clock that ticks half-seconds make from 20 minutes past 8 A. M. on Monday to 12 o'clock noon the following Sunday?

5. How many seconds in 5 weeks 5 days 15 hours 15 minutes 45 seconds?

6. Reduce $\frac{61}{73}$ of a common year to days.

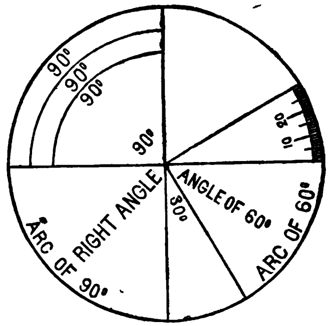
7. How many days less in 9 months of 30 days each than in $\frac{3}{4}$ of a common year?

8. How many weeks, days, hours, minutes and seconds in 9661793 seconds?

9. How much does the time in solar years differ from the time in calendar years, from the beginning of the year 1800 to the beginning of the year 1900?

CIRCULAR MEASURE.

60 seconds (″)	=	1 minute, . . .	′
60 minutes	=	1 degree, . . .	°
360 degrees	=	1 circumference, .	*C.*



NOTE 1.—If the circumference of a circle be divided into 360 equal parts or arcs, each arc is a degree; 90 of these arcs are equal to $\frac{1}{4}$ the entire circumference.

NOTE 2.—If from the ends of an arc of 90°, lines be drawn to the center of the circle, a *right angle* will be formed. Four right angles occupy all the angular extension around the center of the circle.

NOTE 3.—An angle of 1° is always the same, but an arc of 1° varies in length as the size of the circle varies. An arc of 1° of the circumference of the earth at the equator has a length of 69.164 miles very nearly and 1′ is equal to 1.1527+ miles, called a *geographical* or *nautical* mile.

ORAL EXERCISES.

459. *1.* How many angles of 45 degrees in a right angle?

2. How many arcs of 30° in a circumference?

3. If an arc of 1° is 1 foot in length, how long is the circumference? If 60 inches, how long is an arc of 1′?

4. If an arc of 10° is 1 ft. long, how long is the circumference? How long an arc of 90°? Of 45°? Of 60°?

5. If a railroad could be built around the earth on the equator, a passenger train could run along a degree in about two hours; at that rate, how many hours would it require to go around the earth? How many days?

6. If the earth were only one half as many miles in circumference, how many degrees would be in its circumference at the equator?

7. What part of the circumference of the earth is 10°? What part of any circumference is 10°? 20°? 30°? 90°?

8. If a line be drawn north and another west from the same point, how many degrees in the angle formed?

WRITTEN PROBLEMS.

460. *1.* How many minutes in 360°? How many seconds?

2. Reduce 197° 48′ 53″ to seconds.

3. Change 869967″ to degrees, minutes and seconds.

4. What part of a circumference is 864000″?

5. Reduce 299° 31′ 12″ to the decimal of a circumference.

6. If a steamer can go a distance of 7° 30′ each 24 hours, how long at that rate would it require to go a distance equal to the circumference of the earth?

7. It requires 24 hours for the apparent revolution of the sun around the earth; how many degrees is that per hour?

8. If the length of an arc of 22.5° in the circumference of the driving-wheel of a locomotive is 16½ inches, how many feet in the entire circumference?

LONGITUDE AND TIME.

461. The earth turns on its axis once in 24 hours; that is, an entire circumference, or 360° of longitude, passes under the vertical rays of the sun in 24 hours. When the sun's rays are thus vertical on a meridian, it is 12 o'clock noon at every place on that meridian; and since the earth turns from west to east, noon has passed for that day at places east of the meridian, and at places west of it noon has not yet come.

A rotation of 360°	in 24 hours	= 15° in 1 hour.
" " " 15° or 900′	in 60 minutes	= 15′ " 1 minute.
" " " 15′ or 900″	in 60 seconds	= 15″ " 1 second.

That is, *The earth, in a given number of hours, minutes, or seconds, rotates through* 15 *times as many degrees, minutes, or seconds, of longitude, respectively.*

For the subject of Meridian Time, see Appendix, p. 331.

462. To find the difference in time between two places, knowing their difference in longitude.

I. Required to find the difference in time between Greenwich, England, and Boston, Mass., their difference in longitude being 71° 4′ 9″.

Explanation.—Since the difference in longitude is 15 times as many degrees, minutes, and seconds respectively as the difference of time is hours, minutes, and seconds, 71° 4′ 9″ is divided by 15. This gives a result of 4 hours, 44 minutes, $16\frac{3}{5}$ seconds difference of time.

Process.

$$\begin{array}{r|l} 15 & 71^\circ \quad 4' \quad 9'' \\ \hline & \ \ 4 \quad 44 \quad 16\frac{3}{5} \end{array}$$

Rule.—*Divide the difference in longitude by* 15, *and the quotient will express the difference in time.*

463. To find the difference in longitude between two places whose difference in time is given.

I. Required the difference in longitude between New York and San Francisco, their difference in time being 3 h. 13 min. 44.8 sec.

Explanation.—Multiply the difference in time by 15, and the result is the difference in longitude, since the difference in time between two places in hours, minutes and seconds is only $\frac{1}{15}$ as much respectively as the difference in longitude in degrees, minutes, and seconds.

Process.

h.	min.	sec.
3	13	44.8
		15
48°	26′	12″

Rule.—*Multiply the difference in time by* 15, *and the product will express the difference in longitude.*

NOTE 1.—When the longitudes of two places are given, their difference in longitude is found by subtraction if the places are both in east, or both in west longitude, and by addition if one place is in east and the other in west longitude.

NOTE 2.—Clocks should show earlier time at places west of a given place, and later time at places east. Thus, when it is 13 min. 44.8 sec after 3 o'clock P. M. at New York City, it is noon at San Francisco.

WRITTEN PROBLEMS.

464. *1.* The longitude of New York City is 74° 3′ W., and that of Washington 77° 0′ 15″ W.; what is their difference in longitude? In time? When it is noon at New York, what time is it at Washington?

2. It is noon at Rome, Italy, 49 min. $48\frac{14}{15}$ sec. earlier than at Greenwich, England; what is their difference in longitude?

3. St. Louis is in longitude 90° 15′ 15″ W. and St. Petersburg in 30° 19′ E.; what is their difference in time? When it is 9 o'clock A. M. at St. Petersburg, what is the time at St. Louis?

4. The difference in time between New York and Chicago is 54 min. $19\frac{4}{5}$ sec.; the longitude of New York being 74° 3′ W., what is the longitude of Chicago?

5. When it is 30 minutes past 8 P. M. at Philadelphia, the longitude of which is 75° 10′ W., it is 10 min. $20\frac{1}{3}$ sec. past 7 at St. Paul, Minnesota. What is the longitude of the latter place?

6. A gentleman having correct St. Louis time visits Washington, D. C.; by how much will his watch differ from correct Washington time, and which will it be, fast or slow?

7. The longitude of Honolulu is 157° 50′ 36″ W., and that of San Francisco is 122° 26′ 15″ W.; what is their difference in time?

8. A gentleman having correct New York time found on reaching Rio Janeiro, Brazil, that his watch was 2 hrs. 3 min. 36 sec. slow; what is the longitude of Rio Janeiro?

MISCELLANEOUS PROBLEMS IN DENOMINATE NUMBERS.

465. *1.* Reduce 75 lb. 10 oz. 17 dwt. 21 gr. to grains.

2. Change 175364 gr. Troy to higher denominations.

3. A druggist sold ℔ 5 ℥ 11 ʒ 6 ℈ 2 gr. 15 of quinine at one cent a grain; how much did he get for it?

4. Reduce 168000 oz. avoirdupois to higher denominations.

5. A coal dealer bought coal at $3.50 per long ton, and retailed it at $4.75 per ordinary ton; what was his profit on 700 long tons?

6. A druggist retailed 144 lb. avoirdupois of pulverized borax bought at 9 cents a pound, for 20 cents a pound apothecaries' weight; what was his profit?

7. A pedestrian in walking once around a track passes over 24 rds. 5 yds. 1 ft. 6 in.; how many times does he go around the track in walking 100 miles?

8. How many acres in a field 13 ch. 75 l. long, and 7 ch. 80 l. wide?

9. Mr. Hayes paid $75 an acre for a piece of land 176 rds. long and 70 rds. wide; what did it cost him?

10. What will it cost to plaster a room 26 ft. long, 18 ft. wide, 9 ft. high, at 15 cts. a square yard?

11. A man paid $1625 for a city lot 25 ft. front, 100 ft. deep. At that rate, what should he pay for a plot 172 ft. by 200 ft.?

12. How many perch of stone in a wall 49½ ft. long, 6 ft. high, 18 in. thick?

13. What is the capacity in gallons of a cistern 14 ft. long, 11 feet wide, 9 ft. deep?

14. How long will a barrel of syrup containing 41¼ gal. last a family using 2½ gills daily?

15. How many bushels of wheat can be put in a bin 5 ft. by 10 ft. and 3 ft. 7 in. deep? What is it worth at $1.20 per bushel?

16. What is the distance around a wheel if an arc of 14° 24′ of its circumference is 1 ft. 3 in. in length?

17. The heart of a man in good health should beat 75 times per minute; how many times should it beat in a common year?

18. A college student was required to devote to study 5 hrs. 40 min. a day during 5 days of each week; how much did this amount to in 4 years if the college was in session 40 weeks each year?

19. If a field of 45 acres yields 40 bu. 3 pks. 5 qts. of wheat to the acre, how much does the entire field yield?

20. How much is a pile of wood 60 feet long, 4 feet wide, and 6 feet high worth at $4.50 a cord?

21. What is the value of a block of granite 11 ft. long, 9 ft. wide, and 6 ft. 3 in. high, at $75 a perch?

22. A man owned three farms; the first contained 157 A. 129 sq. rds., the second 141 A. 97 sq. rds., and the third 193 A. 80 sq. rds. He sold them all at $68.80 an acre; what did he get for them?

23. What would be the cost of 5 mahogany planks $1\frac{3}{4}$ in thick, 14 ft. long, and 15 in. wide, at 17 cents a foot?

24. One of two places is 18° 30′ 36″ west of the other; what is the time at the place farther west when it is 10 minutes after 10 A. M. at the other place?

25. At 9 o'clock A. M. at Philadelphia, it is 10 minutes after 2 P. M. at Paris; what is their difference in longitude?

26. A teamster contracts to deliver 250 long tons of coal at a wharf for $437.50; how much of this amount has he earned when he has delivered 142 T. 17 cwt. 16 lb.?

27. How many $1\frac{1}{2}$-inch cubes would it take to make a pile containing a cubic yard?

28. A grocer bought 188 Hl. of apples at $2.25 a hektoliter, and retailed them at $1.25 a bushel; did he gain or lose, and how much?

29. What is the gain on 1200 steres of wood bought at $.55 a stere and sold at $3.50 a cord?

30. A cask containing 250 l. of syrup was bought at 15 cents a liter and sold at $.68 a gallon; how much was gained?

31. What is the gain on 12000 m. of silk imported at a cost of $1.75 a meter and sold at $1.95 a yard?

32. A man bought a farm containing 100 hectares at $250 a hectare, and sold it at $195 an acre; did he gain or lose, and how much?

33. Find the cost of building a wall 25.35 m. long, 2.8 m. high, and .45 m. thick, at $6.50 a cubic meter.

REVIEW QUESTIONS.

466. Denominate Numbers.—Define and illustrate denominate number, simple denominate number, compound denominate number. What is a denominate fraction? What is reduction? Reduction descending? Reduction ascending? Tell how to reduce compound denominate numbers to lower denominations; to higher denominations. How is a denominate fraction reduced to lower denominations? To higher denominations? How does the scale of abstract numbers differ from the scales of most compound denominate numbers? How are compound denominate numbers added? Subtracted? Multiplied? Divided? Give the table of surveyors' linear measure. Of metric linear measure. What is the length of the meter? In what respect is the metric system superior to others? How are reduction ascending and reduction descending of metric measures performed? Give the four tables of surface measure. How many sq. yds. in a sq. m.? In an are? How many acres in a hektare? What is the area of a surface? The unit of measure of a surface? What is a rectangle? A square? How is the area of a rectangle found? How is the cost of lumber estimated? Give an example. Give the tables of cubic measure. How many cu. ft. in a cu. m.? When is the cubic meter called a stere? How many cords in a dekastere? What is the volume of a solid? How is it found? What is a cube? Give the tables of capacity measure. How many cubic inches in a bushel? In a gallon? In a liter? How many pints in a liter? Give the tables of weight. What is 18 carat gold? How many grains in a troy ounce? An avoirdupois ounce? How many grains in a gram? How many pounds in a kilo? Tell something of interest about the size and weight of the five-cent nickel. Give the table of English money. What is the value of the £ in gold? Give the table of paper measure. Of time measure. What years are leap years? Give the length of a solar year. Repeat the table of circular measure. What is a nautical mile? When the difference in longitude between two places is known, how do you find the difference in time? When the difference in time is given, how is the difference in longitude found?

PERCENTAGE.

467. Per cent. is an abbreviation of *per centum,* and means *by the hundred.* Any per cent. of a number is *so many hundredths* of it.

Thus, 20 per cent. of 50 is $\frac{20}{100}$ of 50.

468. The sign % is used in place of the words *per cent.*

Thus, $\frac{5}{100}$, .05, 5 per cent., and 5%, all have the same meaning.

ORAL PROBLEMS.

469. *1.* What is $\frac{1}{100}$ of 100? $\frac{3}{100}$ of 100? $\frac{12}{100}$ of 100? $\frac{17}{100}$ of 100? $\frac{35}{100}$ of 100? $\frac{75}{100}$ of 100?

2. What is $\frac{5}{100}$ of 100? Of 200? Of 500? Of 800?

3. What is $\frac{8}{100}$ of 100? .08 of 200? 8 per cent. of 200? 8% of 300? 10% of 300? 15% of 300?

4. A man having 300 chickens sold 12 out of each hundred; how many did he sell? What per cent. did he sell?

5. A gardener planted 500 cabbage plants, and $\frac{8}{100}$ of them died; how many died?

6. A drover sold 12 per cent. of the cattle in his drove; how many did he sell if his drove consisted of 800 cattle?

7. The passengers on a certain ship numbered 600, and 8% of them were Germans; how many were Germans?

8. In an orchard of 400 trees, 10% of them are cherry trees; how many are cherry trees?

9. For grinding 300 bushels of grain, a miller retains 5%; how many bushels does he retain?

10. A farmer raised 450 bushels of potatoes, and kept 10% of them for his own use; how many bushels did he keep?

11. 10% of a number is how many hundredths of the number? $\frac{10}{100}$ of a number is what part of the number? 20% of a number is what part of the number?

470. The following table shows what fractional part of a number is equivalent to a given per cent. of the same number:

4%	= .04	= $\frac{1}{25}$	$16\frac{2}{3}$%	= $.16\frac{2}{3}$	= $\frac{1}{6}$	60%	= .6	= $\frac{3}{5}$
5%	= .05	= $\frac{1}{20}$	20%	= .2	= $\frac{1}{5}$	$62\frac{1}{2}$%	= .625	= $\frac{5}{8}$
$6\frac{1}{4}$%	= .0625	= $\frac{1}{16}$	25%	= .25	= $\frac{1}{4}$	$66\frac{2}{3}$%	= $.66\frac{2}{3}$	= $\frac{2}{3}$
$8\frac{1}{3}$%	= $.08\frac{1}{3}$	= $\frac{1}{12}$	$33\frac{1}{3}$%	= $.33\frac{1}{3}$	= $\frac{1}{3}$	75%	= .75	= $\frac{3}{4}$
10%	= .1	= $\frac{1}{10}$	$37\frac{1}{2}$%	= .375	= $\frac{3}{8}$	80%	= .8	= $\frac{4}{5}$
$12\frac{1}{2}$%	= .125	= $\frac{1}{8}$	50%	= .5	= $\frac{1}{2}$	$87\frac{1}{2}$%	= .875	= $\frac{7}{8}$

ORAL PROBLEMS.

471. *1.* What % of anything is $\frac{1}{100}$ of it? $\frac{5}{100}$? $\frac{12}{100}$? $\frac{25}{100}$? $\frac{100}{100}$?

2. What % of anything is all of it? $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{3}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{3}$ of it? $\frac{2}{3}$ of it? $\frac{5}{8}$ of it? $\frac{3}{8}$ of it?

3. What part of anything is 50% of it? 40%? 30%? 25%? 20%? $33\frac{1}{3}$%? $37\frac{1}{2}$%? $62\frac{1}{2}$%? $8\frac{1}{3}$%? $12\frac{1}{2}$%?

4. What % of a farm is .08 of it? .125? .375? $.14\frac{2}{7}$' $.31\frac{1}{4}$? .3125? $.18\frac{3}{4}$? 1875? .625?

5. What is 10% of 20? Of 30? Of 50? Of 80? Of 800? Of 1600?

6. What is 20% of 40? Of 60? Of 35? Of 75? Of 200? Of 4000? Of 100000?

7. What is $12\frac{1}{2}$% of 48? Of 64? Of 96? Of 160? Of 4000? Of 8000?

8. What is $37\frac{1}{2}$% of 8? Of 24? Of 72? Of 800? Of 16000? Of 40000?

9. A man bought a book for $4.50, and sold it at a gain of $33\frac{1}{3}$%; what did he gain, and what did he get for it?

10. What is $16\frac{2}{3}$% of $1\frac{1}{5}$? Of $4? Of $3\frac{3}{5}$ miles? Of $3\frac{3}{7}$ weeks? Of $9\frac{3}{5}$ acres?

11. A man had a farm of 160 acres, and sold $12\frac{1}{2}$% of it; how many acres did he sell?

12. A grocer bought goods for $450, and sold them at a gain of 20%; how much did he get for them?

472. The **base** is the number on which the percentage is computed.

473. The **rate per cent.**, or the **rate**, is the number denoting how many hundredths of the base are taken.

474. The **percentage** is the result obtained by multiplying the base by the number of hundredths expressed by the rate.

The term *percentage* is used to signify also the operation of computing by hundredths.

The sum of the base and percentage is the *amount;* the *difference* is what is left when the base is diminished by the percentage.

475. The base and the rate being given, to find the percentage.

ORAL PROBLEMS.

1. What is 28% of $150?

Analysis.—28% of a number is $\frac{28}{100}$ or $\frac{7}{25}$ of the number; $\frac{1}{25}$ of $150 is $6, and $\frac{7}{25}$ is 7 times $6, equal to $42.

2. What is 10% of $250? Of $300? Of $350? Of $4.50?

3. A man worked $12\frac{1}{2}$% of 48 days at $3 a day; what did he receive?

4. What is the value of $16\frac{2}{3}$% of 60 bushels of apples at $1.25 per bushel?

5. A man having a drove of 400 sheep sold 20% of them at $4 a head; what did he get for them?

6. What is 25% of 120 pounds? $33\frac{1}{3}$% of 90 bushels? $37\frac{1}{2}$% of 80 yards? 50% of 900? 60% of 555? 75% of 1200? $87\frac{1}{2}$% of $160?

7. A man earned $75 a month, and his expenses amounted to 80% of his earnings; what were his expenses?

8. In a school numbering 160 pupils, $62\frac{1}{2}$% of them study geography; how many study geography?

9. What is $33\frac{1}{3}$% of 900? $37\frac{1}{2}$% of 240? $62\frac{1}{2}$% of 320? $66\frac{2}{3}$% of 180? $87\frac{1}{2}$% of 800?

WRITTEN PROBLEMS.

1. Required to find $39\frac{1}{2}\%$ of \$1867.25.

Explanation.—$39\frac{1}{2}\%$ of a number is $.39\frac{1}{2}$ of the number, equal to .395 of the number. Multiplying the base, \$1867.25, by .395, the rate expressed decimally, and pointing off the result as in multiplication of decimals, gives \$737.56.

Process.

$$
\begin{array}{r}
\$1867.25 \\
.395 \\
\hline
933625 \\
1680525 \\
560175 \\
\hline
\$737.56375
\end{array}
$$

NOTE.—In business calculations, 5 mills or more are counted as 1 cent in final results; less than 5 mills are disregarded.

FORMULA. $\quad Base \times \dfrac{rate}{100} = percentage.$

2. What is 18% of 1395? 25% of 1896? $37\frac{1}{2}\%$ of \$963.84?

3. What is the value of $66\frac{2}{3}\%$ of 7560 acres of land at \$75 an acre?

4. Which is greater, and how much, $87\frac{1}{2}\%$ of \$3848, or $62\frac{1}{2}\%$ of \$5996?

5. A drover sold $16\frac{2}{3}\%$ of 1896 horses at \$175 each; how much did he get for them?

6. A man owing \$9375.68 paid 45% of the debt; how much did he still owe?

7. What is the sum of $56\frac{1}{4}\%$ of 3275, $42\frac{6}{7}\%$ of 6251, and $18\frac{3}{4}\%$ of 84480?

8. What is $31\frac{1}{4}\%$ of 13392 yards? 65% of 46920 kilos? 75% of 83360 inches?

9. A speculator invested \$8350 in a business venture, and gained 15%; how many dollars did he gain?

10. A farmer raised 5375 bushels of grain, and 28% of it was wheat; how many bushels of wheat did he raise?

11. A man owing \$65375 failed, and was able to pay only 68% of his debts; how many dollars did he pay?

12. After a ship had made 56% of a voyage of 8375 miles, how many miles had she yet to make?

13. A man sold a house that cost him $15675 so as to gain 35%; how many dollars did he gain, and what did he get for the house?

14. A grocer bought 75 bushels of walnuts at $2.40 per bushel, and 65 bushels of chestnuts at 25% more per bushel than the price of the walnuts; for which did he pay the greater amount, and how much?

15. A man bought a farm of 98 acres at $160 per acre, and erected buildings on the farm costing 35% as much as the land. He then sold the farm for $18675. Did he gain or lose, and how much?

16. A gentleman dying left an estate worth $37800. His widow received 25% of it and his only son 25% of the remainder. What still remained was divided equally among 5 daughters; how much did each daughter receive?

17. A bought a horse for $240 and sold it to B at a gain of 20%; B afterward sold it at a loss of 25%; what did B get for it?

18. A flour merchant bought 960 barrels of flour at $8.35 per barrel. He sold at one time 40% of it at $8.90 per barrel, at another time he sold 25% of the remainder at $8.75 per barrel, and what still remained he sold at $8 per barrel. Did he gain or lose, and how much?

19. A man owing debts amounting to $18620, paid 20% of what he owed, and afterward 35% of what remained unpaid; how much did he still owe?

20. A company mined during a certain year 88050 tons of coal, and during the next year 37½% more than the year before; what was the product of both years' mining?

21. I paid my butcher $28.60 during a certain month, and my grocer 65% more than I paid my butcher; how much did I pay my grocer?

22. A man whose annual salary is $3600 saves 35% of it the first year, 37½% of it the second year, and 45% of it the third year; how much does he save in the three years?

476. The percentage and the rate being given, to find the base.

ORAL PROBLEMS.

1. 25% of a certain number is 18; what is the number?

Analysis.—25% of a number is $\frac{1}{4}$ of it. If $\frac{1}{4}$ of a certain number is 18, $\frac{4}{4}$, or the whole number, is 4 times 18, equal to 72.

2. 5 is 10% of what number? 20%? 25%? $62\frac{1}{2}$%? $12\frac{1}{2}$%?

3. 24 is 12% of what number? 15%? 20%? 32%? 28%? 30%? 40%? 48%?

4. A boy by selling a ball for 5 cents more than it cost him gained 20%; what did it cost him?

5. $6 is 25% of what a coat cost; how much did it cost?

6. 30 gallons is 60% of what a certain vessel will hold; what is the capacity of the vessel?

7. 12 cents is 6% of what a book cost; what was its cost?

8. A man paid $33\frac{1}{3}$% of his week's wages for a coat that cost him $15; what did he receive per week?

9. After selling 20 gallons of syrup from a hogshead, 60% was left; how many gallons were there at first?

Analysis.—The amount of syrup was reduced from 100% to 60% by the sale of 20 gallons; hence, 20 gallons was $\frac{40}{100}$ or $\frac{2}{5}$ of the syrup at first. If $\frac{2}{5}$ of the syrup was 20 gallons, $\frac{1}{5}$ of it was 10 gallons, and $\frac{5}{5}$, or all of it, was 50 gallons.

WRITTEN PROBLEMS.

1. $37\frac{1}{2}$% of a certain sum is $961.38; what is the sum?

Explanation.—The percentage, $961.38, being the product of two factors, one of which is given, the other factor, which is the base required, is found by dividing $961 38 by .375, the given rate, expressed decimally

Process.

$961.38 ÷ .375 = $2563.68

FORMULA. $\quad \textit{Percentage} \div \frac{\textit{rate}}{100} = \textit{base.}$

2. If 49% of a cargo of grain is 60466 bushels, how many bushels in the cargo?

3. $17655 is 62½% of the cost of my house; find its cost.

4. If 87½% of the pages in Webster's Unabridged Dictionary is 1687, how many pages does it contain?

5. A man owing a bill discharged 35% of it by paying $83.65; what was the entire bill?

6. In a certain school the boys numbered 918, and were 45% of the whole number of pupils; how many pupils were in the school, and how many were girls?

7. A man after selling 37½% of his land had 175.5 acres left; how many acres had he at first?

Analysis.—After selling 37½% of his land, there remained 100% − 37½%, equal to 62½%, or ⅝ of it. If ⅝ of his land was 175.5 acres, ⅛ of his land was ⅕ of 175.5 acres, or 35.1 acres, and ⁸⁄₈, or his land, was 280.8 acres.

8. 33⅓% of the time spent by a ship on a certain voyage was 5 weeks 4 days; how many days was she on the voyage?

9. A drover received $1960 for 28% of a drove of hogs; at that rate what should he receive for them all?

10. In 1880, a certain city was found to have increased 27300 in its population since the census of 1870; the increase being 13%, what was the population in 1870?

11. By selling a lot of goods for $469.20 more than he paid for them, a merchant gained 18¾%; what did they cost him, and what did he get for them?

12. A farmer gained 14²⁄₇% on his farm by selling it for $13960; what did it cost?

Analysis.—Since he gained 14²⁄₇%, or ⅐, and the cost was 100%, or ⁷⁄₇ the selling price was ⁸⁄₇ of the cost. If ⁸⁄₇ of the cost was $13960, etc.

13. A widow received 35% of an estate, and her daughter 23% of it; what was the value of the entire estate if the widow's share was $7695.75 more than the daughter's?

14. From a hogshead of molasses 16% leaked out; how many gallons did it contain at first, if the remainder, sold at $.86 per gallon, brought $90.30?

15. By adding \$695.75 to my money in bank, I increased it 20%; how much did I have in bank before?

16. A stock farmer sold 65% of his sheep at one time, and afterward sold the remainder at the same price per head; from the first sale he received \$1260 more than from the second; what did he get for all?

17. If 12% of the trees in an orchard are pear trees, 15% peach trees, 18% plum trees, and the remainder apple trees, how many trees of each kind are in the orchard if the peach trees and plum trees together number 429?

18. A man dying willed 20% of his property to his wife, 25% of the remainder to his son, $33\frac{1}{3}$% of what still remained to his daughter, and the remainder amounting to \$24800 to found a hospital; what was the value of his entire property?

477. The percentage and the base being given, to find the rate.

ORAL PROBLEMS.

1. What per cent. of 50 is 10?

Analysis.—10 is $\frac{1}{5}$ of 50; $\frac{1}{5}$ of a number is equal to $\frac{20}{100}$, or 20% of the number; therefore, 10 is 20% of 50.

2. What per cent. of 8 is 2? 1? 4? 6? 3? 5? 7?

3. What per cent. of 10 is 1? 2? 3? 4? 5? 8? $2\frac{1}{2}$? $7\frac{1}{2}$?

4. What per cent. of 4 is 4? 5? 6? 8? 7? 12?

5. When a number is increased by its fifth part, what per cent. of the number is the sum? When by its fourth part? Its third part? Its half? Its three eighths?

6. What per cent. of a number is the number less its fourth part? Less its fifth part? Less its third part?

7. A boy had 12 marbles and lost 3; what per cent. did he lose and what per cent. had he remaining?

8. A man who gets \$15 per week has his wages raised to \$18 per week; what per cent. is the increase?

9. A man who has \$10 pays \$$2\frac{1}{2}$ for a hat; what per cent. of his money does he give for the hat, and what per cent. of it remains?

10. John has \$8 and Henry has \$6; what per cent. of Henry's money is equal to John's money, and what per cent. of John's money is equal to Henry's money?

WRITTEN PROBLEMS.

1. Required to find what per cent. \$648 is of \$1728.

Explanation.—The percentage, \$648, being the product of two factors, one of which, the base, is given, the other factor, which is the rate required, is found by dividing \$648 by \$1728.

Process.

$\$648 \div \$1728 = .375 = 37\frac{1}{2}\%$

FORMULA. $(Percentage \div base) \times 100 = rate.$

2. What % of a voyage of 5943 miles is 3962 miles?

3. If 8875 cubic feet of earth is to be removed from a cellar, what per cent. remains after the removal of 2840 cubic feet?

4. A man having \$2025 in bank drew out \$450; what per cent. did he draw out?

5. Mr. Baird bought a house for \$6875 and made a payment of \$2475; what per cent. was still to pay?

6. A teacher whose annual salary is \$875 saves \$490; what per cent. of her salary does she save?

7. A merchant invested \$9050 in business, and at the end of a year his money had increased to \$13122.50; what per cent. of his investment was equal to his gain?

8. A farmer raised 5875 bushels of grain and sold 1410 bushels of it; what per cent. did he sell?

9. A man whose annual salary was \$2350 paid \$376 rent; what per cent. of his salary was the rent?

10. A man bought a house for \$6850 and sold it for \$8494; what per cent. did he gain?

11. I bought 72 tubs of butter, of 45 lb. each, at $.37½ per lb., and sold it all for $1458; what was my gain per cent.?

12. A nurseryman planted 8475 apple trees, of which only 5085 lived; what per cent. of them died?

13. What rate per cent. does a landlord receive who rents a house for $1077 per annum, that cost him $8975?

14. In a city of 89750 inhabitants, 3590 died during an epidemic; what per cent. of the population was the number of deaths?

15. When a ship has sailed 700 miles of a voyage of 3500 miles, what per cent. of the distance has been sailed, and what per cent. remains to be sailed?

16. A merchant deducted $151.74 from a bill of $1896.75; what per cent. of the bill was deducted?

17. A man owing $4675, paid at one time $1870, and at another $1122; what per cent. of the debt remained unpaid?

MISCELLANEOUS PROBLEMS.

478. *1.* What number increased by 15% of itself is equal to 4899?

2. The difference between a number and 35% of the number is 8918; what is the number?

3. 87½% of a certain number exceeds 50% of it by 9186; what is the number?

4. 50% of 90% of a certain number is 81639; what is the number?

5. The sum of two numbers is 18729, which is 225% of the smaller; what are the numbers?

6. The difference between two numbers is 739, and the one is 25% greater than the other; what are the numbers?

7. If ¾ of a number is 240 more than 66⅔% of it, what is the number?

8. A father earns 20% more than his son, and the sum of their wages for one month is $111.65; what does each earn?

9. A butcher bought two cattle for $156, paying for one 40% more than for the other; how much did he pay for each?

10. A farmer raised, during a certain year, an average of 90 bushels of potatoes to the acre on 45 acres; the next year the yield per acre was 10% less, and the number of acres planted in potatoes was increased by 20%; how many did he raise the second year?

11. A man whose salary is $2750 per year, expends $12\frac{1}{2}$% of it for house-rent, and 75% for other uses; how much has he left out of each year's salary?

12. From a farm containing 375 acres there were sold 165 acres; what per cent. of the farm remained unsold?

13. A man having invested $11500 in government bonds, had 54% of his money left; how much had he at first?

14. Of a cargo of flour 45% was thrown overboard during a storm, after which there remained 2915 barrels; how many barrels of flour were thrown overboard?

15. A piano, the catalogue price of which is $650, can be bought for $422.50 cash; what per cent. is deducted?

16. In building a house, 45% of the cost was for the brick-work, 25% for the carpenter-work, 15% for the mason-work, and the remainder, amounting to $3840, was for painting and plastering; what did the house cost?

17. A drover having 8740 sheep, sells 3059 of them; what per cent. of them remains unsold?

18. By selling my farm for $20700 I gain 15%; what did it cost me?

19. A car loaded with coal weighs 42900 pounds, and 43% of the entire weight is the weight of the car; what is the weight of the coal?

20. A man went 979 miles on horseback and by rail; he traveled 20% farther by rail than on horseback; how far did he travel by rail, and how far on horseback?

21. 50% of 675 is 25% of what number?

22. What must a manufacturer get for a piano that cost him \$225, in order to gain 88%?

23. A man sold 336 bales of cotton for \$28800, and by so doing gained $14\frac{2}{7}$%; what did it cost him per bale?

24. In an army of 89172 men, there are 25% more new recruits than veterans; how many of each are there?

25. A bankrupt paid each creditor only 65% of his claim; what was due the creditor that received \$8060?

26. My sheep have increased 125% in number, and I now have 1728; how many had I before the increase?

27. I own 65% of a mill, and sell 40% of my share for \$5538; at that rate, what is the mill worth?

28. What must I get for a farm of $183\frac{3}{8}$ acres that cost me \$122 per acre, in order to gain 20%?

29. A man bought 375 horses in Kentucky for \$73125, he paid \$1125 freight to New York, and then sold them all so as to gain 15% by the transaction; what did he get per head?

30. A man bought two houses, one for \$15625, the other for \$12526; he sold the first at a gain of 20% and the second at a loss of 25%; did he gain or lose, and how much?

31. An army of 27500 men lost in a battle 7425 in killed, wounded and missing; what per cent. remained?

32. A dealer bought 9810 bushels of wheat at $\$1.12\frac{1}{2}$ a bushel, and sold at a loss of $11\frac{1}{9}$%; how much did he lose?

33. A merchant sold a piece of velvet containing 45 yards at a gain of $12\frac{1}{2}$%; how much did he get for the piece if it cost him \$6.40 per yard?

34. Of a cargo of cotton, 35% was sold to one manufacturer, and 60% of the remainder to another; of how many bales did the cargo consist if there remained 858 bales unsold?

35. I bought 275 acres of ground for \$37812.50; I sold 40% of it at a gain of 25%, 60% of the remainder at a gain of $33\frac{1}{3}$%, and on what still remained I lost 50%; did I gain or lose by the transaction, and how much?

36. A butcher bought 9860 lb. of Chicago dressed beef at $8\frac{1}{2}$¢ per pound, paid 1% of the cost for freight to New York, and then sold it so as to gain 25% by the transaction; how much did he get for it?

37. A farmer put a lot of potatoes into his cellar for the winter; he used $1\frac{1}{2}$% of them, sold 20% of them, and 5% of them spoiled; in the spring he had 1176 bushels left; how many bushels were there at first?

38. Mr. Harris, after making three payments on a house, still owed $2673. The first payment was 25% of the price, the second 40% of the remainder, and the third 60% of what still remained; what was the price of the house?

39. A horse and sleigh cost me $500, and the sleigh cost 45% of this sum; what did the horse cost?

40. A gets an annual salary of $2000 and B of $2500, what per cent. does B get more than A, and what per cent. does A get less than B?

41. It costs me $27.50 more to pay for some goods when I get an allowance of 3% for cash than when I get an allowance of 5%; find the amount of the bill.

42. I bought goods which cost $2279.50 after a discount of 3% for cash; what was the amount of the bill before the deduction?

43. By buying a house at 10% less than it cost to build, and selling at a profit of 25%, I gained $675; what did the house cost to build, and what was the selling price?

44. The sum of 20% of the cost of a horse and 25% of the cost of a carriage was $100; how much did the horse cost if the cost of the carriage was $240?

45. What per cent. of a long ton is an ordinary ton?

46. A man agrees to dig a cellar 30 feet long, 24 feet wide, and 6 feet deep; what per cent. of the work has he done when he has removed 16 cubic yards?

47. A has 28% more money than B; what per cent. has B less than A?

48. The profit of a horse-railroad in two years was \$35785, and the profit the second year was $12\frac{1}{2}\%$ more than the first year; what was each year's profits?

49. I sold my farm for \$4680 more than it cost me, and thereby gained 32%; how much did it cost me?

50. A man owned 65% of a factory, and sold 40% of his share for \$7800; what was factory worth at that rate?

51. I sold a mill for \$23100, which was at a loss of $12\frac{1}{2}\%$; what per cent. would I have made if I had sold it for \$33000?

52. A man agreed to excavate a cellar 30 by 40 feet, and 6 feet deep; what per cent. remained after he had excavated a portion 12 by 18 feet to the depth of 4 feet?

53. A man undertook to save enough money to pay for a house. After saving 35% of the amount, he expended 40% of what he then had, and found that he lacked \$6715 of the price of the house. What was the price of the house?

NOTE.—In many varieties of business transactions two or more percentages, called discounts, are deducted in succession from the prices of articles.

Thus, such discounts as 40% and 10%, 30% and 20%, $\frac{1}{3}$ and 5%, are very common. A discount of 40% and 10% means, not 50%, but that 40% is first deducted, leaving 60% of the price, and then 10% of 60% of the price, equal to 6%, is deducted from the remainder, leaving 54% of the price. The discount, though seeming to be 50%, is in reality only 46%.

By a discount of $\frac{1}{3}$ and 10 is meant a discount of $\frac{1}{3}$, that is, $33\frac{1}{3}\%$, from the price, and then another discount of 10% from the remainder.

54. What is received for a \$600 piano sold at a discount of 40% and 10%?

Explanation.—A discount of 40% from \$600 amounts to \$240, which being deducted leaves \$360. Again, 10% of \$360, or \$36, deducted from \$360 leaves \$324, the amount received for the piano.

Process.

$$\$600 \times .40 = \$240;$$
$$\$600 - \$240 = \$360.$$
$$\$360 \times .10 = \$36;$$
$$\$360 - \$36 = \$324.$$

55. A bookseller sells a \$5 book at a discount of 40% and 5%; what does he get for it? What would he get for it if he should give a discount of 45%?

56. A dealer in furniture sells a desk marked \$60 at a discount of $\frac{1}{6}$ and 10%; what does he get for it?

57. A machinist buys a \$300 lathe at a discount of $\frac{1}{3}$ and 10%; what does it cost him?

58. Which is better for the buyer, and how much, a deduction of 20% and 10%, or a deduction of 30%, from a bill of goods amounting to \$9875.50?

59. From a bill of \$1785.66 a deduction of $\frac{1}{6}$ and 10% was made; what was the amount of the discount?

60. How much less is a discount of $\frac{1}{4}$ and 15% than a discount of 40% from a bill of \$346.40?

61. A man bought on 30 days' credit a bill of goods amounting to \$840, less a discount of 15%. On condition that he should pay cash, another discount of 5% was made. What was the amount of the bill after the second deduction had been made?

62. What must I pay for an 875-dollar piano after a discount of 40% and 15%?

63. How much more is a discount of 40% and 10% than a discount of 30% and 20% from a bill of \$1875.50?

64. How much will a discount of 35% and 15% reduce a bill of \$3880.72?

65. A merchant gives a discount of 30% and 20%; what per cent. of the original price does he receive?

66. What discount is equivalent to a deduction of 30% and 10%? 50% and 20%? $\frac{1}{4}$ and 20%? 40% and 20%?

67. A piano manufacturer, after allowing a discount from his catalogue price of 40% and $16\frac{2}{3}\%$, received \$325 for a piano; what was the catalogue price?

68. If I pay \$25 more for a piano when I get a deduction of 40% and 10%, than when I get a deduction of 50%, what is its catalogue price?

PROFIT AND LOSS.

479. Profit and **loss,** as commercial terms, signify the gain or loss in business transactions.

Profit and Loss is one of the applications of Percentage.

The *cost* is the **base.**
The *gain* or *loss per cent.* is the **rate.**
The *profit* or the *loss* is the **percentage.**

480. To find the gain or the loss and the selling price, when the cost and the rate of gain or loss is given.

ORAL PROBLEMS.

481. *1.* A grocer paid 30 cents a pound for butter and sold it at a gain of $16\frac{2}{3}\%$; what was the gain, and what was the selling price?

Analysis.—A gain of $16\frac{2}{3}\%$ is a gain of $\frac{1}{6}$ of the cost; $\frac{1}{6}$ of 30 cents is 5 cents, the gain. The cost, 30 cents, plus the gain, 5 cents, is 35 cents, the selling price.

2. Bought potatoes at $4 per barrel and sold them at a gain of 25%; what was the gain, and what was the selling price?

3. Bought goods for $10 and sold them at a loss of 20%; what was the selling price?

4. For how much must cheese bought at 20 cents a pound be sold to gain 20%? 10%? 25%? 50%? 75%?

5. What is the loss, and what is the selling price of eggs bought at 24 cents a dozen and sold at a loss of $12\frac{1}{2}\%$?

6. A grocer bought apples at $1.20 a bushel and sold them at a gain of $16\frac{2}{3}\%$; what did he get for them per bushel?

7. I sold a book that cost me $3.20 at a loss of $37\frac{1}{2}\%$; what did I lose, and what did I get for it?

WRITTEN PROBLEMS.

1. A speculator bought grain for \$18675, and sold it at a gain of 28%; what was the gain, and what did he get for it?

Explanation.—A gain of 28% is a gain of .28 of the cost, equal to \$5229. The cost, \$18675, plus the gain, \$5229, gives the selling price, \$23904.

Process.

$$\$18675 \times .28 = \$5229.$$
$$\$18675 + \$5229 = \$23904.$$

2. A merchant bought goods for \$1875 and sold them at a gain of 20%; what did he gain, and what was the selling price?

3. Potatoes were bought at \$1.60 a bushel and sold at a loss of 25%; find the loss and the selling price.

4. I bought a horse and sleigh for \$465 and sold them so as to gain $33\frac{1}{3}$%; what did I get for them?

5. A dealer invests \$9864 in shoes; what must he get for them to gain $37\frac{1}{2}$%?

6. A butcher pays \$3930 for cattle and sells them at a loss of $16\frac{2}{3}$%; what is the loss and what is the selling price?

7. A ship bought for \$125800 was sold at a loss of 15%; what was lost and for how much was it sold?

8. An ice-house was filled with ice at a cost of \$875, and the expense of delivering it to customers was \$1985; if the gain on the entire investment was 275%, what was received for the ice?

9. I bought a farm of 378 acres at \$95 per acre and sold at a gain of 36%; what did I get for it, and what did I gain?

10. A piano costing \$360 was sold at a gain of 45%; what were the gain and the selling price?

11. A contractor built a row of 9 houses at a cost of \$41760; he sold them at a gain of 35%; what was the total gain, and the average price received for them?

12. I bought 75 horses at \$125 each and sold them at a gain of 36%; what did I get for them, and what did I gain?

482. To find the rate of gain or loss when the cost and selling price are given.

ORAL PROBLEMS.

1. A grocer buys butter at 20 cents a pound and sells it at 25 cents a pound; what is his gain per cent.?

Analysis.—He gains on one pound the difference between 25 cents and 20 cents, or 5 cents; 5 cents is $\frac{1}{4}$ or $\frac{25}{100}$ of the cost; hence, his gain is 25 per cent.

2. What is the gain per cent. on sugar bought at 6 cents a pound and sold at 8 cents a pound?

3. What gain per cent. is a gain of 2 cents on 4 cents? A gain of 3 cents on 12 cents? \$4 on \$20? \$6 on \$30?

4. What do I lose per cent. to buy at 15 cents and sell at 12 cents? To buy at 20 cents and sell at 18 cents? At 16 cents? At 15 cents?

5. A grocer bought flour at \$6 a barrel and sold it at \$7, what did he gain per cent.?

WRITTEN PROBLEMS.

1. A man bought a house for \$8800 and sold it for \$10120; what did he gain per cent.?

Process.

$$\$10120 - \$8800 = \$1320$$
$$\$1320 \div \$8800 = .15 = 15\%$$

Explanation. — Subtracting the cost from the selling price gives \$1320, the gain. Dividing \$1320 by \$8800 gives .15, the decimal part of the cost that is equal to the gain. A gain of .15 is a gain of 15 on 100, equal to a gain of 15%.

2. A merchant sold goods that cost him \$7500 so as to gain \$1875; what did he gain per cent.?

3. Bought 925 barrels of flour at \$8.20 per barrel and sold it for \$9102; what was gained per cent.?

4. A farm was bought for \$12375 and sold for \$13365; what was the gain per cent.?

5. Bought 2350 yards of brocade at \$3.25, and sold at a gain on the whole of \$6721; what per cent. did I gain?

6. Cloth costing \$1.75 a yard is marked \$2.50; before selling, the merchant deducts 20%; what does he gain per cent.?

7. A man bought two houses, one for \$8380, the other for \$12600; he sold both for \$24127; what was his gain per cent.?

8. A man whose salary was \$1800 had it increased to \$2250; what was the per cent. of increase?

9. By selling for \$11422.40 goods that cost me \$9680, what per cent. did I gain?

10. I bought a farm for \$3680 and sold it for \$4232; what did I gain %?

11. A speculator bought 360 barrels of flour at \$8.40 and sold it all for \$3477.60; what was the gain or loss %?

483. To find the cost when the selling price, or the loss or gain, and the loss or gain per cent. are given.

ORAL PROBLEMS.

1. A grocer sold coffee at a profit of 4 cents per pound, and gained $16\frac{2}{3}$%; what was the cost and the selling price?

Analysis—A gain of $16\frac{2}{3}$% or $\frac{1}{6}$ of the cost, is equal to 4 cents; the whole cost is, therefore, 6 times 4 cents, or 24 cents; and the selling price, 24 cents plus 4 cents, or 28 cents.

2. When a gain of \$8 is a gain of 20%, what is the cost, and what the selling price?

3. Sold peaches at a profit of 20 cents on a basket, thereby gaining $16\frac{2}{3}$%; what did they cost, and what was the selling price?

4. What is the cost, and what the loss when a loss of 40 cents is equivalent to a loss of 10%? $12\frac{1}{2}$%? 20%? 25%? 40%? $62\frac{1}{2}$%?

5. What is the cost if I gain 25% by selling at a gain of 4 cents on each article sold?

6. I gain $37\frac{1}{2}\%$ by selling apples at \$2 a barrel; what did they cost?

7. By selling velvet at \$3.60 per yard a merchant loses 10%; what did it cost him?

WRITTEN PROBLEMS.

1. By selling a house for \$750 more than it cost me I gained 15%; what did it cost?

Explanation.—Since \$750 is 15% of the cost, the cost is found by dividing \$750 by .15, which gives \$5000.

Process.

$$\$750 \div .15 = \$5000$$

2. A merchant sold goods at a profit of \$3870, thereby gaining 15%; what did he pay for them?

3. Mr. Brown sold some lots at a profit of \$1440; if his gain was equal to $33\frac{1}{3}\%$ of the cost, what did he pay for them?

4. At the close of a year's business A had realized a profit of \$5643, which was $37\frac{1}{2}\%$ of his capital at the beginning of the year; what was his capital?

5. By selling a property for \$87100 a man gained 30%; what did it cost him?

6. A man gained 8% more by selling his farm at a gain of \$3780 than if he had gained only \$2700; what did the farm cost him?

7. A merchant sold a bill of goods for \$3660, thereby gaining 20%; what did they cost him?

8. I sold my house for \$3570, which was at a loss of 15%; what did it cost me?

9. A speculator sold two houses, one for \$9680, and the other for \$3190; on the first he gained $37\frac{1}{2}\%$, and on the second he lost 45%; did he gain or lose by the transaction, and how much?

10. I bought a store for 10% less than it cost to build, and sold it for \$22500, thereby gaining 25% on my investment; what did it cost the builder, and what did I pay for it?

COMMISSION.

484. **Commission** is the compensation received by an agent.

485. An **agent** is a person that transacts business for another.

According to the nature or circumstances of the business, the agent is called a *broker, factor, correspondent, consignee, collector,* or *commission merchant,* etc.

486. The principles of percentage are applicable to the operations of commission.

The *sum on which the commission is paid* is the **base.**
The *per cent. of commission* is the **rate.**
The *commission* is the **percentage.**

487. **To find the commission, when the rate and the sum on which the commission is paid is given.**

ORAL PROBLEMS.

1. What should be paid to an agent who charges 5% for collecting a debt of $300?

Analysis.—A charge of 5% is equal to .05, or $\frac{1}{20}$ of the amount collected; $\frac{1}{20}$ of $300 is $15, the commission.

2. How much must be paid to an agent for selling goods for $100, his commission being 3%? How much, if the sales amount to $200? $300? $500?

3. A lawyer collects for me bills amounting to $400, on a commission of 5%; how much of the money should he retain for his services? How much, if his commission is $4\frac{1}{2}$%? $3\frac{1}{2}$%? $2\frac{1}{2}$%?

WRITTEN PROBLEMS.

1. A commission merchant in New York City sells, for a fruit grower in Delaware, 1875 baskets of peaches at $.90 per basket; what is his commission at 8% ?

2. A correspondent in Boston sold for a merchant in Brazil, on a commission of 12%, 3750 bags of coffee at $18.75 per bag; find the amount of the commission.

3. A book agent sold 976 copies of a book at $4.60 per copy, receiving a commission of 28%; what amount should he remit to the publishers?

4. A traveling agent sold, on a commission of 5%, goods amounting to $21375; what was his commission?

5. A buyer for a wholesale merchant expended $198375 for silks on a commission of 4½%; what was the amount of his commission?

488. To find the sum on which the commission is charged, when the rate and the commission are given.

ORAL PROBLEMS.

1. An agent charged me $25 for collecting a bill; what was the amount collected if his rate for collecting was 5%?

Analysis.—A rate of 5% is equivalent to $5 on each $100 collected, hence, he collected as many times $100 as $5 is contained times in $25, equal to 5 times $100, or $500.

2. A merchant paid a traveling salesman $100 for selling a bill of goods; what was the amount of the bill if the salesman received 4% of the bill?

3. A real estate agent was paid in a year $400 by a landlord for collecting rents; how much did he collect, his commission being 5%?

4. A collector whose commission was 4% received $6 for collecting a bill; what was the amount of the bill?

5. If an agent charges \$20 for purchasing goods at 4% commission, what did he pay for the goods?

WRITTEN PROBLEMS.

1. An agent whose commission was 1¾% charged \$551.74 for purchasing molasses in New Orleans; what did he pay for it?

2. My city agent sold a shipment of butter for me on a commission of 4¾%; what did he get for it if his commission was \$39.90?

3. An agent in Liverpool sold some cotton for a southern planter on a commission of 4½%. For how much was the cotton sold if the agent charged \$954.72?

4. A tax collector whose commission for collecting was 5⅓% received \$374.88; how much did he collect?

5. A commission merchant in New York sold a consignment of oranges for a grower in Florida on a commission of 12½%, and received for his services \$396.48; what did he get for them?

6. A lawyer distributed a trust fund at a commission of 2½%, and his bill for so doing was \$1565.85; what was the amount of the fund?

489. To find the base on which commission is charged when the rate and the sum or the difference of the base and commission are given.

ORAL PROBLEMS.

1. A grocer sent his agent in the country \$315 with which to buy butter, agreeing to pay him 5% on the amount expended; how much can the agent expend and retain the amount of his commission?

Analysis.—For each \$1 expended for butter the agent charges 5 cents, making the expense to the grocer \$1.05; hence, the agent will expend in all for butter as many dollars as \$1.05 is contained times in \$315, or \$300.

2. I sent my correspondent \$102 to buy goods for me after deducting his commission of 2%; how much should he retain for himself, and how much should he expend for me?

3. A salesman, after retaining his commission at 4%, pays over to the merchant employing him \$96; what was the amount of his sales?

Analysis.—Since he retains 4% of the amount of sales, he returns 100%—4%, or 96%, equal to $\frac{96}{100}$. Hence, $\frac{96}{100}$ of the amount of sales equals \$96, etc.

4. What rent is paid for a house that yields \$475 a year, after the agent retains his commission of 5%?

WRITTEN PROBLEMS.

1. What amount of goods can an agent purchase with \$44843.75, after deducting a commission of $2\frac{1}{2}$%?

2. A dealer in Chicago sent to his agent in Texas \$98759.50 for the purchase of cattle. How much could be expended for cattle after deducting a commission of $1\frac{1}{2}$%?

3. After retaining his commission at 5%, a collector of taxes for a certain county pays into the treasury \$8675.40; what was his commission, and how much did he collect?

4. A merchant sent \$5258.25 to his agent in Minneapolis to invest in flour at \$4.75 per barrel after deducting his commission of $2\frac{1}{2}$%; how many barrels should the merchant receive, and how much was the agent's commission?

5. A Liverpool merchant sent his correspondent in Mobile \$73150 with which to buy cotton; how many bales at \$175 each can be bought after deducting a commission of $4\frac{1}{2}$%?

6. I sent my agent \$48970, with which he purchased city lots at \$295 each, on a commission of $3\frac{3}{4}$%; how many lots did he buy, after reserving his commission?

7. A real estate agent remitted to the owner rents amounting to \$8039.10, after deducting $4\frac{3}{4}$% for making the collections; what was the amount of his collections?

INSURANCE.

490. Insurance is an agreement by one party to indemnify another in case of loss.

491. The **premium** is the amount paid for the insurance.

492. The **policy** is the writing or instrument that embodies the contract between the insurer and the insured.

The terms correspond with those in Percentage as follows:

The *sum insured*, or the *face of the policy*, is the **base**.
The *rate of the premium* is the **rate**.
The *premium* is the **percentage**.

493. To find the premium, the sum insured and the rate of premium being given.

WRITTEN PROBLEMS.

1. What must I pay to insure my house for $5000 against loss by fire for 3 years, at 2%?

Explanation.—Since the rate of premium is 2% for the given time, .02 of the face of the policy equals the premium, or $100.

Process.

$\$5000 \times .02 = \$100.$

2. What is the premium for insuring a mill for $65000 at $1\frac{1}{2}\%$?

3. What will it cost to insure during transportation a large mirror for $850, the rate of premium being $\frac{3}{4}\%$?

4. A ship's cargo was insured for $185000 at a premium of $5\frac{1}{4}\%$ on the amount insured; what was the premium?

5. Mr. Ellis insured his store for $35000 and his stock for

$85000 ; if the rate on the store was 1½% and on the stock 2½% what was the total premium ?

6. What must I pay for a policy of $3500 on the furniture in my house at ½% ?

7. A ship was insured for $325000 at 4½% for one year; what was the premium ?

8. A gentleman insures a row of 12 houses at $6500 each, paying an annual premium of 1¼%; what does it cost him ?

9. What must be paid for an insurance of $165000 on a theatre, at 1⅝% premium ?

494. To find the sum insured when the premium and the rate are given.

WRITTEN PROBLEMS.

1. I paid $55.50 for insuring my house for 2 years at 1½%; what was the face of the policy ?

Explanation.—Since $55.50 is 1½% of the face of the policy, the face of the policy is found by dividing $55.50 by .015, which gives $3700.

Process.

$\$55.50 \div .015 = \$3700.$

2. When $198.33 is paid for insurance at 2¾%, what is the amount insured ?

3. To insure a cargo from New York to Rio Janeiro at 5¼% cost $12337.50 ; what was the face of the policy ?

4. A manufacturer paid an annual premium of $3134.90 on his mill insured at 2⅞% ; for what was the mill insured ?

5. The premium paid for insuring Mr. Ackerman's store and stock at 1¼% was $123.75 annually ; find the sum insured.

6. A drover paid $924.60 to insure several car-loads of cattle at 2⅞% ; for what were they insured ?

7. A barn was insured at 1¼% and the annual premium was $29.45 ; for how much was it insured ?

8. A business block was insured at 1⅜%, and the premium was $5445 ; what was the face of the policy ?

495. To find the rate when the sum insured and the premium are given.

WRITTEN PROBLEMS.

1. At what rate is a factory insured, if the amount insured is \$32500 and the premium is \$893.75?

Explanation.—Dividing the \$893.75 by \$32500 gives .0275, the fractional part of the sum insured that equals the premium; .0275 of a number is the same as $.02\frac{3}{4}$ or $2\frac{3}{4}\%$.

Process.

$$\$893.75 \div \$32500 = .0275 = 2\tfrac{3}{4}\%$$

2. When the sum insured is \$63500 and the premium is \$1270, what is the rate?

3. What is the rate if the premium is $\$351.37\frac{1}{2}$ and the face of the policy \$46850?

4. My life is insured for \$15000, at an annual cost of \$525; what is the rate of premium?

5. To insure a mill for \$185000 cost \$4162.50; what was the rate?

6. A ship was insured for \$135000 for a premium of \$4387.50; what was the rate?

7. A theatre was insured for \$110000, and the annual premium was \$2062.50; what was the rate?

TAXES.

496. A **tax** is a sum of money raised for public use, assessed upon the person or the property of individuals, or upon the property of corporations.

497. A **poll tax** is a tax upon the person.

The *value assigned to the property by the assessor* is the **base.**

The *tax on* \$1 is the **rate.**

The *tax levied on the property* is the **percentage.**

498. To find the rate of taxation, and the tax of each individual.

WRITTEN PROBLEMS.

1. A certain town is to be taxed \$30800. The assessed valuation of the real estate is \$1700000 and of the personal property \$300000. There are 500 polls taxed at \$1.60 each. What is the tax of a citizen whose real estate is assessed at \$12000 and his personal property at \$1860, if he pays one poll tax?

Explanation. — Multiplying \$1.60 by 500 gives \$800, the amount of poll tax. The entire tax to be raised, less the poll tax, is \$30000. This amount divided by \$2000000, the valuation of the taxable property, gives .015, the rate. The property, real and personal, of the citizen in question is valued at \$13860; this sum multiplied by the rate, and the result increased by the amount of one poll tax, is \$209.50, the tax to be paid by the citizen.

Process.

$$\$1.60 \times 500 = \$800.$$
$$\$30800 - \$800 = \$30000.$$
$$\$1700000 + \$300000 = \$2000000.$$
$$\$30000 \div \$2000000 = .015.$$
$$\$12000 + \$1860 = \$13860.$$
$$\$13860 \times .015 = \$207.90.$$
$$\$207.90 + \$1.60 = \$209.50.$$

Rule.—I. *From the entire tax to be raised subtract the amount of the poll tax, and divide the remainder by the assessed valuation of all the taxable property, real and personal. The result is the rate.*

II. *Multiply the assessed valuation of each individual's property, real and personal, by the rate, and to the product add his poll tax. The result is the tax of the individual.*

2. In a township where the assessed value of the taxable property was \$975000, a tax of \$15600 was raised to build a school-house; what was the rate, and what was the tax of Mr. A, whose property was assessed at \$5375?

3. If in a certain town the taxable property is assessed at \$3875500, and the polls are 1241, assessed at \$1 each, what

rate will enable the people of the town to pay for sewers costing $71000?

4. In the above town, what was A's tax, if his property was assessed at $5375, and he paid for one poll?

5. A, B and C occupy adjoining properties on the same street; they unite in paying $5600 for a sewer in the street; A's property is worth $8700, B's $22900, and C's $35600; how much should each pay?

499. In actual practice, the tax-rate is first determined, and the tax of each individual is then found by using a table constructed in accordance with that rate.

For a tax of 18 mills on $1, or $.018, the following is such a

TABLE.

Assessment,	$1	$2	$3	$4	$5	$6	$7	$8	$9
Tax, . . .	$.018	$.036	$.054	$.072	$.09	$.108	$.126	$.144	$.162

1. Required to find the tax on $9875.50 at the rate of $.018

Explanation.—The tax on $9000 is 1000 times the tax on $9, equal to $162. The tax on $800 is 100 times the tax on $8, equal to $14.40. The tax on $70 is 10 times the tax on $7, equal to $1.26. In the same way the tax is found on $5 and on 50 cents, after which the addition of both columns gives the result desired.

Process.

Tax on		$9000	=	$162.00
"	"	800	=	14.40
"	"	70	=	1.26
"	"	5	=	.09
"	"	0.50	=	.009
"	"	$9875.50	=	$177.76

At 18 mills on $1, find the tax on:

2. $891.	*6.* $11650.	*10.* $729.80.	*14.* $12869.80.
3. $1525.	*7.* $18212.	*11.* $831.40.	*15.* $32131.20.
4. $8739.	*8.* $24863.	*12.* $1213.75.	*16.* $92386.25.
5. $12345.	*9.* $83965.	*13.* $3145.60.	*17.* $18139.90.

STOCKS AND BROKERAGE.

500. A **corporation** is a society authorized by law to transact business under a company name, and possessed of the same legal rights and responsibilities as an individual.

501. **Stock** is the capital of a corporation or company. It is generally divided into equal shares, usually $100, which are transferable like other property. The interest-bearing securities of corporations and governments are called **bonds.**

502. **Coupons** are certificates of interest attached to bonds.

503. Stock is **at par** when it may be sold in open market for its face value; when it sells for less, it is **below par,** or **at a discount**; when it sells for more, it is **above par,** or **at a premium.**

504. **A stock broker** is one who is authorized to buy and sell stocks for others. His commission is called **brokerage.** It is usually $\frac{1}{4}\%$ or $\frac{1}{8}\%$ on the par value of the stock.

505. To find the cost of any number of shares of stock when the market value and rate of brokerage are given.

WRITTEN PROBLEMS.

1. What will be the cost of 180 shares of telegraph stock, the market value being $113\frac{3}{4}$ and brokerage $\frac{1}{4}\%$?

Explanation.—The cost of each share, including the brokerage, will be $114, and the cost of 180 shares will be 180 times $114, equal to $20520.

Process.

$$\$113\tfrac{3}{4} + \$\tfrac{1}{4} = \$114$$
$$\$114 \times 180 = \$20520$$

2. What must be paid for 575 shares of Delaware, Lackawanna and Western Railroad stock at 108, brokerage being $\frac{1}{4}$%?

3. What will 500 shares of Central Pacific cost at 102$\frac{1}{4}$, brokerage $\frac{1}{8}$%?

4. What must I pay for 175 Paterson city bonds at 104$\frac{1}{4}$, brokerage $\frac{1}{8}$%?

5. What must be paid for 125 shares of Union Pacific stock at 118$\frac{1}{2}$, brokerage being $\frac{1}{4}$%?

6. My broker charged me $\frac{1}{8}$% for buying 396 shares of Pennsylvania Central stock at 113; what was the entire cost?

7. A broker sold for me for $\frac{1}{8}$% 1876 shares of telephone stock at 92; what sum should he remit to me?

8. I sold through a broker 1280 shares of stock at 113$\frac{5}{8}$, brokerage $\frac{1}{4}$%; what should he remit to me?

9. What should I get for 3756 shares of bank stock sold at 107$\frac{3}{8}$, brokerage $\frac{1}{4}$%?

506. To find the number of shares that can be purchased for a given amount.

WRITTEN PROBLEMS.

1. When the brokerage is $\frac{1}{8}$%, how many shares of stock at 98$\frac{7}{8}$ can be purchased for $7425?

Explanation.—The cost of one share with the brokerage will be $99. Dividing $7425 by $99 gives 75 shares.

Process.

$$\$98\tfrac{7}{8} + \$\tfrac{1}{8} = \$99$$
$$\$7425 \div \$99 = 75$$

2. How many shares of Old Colony Railroad stock at 102$\frac{1}{4}$ can be bought for $29484, brokerage $\frac{1}{8}$%?

3. My broker expended $91300 in the purchase of government bonds at 103$\frac{1}{2}$, brokerage being $\frac{1}{4}$%; how many bonds did he buy?

4. How many shares of Philadelphia and Erie Railroad stock at 92$\frac{5}{8}$ can be bought for $7049, brokerage $\frac{1}{8}$%?

5. How many shares of bank stock at a discount of 8% can be bought for \$33948 if $\frac{1}{4}$% is paid for brokerage ?

6. What number of U. S. bonds at 107$\frac{3}{4}$ can be bought with \$69903, when $\frac{1}{8}$% is paid for brokerage ?

7. I invested \$9717 in mining stocks at 78$\frac{7}{8}$, brokerage $\frac{1}{8}$%; how many shares did I get ?

8. A broker invested for me \$12789 in street railroad bonds at 86$\frac{3}{4}$, brokerage $\frac{1}{4}$%; how many shares did he buy ?

507. To find the amount of an investment when the income and rate are given.

WRITTEN PROBLEMS.

1. How much must be invested in U. S. government 5% bonds at 106$\frac{3}{4}$, brokerage at $\frac{1}{8}$%, to yield an annual income of \$2690 ?

Process.

$$\$2690 \div \$5 = 538 \text{ shares.}$$

$$(\$106\tfrac{3}{4} + \$\tfrac{1}{8}) \times 538 = \$57498.75.$$

Explanation.—Bonds paying 5% yield \$5 each per annum ; hence, \$2690 divided by \$5 gives the number of bonds to be purchased. Each bond will cost \$106$\frac{3}{4}$ + \$$\frac{1}{8}$ = \$106$\frac{7}{8}$, and 538 will cost 538 times \$106$\frac{7}{8}$, equal to \$57498.75.

2. A gentleman wishes to endow a professorship in a college with an annual income of \$3000 ; what amount must he invest in U. S. 4% bonds bought at 106$\frac{1}{4}$, brokerage $\frac{1}{8}$% ?

3. What amount must a man invest in railroad bonds paying an annual dividend of 10%, if he buys them at 98, brokerage $\frac{1}{8}$%, in order to have from them an income of \$5000 a year ?

4. My income from bank stock bought at 107$\frac{1}{2}$, brokerage $\frac{1}{4}$%, is \$8400. What did it cost me if it pays an annual dividend of 6% ?

5. How much must be invested in 7% city bonds bought at 101$\frac{1}{4}$, brokerage $\frac{1}{4}$%, to yield an annual income of \$9800 ?

6. How much must be invested in 5% bonds bought at 94$\frac{1}{8}$, brokerage $\frac{1}{8}$%, to yield an annual income of \$8400 ?

7. I have an income of \$1120 per annum from stocks that pay a quarterly dividend of $3\frac{1}{2}\%$; what did I pay for them at $109\frac{5}{8}$, brokerage $\frac{1}{4}\%$?

8. The salary of a college professor is \$3500 a year, and is paid with the income from bonds yielding a semi-annual dividend of $1\frac{3}{4}\%$; what did they cost at $94\frac{1}{8}$, brokerage $\frac{1}{8}\%$?

9. A man directed in his will that enough 5% stocks should be bought to pay his wife an annual income of \$4500; how much would they cost at $103\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

508. To find the rate % of income from money invested in bonds.

WRITTEN PROBLEMS.

1. What % do I get from money invested in bonds paying 5%, if I buy them at $94\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

Explanation.—The cost of one bond is \$95, and it yields me \$5 annual income. If \$95 yields an annual income of \$5, \$1 will yield $\frac{1}{95}$ of \$5, equal to $\$\frac{5}{95}$ or $\$\frac{1}{19}$; and \$100 will yield $\$\frac{100}{19}$, or $\$5\frac{5}{19}$. Hence, the investment pays $5\frac{5}{19}\%$.

Process.

$$\$5 \div (\$94\tfrac{3}{4} + \$\tfrac{1}{4}) = .05\tfrac{5}{19} = 5\tfrac{5}{19}\%$$

2. What % income is yielded by 7% municipal bonds bought for $139\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

3. What per cent. do I realize from railroad stock paying an annual dividend of 8%, if it costs me $159\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

4. A certain bank stock is bought at $119\frac{7}{8}$, brokerage $\frac{1}{8}$, and pays an annual dividend of 12%; what % income is yielded by money invested in it?

5. Which will pay the better per cent. on the investment, and how much; 8% stock bought at $79\frac{7}{8}$, or 5% stock bought at $62\frac{3}{8}$, brokerage in each case being $\frac{1}{8}\%$?

6. How much more is my rate of income on 6% bonds bought at $74\frac{3}{4}$, than on 4% bonds bought at $53\frac{1}{2}$, brokerage in each case being $\frac{1}{4}\%$?

CUSTOMS OR DUTIES.

509. Customs or **duties** are taxes levied by governments on imported goods.

510. Duties are of two kinds, **specific** and **ad valorem.**

511. A **specific duty** is a tax of a specified amount levied without regard to the value of goods.

512. An **ad valorem duty** is a tax determined by the cost of goods in the country from which they come.

513. Tare is an allowance for the weight of the boxes, bags, etc., containing the goods.

514. Leakage and **breakage** are allowances for the leakage of liquids and the breakage of bottles.

515. Cities where customs are collected are **ports of entry.**

516. Gross weight is the weight before deductions are made; **net weight** is the weight after deductions are made.

517. To find the duty on goods.

WRITTEN PROBLEMS.

1. What is the duty at 60% *ad valorem* on 15640 yards of silk that cost in France $1.30 per yard ?

Explanation. — At $1.30 per yard, the cost in France was $20332. 60%, or .60 of this amount is equal to the duty.

Process.

$$\$1.30 \times 15640 = \$20332$$
$$\$20332 \times .60 = \$12199.20$$

2. Required the duty at $1\frac{1}{2}$ cents a pound on 57 hogsheads of sugar imported from Havana, each hogshead weighing 790 lb., and tare being 75 lb. on each hogshead.

Explanation.—The gross weight, 45030 pounds, diminished by the tare, 4275 pounds, gives 40755 pounds, the amount on which duty is charged. Multiplying the duty per pound by the number of pounds, gives the duty, \$611.33.

Process.

$$790 \times 57 = 45030 \text{ lb.}$$
$$75 \times 57 = \underline{4275} \text{ lb.}$$
$$40755 \text{ lb.}$$
$$\$.015 \times 40755 = \$611.325$$

3. What is the duty, at 65% *ad valorem,* on 500 cases of champagne invoiced at \$23 a case, there being an allowance of 2% for breakage?

4. What is the duty at 36 cents a pound on 75 cases of tobacco, each weighing 190 pounds?

5. Find the duty at 25% *ad valorem* on 360 chests of tea, each containing 86 pounds invoiced at 40 cents a pound?

6. What is the duty on 860 tons of steel at five cents a pound, duty being 25% *ad valorem?*

7. At a specific duty of \$.50 per pound, and an *ad valorem* duty of 25%, what is the duty on tobacco weighing 600 pounds, valued at \$1.20 a pound?

8. Find the duty on lace valued at \$1575, duty being 25% *ad valorem.*

9. What is the amount of an *ad valorem* duty of 15% on goods invoiced at \$36875?

10. What is the amount of a specific duty of 45 cents a yard and an *ad valorem* duty of 20%, on 8500 yards of French broadcloth invoiced at \$3.75 a yard?

11. An importer at New York received a cargo consisting of 875 hogsheads of molasses of 126 gallons each, and 800 hogsheads of sugar each weighing 525 lb. After an allowance of 14% on the sugar for tare and of 12% on the molasses for leakage, he paid a duty of $2\frac{1}{4}$ cents a pound on the sugar and 9 cents a gallon on the molasses; what was the duty?

INTEREST.

518. **Interest** is money paid for the use of money.

519. The **principal** is the sum for the use of which interest is paid.

520. The **amount** is the sum of the principal and interest.

521. The **rate** is the per cent. of the principal paid for its use for a given time. The time is one year, unless a longer or shorter period is specified.

522. The **legal rate** is the rate established by law.

523. **Usury** is a rate greater than the legal rate.

For a table showing the legal rate in the different states, see Appendix, page 341.

The terms in Interest correspond with those in Percentage, as follows:

The *principal* is the **base.**
The *interest* is the **percentage.**
The *rate per year* is the **rate.**

Strictly, *the product of the rate per year and the time in years* corresponds with the rate in Percentage.

Thus, if the rate is 6% a year, and the time is 2 years 6 months, the rate is $2\frac{1}{2}$ times 6%, or 15%; that is, .15 of the principal equals the interest.

524. There are several methods of computing interest, but from the explanation given above of the correspondence of terms in Interest and Percentage, it follows that all methods depend on the fact expressed in the following formula:

$$\textit{Interest} = \textit{Principal} \times \frac{\textit{rate}}{100} \times \textit{time in years.}$$

METHOD BY ALIQUOT PARTS.

WRITTEN PROBLEMS.

525. *1.* What is the interest on $375 for 3 years 9 months 24 days at 8%?

Explanation.—The principal multiplied by .08 gives the interest for 1 year, and the result multiplied by 3 gives the interest for 3 years. Dividing the interest for 1 year by 2 gives $15, the interest for 6 of the 9 months, and for the remaining 3 months the interest is half as much as the interest for 6 months. Since 15 days is $\frac{1}{6}$ of 3 months, $\frac{1}{6}$ of $7.50, or $1.25, is the interest for 15 days. For the remaining 9 days, $\frac{1}{10}$ of 3 months' interest is taken, since 9 days is $\frac{1}{10}$ of 3 months, or 90 days. The sum of all these partial results is the interest required.

Process.

$375	
.08	
$30.00	= Int. for 1 year.
3	
$90.00	= Int. for 3 years
15.00	= " " 6 months.
7.50	= " " 3 months.
1.25	= " " 15 days.
.75	= " " 9 days.
$114.50	= Int. for 3 yr. 9 mo. 24 da.

Any other convenient aliquot parts might have been taken. Thus, 9 months is $\frac{1}{4}$ of 3 years, 15 days is $\frac{1}{18}$ of 9 months, and 9 days is $\frac{1}{30}$ of 9 months.

2. What is the interest on $1600 for 2 months 25 days at $7\frac{1}{2}\%$?

Explanation.—At $7\frac{1}{2}\%$ the interest on $1600 for 1 yr. is $120, and $\frac{1}{6}$ of this is the interest for 2 months. The interest for 20 of the 25 days is $\frac{1}{3}$ of $20, since 20 days is $\frac{1}{3}$ of 2 months. Since the remaining 5 days is $\frac{1}{4}$ of 20 days, $6.667 is divided by 4. The sum of the partial results is the interest required.

Process.

$1600	
.075	
$120.00	= Int. for 1 year.
$20.00	= Int. for 2 months.
6.667	= " " 20 days.
1.666	= " " 5 days.
$28.33	= " " 2 mo. 25 da.

Rule.—I. *Find the interest on the principal for one year at the given rate, and multiply this result by the time in years, and then find by aliquot parts the interest for the months and days given.*

II. *Add all these partial results, and the sum will be the interest required.*

Find the interest on :

3. \$890 for 1 year 2 months 15 days at 6%.
4. \$625 for 2 years 5 months 27 days at 8%.
5. \$415.50 for 1 year 7 months 12 days at 10%.
6. \$624.25 for 5 years 5 months 15 days at 4½%.
7. \$2250 for 2 years 7 months at 9%.
8. \$618 for 1 year 1 month 20 days at 7½%.
9. \$925 for 3 years 5 months 6 days at 5½%.
10. \$580 for 4 years 1 month 24 days at 10%.
11. \$912.60 for 1 year 9 months 10 days at 7%.
12. \$515.80 for 2 years 3 months 3 days at 3½%.
13. \$895 for 7 months 27 days at 9%.
14. \$36000 for 24 days at 7½%.

Find the amount of :

15. \$2787.20 for 5 years 8 months 26 days at 5⅝%.
16. \$4363.20 for 6 years 7 months 25 days at 5⅔%.
17. \$5760 for 5 years 6 months 24 days at 8¾%.
18. \$715.50 for 4 years 5 months 23 days at 7⅕%.
19. \$604.80 for 7 years 3 months 20 days at 6½%.
20. \$1120 for 6 years 5 months 24 days at 3¾%.
21. \$118.80 for 5 years 6 months 10 days at 5⅔%.
22. \$843.75 for 4 years 7 months 16 days at 6½%.
23. \$2952 for 3 years 8 months 27 days at 5½%.
24. \$1296 for 11 years 7 months 17 days at 6¾%.
25. \$8760 for 3 years 5 months 29 days at 7½%.
26. \$15675 for 2 years 7 months 15 days at 8%.
27. \$26850 for 3 years 3 months 12 days at 9%.

SIX PER CENT. METHOD.

WRITTEN PROBLEMS.

526. *1.* What is the interest on $1560 for 5 years 7 months 28 days at 6%?

Explanation.—The interest on $1 for 1 year at 6% is $.06, and for 5 years it is $.30. Since the interest on $1 for 1 year, or 12 months, is $.06, for 1 month it is $\frac{1}{12}$ of $.06, or $.005, and for 7 months it is $.035. Since the interest on $1 for 1 month, or 30 days, is $.005, for 1 day it is $\frac{1}{30}$ of $.005, or $.000$\frac{1}{6}$, and for 28 days it is $.004$\frac{2}{3}$. The sum of these results, or the interest on $1 for 5 years 7 months 28 days, is $.339$\frac{2}{3}$, and the interest on $1560 is equal to 1560 times $.339$\frac{2}{3}$, or what is the same, $1560 × .339$\frac{2}{3}$, equal to $529.88.

Process.

Int. on $1 for 5 yr. =	$.30
" " " 7 mo. =	.035
" " " 28 da. =	.004$\frac{2}{3}$
	$.339$\frac{2}{3}$

$1560 × .339$\frac{2}{3}$ = $529.88

Rule.—I. *Take 6 cents for each year, 5 mills for each month, and $\frac{1}{6}$ of a mill for each day; the sum of these will be the interest on $1 for the given time at 6%. As many times this quantity as there are dollars in the principal will be the interest on the principal for the given time at 6%.*

II. *For other rates than 6%, divide the interest at 6% by 6, and multiply the quotient by the given rate.*

Find the interest on:

2. $560 for 1 year 6 months at 6%.
3. $1800 for 1 year 2 months 20 days at 6%.
4. $2500 for 2 years 4 months 24 days at 8%.
5. $556.50 for 5 years 2 months 12 days at 9%.
6. $1080 for 6 years 1 month 10 days at 5%.
7. $4096.64 for 3 years 1 month 15 days at 7$\frac{1}{2}$%.
8. $5840 for 4 years 3 months 18 days at 8$\frac{1}{2}$%.

9. \$650 for 7 years 7 months 6 days at $5\frac{1}{2}\%$.

10. \$840 for 3 years 5 months 17 days at 6%.

11. \$3604.50 for 5 years 5 months 26 days at 10%.

12. \$337.50 for 4 years 2 months 20 days at 12%.

13. \$8800 for 3 years 5 months 21 days at $8\frac{1}{2}\%$.

14. \$1464 for 4 years 4 months 27 days at 7%.

15. \$181.20 for 6 years 5 months 24 days at $6\frac{1}{2}\%$.

16. \$360.60 for 1 year 1 month 18 days at $5\frac{3}{4}\%$.

17. \$2880 for 2 years 11 months 29 days at $3\frac{1}{2}\%$.

18. \$156.25 for 3 years 10 months 28 days at $4\frac{1}{2}\%$.

19. \$875 for 4 years 9 months 27 days at 4%.

20. \$9850 for 2 years 5 months 17 days at 8%.

SPECIAL METHOD FOR DAYS.

527. The following short method of computing interest may be employed to advantage when large sums are borrowed for brief periods.

WRITTEN PROBLEMS.

1. What is the interest on \$60000 for 46 days at 6%?

Explanation.—At 6% a year the interest for 2 months is $\frac{1}{6}$ of 6%, or 1%. Hence, 1%, or $\frac{1}{100}$ of the principal equals the interest for 60 days. $\frac{1}{100}$ of \$60000 is \$600. For 30 days the interest is $\frac{1}{2}$ of \$600, or \$300. For 10 of the remaining 16 days it is $\frac{1}{3}$ of \$300, equal to \$100, and for the remaining 6 days it is $\frac{1}{10}$ as much as for 60 days, equal to \$60. The sum, \$460, is the interest for 46 days.

Process.

\$600.00	=	Int. for 60 da.
\$300.00	=	Int. for 30 da.
100.00	=	" " 10 da.
60.00	=	" " 6 da.
\$460.00	=	Int. for 46 da.

Rule.—*Take $\frac{1}{100}$ of the principal as the interest at 6% for 60 days, or $\frac{1}{1000}$ of the principal as the interest for 6 days, and then by aliquot parts find the interest for the given time.*

Note.—For other rates than 6%, proceed as already explained.

Find the interest on:

2. \$35000 for 90 da. at 6%.
3. \$75000 for 45 da. at 6%.
4. \$100000 for 80 da. at 7%.
5. \$45000 for 96 da. at 8%.
6. \$68000 for 24 da. at 5%.
7. \$54000 for 15 da. at 6%.
8. \$83400 for 12 da. at 3%.
9. \$92500 for 10 da. at 4%.
10. \$86500 for 75 da. at 6%.
11. \$975500 for 86 da. at 4%.
12. \$850600 for 112 da. at 4%.
13. \$956875 for 23 da. at 6%.
14. \$3753.75 for 37 da. at 8%.
15. \$168000 for 4 da. at 5%.
16. \$427000 for 9 da. at 3%.
17. \$365000 for 14 da. at 4%.

EXACT INTEREST.

528. To compute interest on the supposition that a year consists of 360 days, gives results too large. Since 360 days is $\frac{360}{365}$, or $\frac{72}{73}$ of a common year, and $\frac{360}{366}$, or $\frac{60}{61}$ of a leap year, the interest as found by the ordinary methods must be diminished by $\frac{1}{73}$ or $\frac{1}{61}$ of itself, in order to obtain the *exact interest.*

Thus, the interest at 6% on \$7300 for 60 days amounts, by the Special Method for Days, to \$73; but 60 days is really $\frac{60}{365}$ or $\frac{12}{73}$ of a year. Hence, the equitable interest is $\$7300 \times .06 \times \frac{12}{73} = \$72$. That is, when the time is in days, each \$73 interest as found by the ordinary methods, is \$72 by the method of exact interest.

NOTE.—The ordinary methods give the exact interest when the time is expressed in years, or in years and aliquot parts of a full year.

Rule.—I. *When the time is in days, diminish the interest as found by the ordinary method by $\frac{1}{73}$ of itself if the time is a portion of a common year, or by $\frac{1}{61}$ of itself if the time is a portion of a leap year.*

II. *When the time is in years, months, and days, or in months and days, add the exact interest for the days to the interest for the remaining time as found by the ordinary method.*

WRITTEN PROBLEMS.

529. *1.* What is the exact interest on \$28560 for 138 days at 6%?

Explanation.—Finding the interest for the given time and rate by the Special Method for Days gives \$656.88. Diminishing this by $\frac{1}{73}$ of itself gives the exact interest \$647.88.

Process.

	\$285.60	= Int. for	60 da.
	285.60	= " "	60 da.
	71.40	= " "	15 da.
	14.28	= " "	3 da.
73)	\$656.88	= Int. for	138 da.
	8.998		
	\$647.882	= Exact interest.	

Find the exact interest on:

2. \$95000 for 24 da. at 6%.
3. \$83500 for 38 da. at 6%.
4. \$567500 for 73 da. at 4%.
5. \$186564 for 138 da. at 7%.
6. \$193724 for 197 da. at 4%.
7. \$621400 for 238 da. at 5%.

8. \$8640 for 2 years 3 months 29 days at 6%.
9. \$6240 for 3 years 5 months 27 days at 6%.
10. \$36000 from Feb. 2, 1882, to Oct. 1, 1885, at 6%.

Explanation. — The interest on \$36000 for 3 yrs. 7 mo. at 6% is \$7740. The interest on \$36000 for 29 days, found by the Special Method for Days, is \$174. Diminishing \$174 by $\frac{1}{73}$ of itself gives \$171.62, the exact interest. Adding these two amounts together gives the exact interest on \$36000 for 3 years 7 months 29 days.

Process.

yrs.	mo.	da.
1885	10	1
1882	2	2
3	7	29

\$7740	= Int. for 3 yrs. 7 mo.
171.62	= Exact int. for 29 da.
\$7911.62	= Exact interest.

NOTE.—In the foregoing example, the difference between the dates given is found by subtracting as in denominate numbers. For other methods, see Appendix, page 337.

11. \$6878400 from Sept. 19, 1885, to July 7, 1886, at 5%.
12. \$65000 from Apr. 13, 1881, to July 19, 1881, at 4%.
13. \$14900 from Feb. 13, 1884, to March 19, 1885, at 5%.
14. \$23960 from Oct. 20, 1880, to Nov. 8, 1883, at 7%.

PROBLEMS IN INTEREST.

530. To find the principal when the interest, rate, and time are given.

WRITTEN PROBLEMS.

1. What principal in 3 years 4 months 24 days at 7% will give $321.30 interest?

Explanation.—The interest on $1 for 3 years at 6% is $.18; for 4 months it is $.02; and for 24 da. it is $.004. Hence, the interest on $1 for 3 yrs. 4 mo. 24 da. at 6% is $.18 + $.02 + $.006 = $.204. At 7% it is $\frac{1}{6}$ more, or $.238. Since $1 in the given time at 7% will produce $.238 interest, it will take as many dollars to produce $321.30 as $.238 is contained times in $321.30, equal to $1350.

Process.

Int. on $1 = $.238.

$321.30 ÷ .238 = $1350.

Rule.—*Divide the given interest by the interest on $1 for the given time, and the quotient will express the number of dollars in the principal required.*

NOTE.—If the amount is given, and not the interest, divide the given amount by the amount of $1 at the given rate for the given time.

2. What principal in 1 year 1 month 12 days at 6% will give $104.52 interest?

3. What principal in 2 years 2 months 18 days at 6% will give $232.75 interest?

4. What principal in 5 years 9 months 18 days at 8% will gain $556.80 interest?

5. What sum in 108 days at 5% will gain $195 interest?

6. How much money in 2 years 5 months 18 days at 6% will give an amount of $16117.92?

7. What principal will in 90 days at $5\frac{1}{2}$% give $151.25 interest?

8. My money in bank at 4% yielded me $271.25 interest for 1 year 3 months 15 days; how much have I in bank?

531. To find the rate when the principal, interest, and time are given.

WRITTEN PROBLEMS

1. At what rate will $675 give $172.80 in 3 years 2 months 12 days?

Explanation. — The interest on $1 for 3 years 2 months 12 days at 6% is $.192, and at 1% it is $\frac{1}{6}$ as much, or $.032. On $675 for the given time the interest at 1% will be $.032 × 675, or $21.60. If $675 produces in the given time $21.60 at 1%, to produce $172.80 in the same time will require as many times 1% as $21.60 is contained times in $172.80, equal to 8%.

Process.

Int. at 6% = $.192.
" " 1% = $.032.
$.032 × 675 = $21.60.
$172.80 ÷ $21.60 = 8.

Rule.—*Divide the given interest by the interest on the given principal at 1% for the given time; the quotient will express the rate required.*

2. At what rate % will $7600 in 2 years 5 months 21 days give $1692.90 interest?

3. A principal of $5885 on interest for 1 year 3 months 6 days gained $447.26 interest; what was the rate?

4. If $14400 in 5 months 8 days yielded $347.60 interest, what was the rate per cent.?

5. At what rate per cent. will $17280 in 66 days gain $205.92 interest?

6. At what rate per cent. will $59860 in 186 days gain $1525.20, exact interest?

7. When the exact interest on $136510 for 216 days is $3635.28, what is the rate per cent.?

8. At what rate per cent. will $1240 in 3 years 3 months 24 days amount to $1486.76?

9. At what rate per cent. will $240000 in 93 days give $3100 interest?

532. To find the time when the principal, interest, and rate are given.

WRITTEN PROBLEMS.

1. In what time will $1960 at 7% give $363.58 interest?

Explanation.—At 7%, the interest on $1960 for one year is $137.20. Dividing the given interest, $363.58, by the interest for one year gives the number of years required, equal to $2\frac{8918}{13720}$ years. Reducing $\frac{8918}{13720}$ of a year to integers of lower denomination by the method explained in Art. 420 gives 7 months 24 days.

Process.

$$\$1960 \times .07 = \$137.20$$

$$\$363.58 \div \$137.20 = 2\tfrac{8918}{13720}$$

$$2\tfrac{8918}{13720} \text{ yrs.} = 2 \text{ yrs. } 7 \text{ mo. } 24 \text{ da.}$$

Rule.—*Find the interest on the given principal for one year at the given rate; divide the given interest by this interest, and the quotient will be the required time in years. Reduce the fraction of a year, if there be any, to months and days.*

2. In what time will $284 at 6% give $38.34 interest?

3. How long will $3640 at 7½% require to yield $982.80 interest?

4. In what time will a principal of $3600 at 5% amount to $4875?

5. I have $7080 in bank at 4⅛%; in what time will it amount to $8850?

6. My father at his death bequeathed me $5600, with the condition that I should not receive it until at 5½% it should amount to $9450; how long before I received it?

7. Mr. Brown owed a debt of $1260, on which he was paying interest at 6⅛%; he did not pay the debt until the interest on it was $445.90; how long did he owe the money?

8. In what time will a mortgage of $3960 on my house yield $770 interest, the rate being 5%?

9. In how many days will $1825 at 5% give $18.25, exact interest?

PROMISSORY NOTES.

533. A **promissory note** is a written promise to pay a specified sum of money.

There are several varieties of promissory notes, as **demand** notes, **time** notes, **negotiable** notes, etc.

The following is a common form of promissory note:

$875\frac{60}{100}$. New York City, Jan. 29, 1886.

Ninety days after date I promise to pay William Ellis, or order, eight hundred seventy-five and $\frac{60}{100}$ dollars, value received. Richard Proctor.

NOTE 1.—In the foregoing note, the words *or order* make the note *negotiable*; the absence of the words *or order* renders a note *non-negotiable*. A person wishing to transfer a negotiable promissory note to another person must indorse it,—that is, he must write his name across the back of the note. Sometimes the words *or bearer* are used instead of *or order*. Such a note may be transferred without indorsement.

NOTE 2.—A promissory note should contain the words *value received*; otherwise, the holder may find it necessary to prove that value to the amount of its face was given for it.

The following is the form of a note payable at a bank:

$2000. New York City, Mar. 5, 1886.

Sixty days after date, value received, I promise to pay to Thomas Hayes, or order, two thousand dollars at the Chemical Bank. Edward Phelps.

NOTE 1.—The holder of the foregoing note sends it to the Chemical Bank for collection. Edward Phelps is notified that the note is there, and of the date of its maturity. He is expected to call and pay the amount, but is allowed *three days of grace* beyond the 60 days.

NOTE 2.—In most states a note is not legally due until the expiration of three days of grace, unless the third is Sunday or a legal holiday; in such case, it is legally due the day before. See Interest Laws, p. 341. A note without interest, payable on demand, bears interest after demand, and a note without interest, due at a specified time, bears interest from maturity.

534. ***Find the amounts due on the following notes:***

1. A note for \$3000, due in 90 days, and paid at maturity, with interest at 5%.

2. A note for \$5600, due in 60 days, bearing interest at 6%, and paid at maturity.

3. A note for \$7500, bearing interest at 6%, dated July 1, 1882, due in 30 days, and paid October 16, 1882.

4. A note for \$960.50, dated March 15, 1884, due in 6 months, and paid March 15, 1885, rate of interest after maturity being 6%.

5. A note for \$12000, bearing interest at $5\frac{1}{2}$%, dated May 1, 1886, due in 90 days, and paid at maturity.

PARTIAL PAYMENTS.

535. Partial payments are payments in part of a debt or other obligation.

536. When partial payments are made on a note, the amounts and dates of payment, written on the back of the note, are called **indorsements.**

Unless a note contains the words *with interest,* it does not begin to draw interest until the note is due.

537. The Supreme Court of the United States has adopted the following rule for Partial Payments:

UNITED STATES RULE.

I. *Find the amount of the principal to the time when the payment, or the sum of the payments, is greater than the interest then due. From the amount subtract the payment or the sum of the payments, and treat the remainder as a new principal.*

II. *Proceed in this manner to the date of settlement and the last amount will be the sum still due.*

538. To compute by the United States Rule for partial payments.

(I.)

$1000. *New York, Jan. 1, 1884.*

On demand I promise to pay Sheldon & Co., or order, one thousand dollars, with interest at 6%, value received.

William R. Morris.

Indorsements: July 1, 1884, $20; Oct. 1, 1884, $375; Jan. 1, 1885, $215.75; Nov. 1, 1885, $320; March 1, 1886, $100; May 1, 1886, $25. What was due July 1, 1886?

	Process.
Principal,	$1000.
Int. from Jan. 1, '84, to Oct. 1, '84 (9 mo.),	45.
Amount,	1045.
Sum of 1st and 2d payments,	395.
New principal,	650.
Int. from Oct. 1, '84, to Jan. 1, '85 (3 mo.),	9.75
Amount,	659.75
3d payment,	215.75
New principal,	444.
Int. from Jan. 1, '85, to Nov. 1, '85 (10 mo.),	22.20
Amount,	466.20
4th payment,	320.
New principal,	146.20
Int. from Nov. 1, '85, to Mar. 1, '86 (4 mo.),	2.92
Amount,	149.12
5th payment,	100.
New principal,	49.12
Int. from Mar. 1, '86, to May 1, '86 (2 mo.),	.49
Amount,	49.61
6th payment,	25.
New principal,	24.61
Int. from May 1, '86, to July 1, '86 (2 mo.),	.25
Bal. due July 1, '86,	$24.86

WRITTEN PROBLEMS.

539. (*1.*)

$2500. *St. Louis, July 1, 1880.*

One year after date, I promise to pay John Holmes, or order, two thousand five hundred dollars, value received. Interest at 6%. *Henry Smith.*

Indorsements: Dec. 1, '81, $750; Jan. 1, '82, $465; June 16, '83, $390. What was due Jan. 1, 1884?

2. On a note for $3650, dated Sept. 1, 1882, were the following indorsements: Jan. 1, '83, $1250; July 25, '83, $45; Dec. 25, '83, $975; May 1, '84, $600. What was due Jan. 1, 1885, interest at 6%?

3. A note for $1175, dated July 1, 1885, and bearing interest at 6%, was indorsed as follows: Oct. 1, '85, $25; Jan. 25, '86, $195; Mar. 1, '86, $465; April 16, '86, $400. What was due May 1, 1886?

4. A note for $8960, dated April 15, 1880, and bearing interest at 5%, was indorsed as follows: Dec. 1, '80, $1000; May 16, '81, $950; Jan. 28, '82, $1200; April 30, '83, $2560; Dec. 1, '83, $1875. What was due April 15, 1884?

5. Find what would be due on the note mentioned in the preceding example, if the rate were 8%.

6. A mortgage for $9650, dated May 1, 1875, bearing interest at 5%, was indorsed as follows: Jan. 1, '76, $300; June 1, '76, $800; Feb. 1, '77, $1250; July 25, '78, $2250; Sept. 22, '80, $2000. What was due, Jan. 1, 1881?

540. To compute by the Merchants' Rule for partial payments.

541. In the settlement of notes and other obligations bearing interest, when the time is less than a year, the computations are frequently made in accordance with the

MERCHANTS' RULE.

I. *Find the amount of each of the several payments from the time each is made to the date of settlement.*

II. *Subtract the sum of these amounts from the amount of the obligation from its date to the time of settlement. The remainder will be the amount still due.*

NOTE.—In this method the *exact interest* is usually taken for the precise number of days between dates. In the illustrative example the ordinary method is employed.

I. A note for $1200, dated Jan. 1, 1886, and bearing interest at 6%, had the following indorsements: Feb. 12, '86, $275; March 19, '86, $320; June 16, '86, $412. What was due July 1, 1886?

		Process.
Principal, ...		$1200.00
Int. from Jan. 1, '86, to July 1, '86 (6 mo.), . . .		36.00
Am't of 1st paym't on July 1, '86 (139 da.),	$281.37	
Am't of 2d paym't on July 1, '86 (104 da.),	325.55	
Am't of 3d paym't on July 1, '86 (15 da.),	413.12	$1020.04
Balance due at settlement, ...		$215.96

WRITTEN PROBLEMS.

542. *1.* A note for $2500, bearing interest at 6%, and dated Jan. 1, 1885, was indorsed as follows: Feb. 23, '85, $750; Apr. 29, '85, $500; July 18, '85, $600. How much remained due, Sept. 1, 1885?

2. A note was dated Oct. 17, '85; face of note, $12500; rate 6%; payments Dec. 12, '85, $4600; Jan. 11, '86, $3600; Jan. 29, '86, $4000. How much was due July 15, 1886?

3. A note for $3620, dated Jan. 1, 1886, exact interest at 6%, bore the following indorsements: Feb. 19, $1200; Mar. 24, $960. What is due May 1, 1886?

4. A merchant on Dec. 1, 1880, incurred a debt for goods amounting to \$6860, and agreed to pay interest at 6%. He paid on account, Jan. 16, '81, \$2600; March 30, '81, \$2000, May 12, '81, \$1600. How much was due Sept. 10, 1881, allowing him interest on his payments?

5. By the method of exact interest, find the amount due May 1, 1886, on a note for \$8350, bearing interest at 5% and dated Sept. 23, 1885.

Indorsements: Oct. 30, '85, \$1800; Dec. 31, '85, \$4400.

ANNUAL INTEREST.

543. If it is stipulated in a note that interest shall be paid *annually*, and it is not so paid, some states allow the collection of interest on this unpaid interest.

(I.)

\$1200. *Albany, N. Y., Oct. 1, 1880.*

On demand, for value received, I promise to pay John Harvey, or order, twelve hundred dollars, with interest at 6%, payable annually. *Cyrus Smith.*

How much is due Oct. 1, 1885?

Process.

Prin., ...	\$1200
Int. for 5 years, . .	360
Int. on \$72 for 1 yr. × 10,	43.20
Am't due,	\$1603.20

Explanation.—The interest on \$1200 each year is \$72; hence, for 5 years it is \$360. The first year's interest, \$72, is on interest for 4 years. In like manner, the 2d year's interest draws interest for 3 years, the third year's interest draws interest for 2 years, and the 4th year's interest draws interest for 1 year. The interest on \$72 for 4 years and 3 years and 2 years and 1 year is equal to the interest on \$72 for $(4+3+2+1)$ years, or 10 years. The interest on \$72 for 10 years at 6% is \$43.20. The sum of the principal, \$1200, the interest on it for 5 years, \$360, and the interest on \$72 for 10 years, \$43.20, is \$1603.20, the amount due at settlement.

WRITTEN PROBLEMS.

544. *1.* Annual interest being unpaid, what is due on \$4800 in 4 years at 6% ?

2. What is the amount, annual interest being unpaid, on a debt of \$6000 in 6 years at 7% ?

3. Annual interest being unpaid, what is the amount of a note for \$5656 due in 7 years at $5\frac{1}{2}$% ?

4. What will be the amount in 8 years, at $6\frac{1}{2}$%, of \$427.50, with annual interest unpaid ?

COMPOUND INTEREST.

545. Compound interest is interest on both principal and unpaid interest.

546. Interest is generally compounded annually ; that is, the accrued interest is added to the principal annually to form a new principal.

Interest may, however, be compounded semi-annually, quarterly, or after other periods agreed upon.

547. To find the compound interest on a given principal.

I. Required the compound interest on \$500 for 2 years 5 months 18 days at 6%, interest compounded annually.

	Process.
Principal for 1st year, ...	\$500
Interest " " "	30
Principal for 2d year, ...	\$530
Interest " " "	31.80
Principal for 5 mo. 18 da., ...	\$561.80
Interest " " "	15.73
Amount of \$500 at compound interest, . .	\$577.53
Original principal, ...	500.00
Interest of \$500 for 2 yrs. 5 mo. 18 da., .	\$77.53

Rule.—I. *Find the amount of the given principal for the first period for which interest is due.*

II. *Find the amount of this sum treated as a principal for the second period, and so continue for each successive period, and for the final portion of a period, if any.*

III. *From the final amount subtract the original principal, and the remainder will be the compound interest required.*

NOTE.—Although compound interest is illegal, it is frequently paid in actual business. Savings-banks, for example, add accrued interest to money deposited, and regard the sum as a new principal. Annual interest, also, is a form of compound interest, but it has been legalized in some states.

WRITTEN PROBLEMS.

548. *1.* Find the compound interest on $750 for 4 years at 7%, interest compounded annually.

2. What is the compound interest on $460 for 3 years at 8%, interest compounded semi-annually?

3. What is the interest on $1500 for 2 years at 6%, interest compounded semi-annually?

4. Mr. Jones deposited $350 in a savings bank, compounding interest semi-annually; what amount was due him in 2½ years at 4% per annum?

5. What is the amount of $148.50 at compound interest for 4 yrs. 7 mo. 12 da. at 7%, interest compounded annually?

6. Find the amount of $735.60 for 2 yrs. 5 mo. 24 da. at 8%, interest compounded quarterly.

7. What is the amount of $1200, at 8%, for 2 yrs. 8 mo. 12 da., compounded semi-annually?

Compound interest in actual business is found by means of a table showing the amount of $1 for various times at different rates. For such table, see Appendix, page 342.

MISCELLANEOUS PROBLEMS.

549. *1.* What is the exact interest, at $5\frac{1}{2}\%$, on a note foi \$9600, due in 93 days?

2. What principal will give \$1701.36 interest in 2 years 4 months 24 days at $8\frac{1}{2}\%$?

3. What is the amount, at 5%, of a note for \$18640, from June 15, 1881, to April 29, 1885?

NOTE.—Interest on the basis of 360 days to a year is meant, unless otherwise stated.

4. My son will be of age in 7 years 5 months 21 days; how much money on interest at 6% will amount to \$10000 when he is of age?

5. At what rate per cent. will \$57600 in 3 yrs. 5 mo. 27 da. amount to \$68661.60?

6. A debt I incurred 5 years 3 months 15 days ago, on which interest at 7% has been accruing, amounts to \$2548.975; what was the original debt?

7. How much does the interest at 6% by the Six Per Cent Method exceed the exact interest, on \$146000 for 245 days?

8. By the method of exact interest, find how much is due on \$6400, in 4 years 3 months 25 days, at 6%.

9. A man borrowed Jan. 15, 1884, a sum of money which at 6% amounted to \$1882.72 on the 3d of July, 1886; how much did he borrow?

10. By the United States Rule, find how much will remain due, Jan. 1, 1886, on a note, at 6%, for \$6500, dated June 9, 1881, and bearing the following indorsements: Dec. 4, 1881, \$875; Sept. 27, 1882, \$1200; Jan. 16, 1883, \$3750.

11. What principal will give \$477.10 interest in 3 years 5 months 18 days at $7\frac{1}{2}\%$?

12. In what time will \$4000 at 6% amount to \$4146?

TRUE DISCOUNT.

The term discount has already been used to mean a deduction from the price asked. When the element of time is considered, the process is called true discount or bank discount.

550. The **true present worth** of a debt or other obligation is such a sum as will, at legal interest, amount to the debt or obligation in the given time.

551. True discount is the amount that must be deducted from the face of a note or other obligation in order to give the true present worth.

The terms in Discount correspond to those in Interest as follows:

> *The* **present worth** *is the* **principal.**
> *The* **rate of discount** *is the* **rate.**
> *The* **discount** *is the* **interest.**
> *The* **entire debt** *or* **obligation** *is the* **amount.**

552. To find the discount and present worth, when the debt, the rate of discount and the time are given.

I. Required the present worth and discount of a debt of \$1380 due in 2 years 6 months, discounted at 6%.

Process.

$$6\% \times 2\tfrac{1}{2} = 15\%.$$
$$1.00 + .15 = 1.15.$$
$$\$1380 \div 1.15 = \$1200.$$
$$\$1380 - \$1200 = \$180.$$

Explanation.—In $2\frac{1}{2}$ years, at 6%, the amount of \$1 is \$1.15; that is, the present worth of \$1.15 due in $2\frac{1}{2}$ years at 6% is \$1. The present worth of \$1380 due in $2\frac{1}{2}$ years at 6% is equal to as many dollars as \$1.15 is contained times in \$1380, equal to \$1200. Hence, \$1380 less \$1200 gives the discount, \$180.

Rule.—I. *Divide the debt by the amount of $1 at the given rate for the given time, and the quotient will express the number of dollars in the true present worth.*

II. *Subtract the present worth from the entire debt and the remainder will be the true discount.*

NOTE.—When the obligation bears interest, the amount is first found, and that sum is discounted.

WRITTEN PROBLEMS.

553. *1.* Find the present worth and the discount of a debt of $2000 due in 1 year and 6 months without interest, the rate of discount being 6%.

2. What is the cash value of a note for $3600 due in 9 months without interest, money being worth 5%?

3. How much should I pay for a claim for $6780, due in 1 yr. 9 mo., without interest, when money is worth 6%?

4. A note for $6000, bearing interest at 7% and due in 2 years, is discounted at 5%; what is its present value?

5. What is the present worth of a note for $7500, bearing interest at 6%, and due in 3 yrs. 4 mo., discounted at 5%?

6. A merchant sells a bill of goods amounting to $9650.80 on 6 mo. credit; if money is worth 8%, what should be the deduction for cash payment?

7. Mr. Brown can buy a house for $7000 in cash, or for $7400 on 1 year's credit without interest; which is better, and how much, money being worth 6%?

8. I owe a note of $3425.60, payable in 9 months, bearing interest at 6%; if the party I owe will discount the note at 8% for cash payment, what do I gain by accepting his offer, money being worth 7%?

9. I hold Mr. Carroll's note for $3600, dated January 8, 1886, due in one year without interest; what is its cash value July 20, money being worth 6%?

10. A note at 6% for $6400 due in one year is discounted at 8% six months before it is due; how much is it worth?

BANK DISCOUNT.

554. Bank discount is the amount charged by a bank for discounting a note or other obligation, and is equal to the interest on the obligation from the date of discount to the date of maturity.

Bank discount is inequitable in the fact that interest is charged not on the money actually paid for a note, but on the entire face of the note.

This form of discount is called Bank Discount to distinguish it from True Discount.

555. The **proceeds** is the amount of the obligation less the bank discount.

556. After the time mentioned in a note for its payment, **three days of grace** are allowed before the note is legally payable.

A note is not **payable** at the end of the time specified in the note, but at its **maturity** three days later.

NOTE.—When the date of maturity falls on a legal holiday, it is due the day before, but in such case, interest or discount is calculated for *three*, not *two*, days of grace.

If the time is given in months, calendar months are meant; and when a note due in a certain number of months is dated on the first or the last of a month, it is payable on the first or the last of the month, and its maturity is 3 days later, without regard to the varying number of days in a month.

557. The **term of discount** is the time from the date of discount to the date of maturity.

558. To find the bank discount and proceeds of a note.

I. Required the bank discount and proceeds of a note for \$1800, due in 60 days, discounted at 6%.

Explanation.—Finding the interest on \$1800, the face of the note, for 60 days plus 3 days of grace, gives \$18.90, the bank discount. Subtracting the bank discount from the face of the note gives \$1781.10, the proceeds.

Process.

\$18.00	=	Int. for 60 da.
.90	=	" " 3 da.
\$18.90	=	Bank discount.

\$1800—\$18.90 = \$1781.10

Rule.—I. *Find the interest on the face of the note at the given rate of discount for 3 days more than the note has to run, and the result will be the bank discount.*

II. *Subtract the bank discount from the face of the note and the remainder will be the proceeds.*

NOTE.—In the case of an interest-bearing note, treat the amount of the note at maturity as the sum to be discounted.

Some banks employ the method of exact interest.

WRITTEN PROBLEMS.

559. *1.* Find the bank discount and the proceeds of a note for \$3600, due in 90 days, discounted at 7%.

2. Find the bank discount and the proceeds of a thirty-day note for \$1375, discounted at 6%.

3. Required the proceeds of a four-months note for \$7500, discounted at 8%.

4. Find the bank discount on a note for \$1650.80, due in 90 days, dated July 10, and discounted September 5, at 8%.

(*5.*)

\$2560.50. *New York, Jan. 2, 1886.*

Ninety days after date I promise to pay Charles Lovell, or order, two thousand five hundred sixty dollars and fifty cents, for value received.

James Burnett.

Find the proceeds of the foregoing note, if discounted Feb. 16, 1886, at 5½%.

6. Find the proceeds of the following note, if discounted Apr. 12, at 7%.

\$7805.60. *Brooklyn, N. Y., Mar. 5, 1886.*

Four months after date, for value received, I promise to pay Cyrus Eustis, or order, seven thousand eight hundred five and $\frac{60}{100}$ dollars, with interest at 7%.

Wm. Edson.

7. The following note was discounted Jan. 7, 1886, at 8%. Find the proceeds.

\$9375. *St. Louis, July 1, 1885.*

One year after date, for value received, I promise to pay Henry Schulz, or order, nine thousand three hundred seventy-five dollars, with interest at 8%.

James Rogers.

8. Find the proceeds and the bank discount of the following note, discounted Apr. 8, 1886, at 7%.

\$3675.40. *Boston, Mass., Oct. 12, 1885.*

Nine months after date, I promise to pay Hart, Lyons & Co., or order, three thousand six hundred seventy-five and $\frac{40}{100}$ dollars, with interest at 7%. Value received. Payable at First National Bank.

David Skillman.

560. To find the face of the note, when the proceeds, rate, and time are given.

I. For what amount must a note be drawn, payable in 90 days, so as to yield \$1500 proceeds when discounted at 6%?

Process.

\$.0155 = discount on \$1 for 93 da.

\$1 — \$.0155 = \$.9845 = proceeds of \$1.

\$1500 ÷ .9845 = \$1523.616, face.

Explanation. — The bank discount on \$1 for 93 days is \$.0155, and the proceeds, \$.9845. Dividing the given proceeds, \$1500, by the proceeds of \$1, gives a quotient expressing the number of dollars in the face of the note to be drawn, equal to \$1523.62.

Rule.—*Divide the given proceeds by the proceeds of* $1 *for* 3 *days more than the given time, and the quotient will express the number of dollars in the face of the note.*

WRITTEN PROBLEMS.

561. *1.* Find the face of a note due in 60 days, such that if discounted at 6% the proceeds shall be $2400.

2. Find the face of a note due in 6 mo., whose proceeds when discounted at 9% will be $896.

3. I owe a debt of $3500, which I wish to pay with the proceeds of a note for 30 days, discounted at a bank where the method of exact interest is employed, and whose rate of discount is 6%; what must be the face of the note?

4. Find the face of the note whose date is Jan. 7, 1886. time, 90 days; proceeds, $1875; date of discount, Feb. 27, rate, 6%.

5. What is the face when the date of the note is Feb. 17, 1886; proceeds, $9260; date of discount, Apr. 7; time, 3 months; rate, 6%?

6. The proceeds being $4236; time, 1 year; date, Oct. 18, '85; date of discount, April 29; rate of discount, 10%; find the face of the note.

7. How much less is the face of a note without grace, which will yield by equitable discount $3000, than one whose proceeds when discounted at a bank shall be the same sum, if the time in both cases is 1 year, and the rate of discount 9%?

8. Write a note due in 90 days which if discounted at a bank at $\frac{3}{4}$% a month will yield $2750 proceeds.

9. A note bearing interest at 5%, due in one year, and dated January 1, 1886, yields $8375.60 when discounted at a bank May 1, at 6%. Find the face of the note, and write the note.

10. What is the face of a note bearing interest at 7%, and having 60 days to run, that will yield $4800 if discounted at date at 6%?

EXCHANGE.

562. Exchange is a method of paying debts in distant places by **bills of exchange** or **drafts.**

Exchange is of two kinds—*domestic exchange* and *foreign exchange.*

563. Domestic or **inland exchange** is exchange between different parts of the same country. It is accomplished by means of **drafts.**

564. Foreign exchange is exchange between different countries. The instrument employed is called a **bill of exchange.**

[For foreign exchange, see Appendix, p. 344.]

565. A **sight draft** is a draft payable on presentation.

566. A **time draft** is a draft payable after a specified time, usually 30, 60, or 90 days. In some places three days of grace are allowed on time drafts.

Exchange is *at a premium* when a draft costs more than its face, and *at a discount* when a draft costs less than its face.

567. The following is the usual form of a sight draft:

$900. *New York, Jan. 5, 1886.*

At sight, pay to the order of Henry Hayes, nine hundred dollars, value received, and charge to the account of

Charles Spencer.

To William J. Rose, San Francisco, Cal.

568. Discount is allowed on time drafts as on promissory notes. It is computed on the amount or face of the draft.

569. To find the cost of a sight draft.

I. Required to find the cost of the following draft at 1% premium:

$2000. *Philadelphia, Mar. 1, 1886.*

At sight, pay to the order of Joseph Card, two thousand dollars, value received, and charge to my account.

Richard Felton.

To John S. Lang, Chicago, Ill.

Explanation. — The face of the draft, $2000, multiplied by .01, gives the premium, $20. The premium added to the face of the draft gives $2020, the cost of the draft.

Process.

$$\$2000 \times .01 = \$20$$
$$\$2000 + \$20 = \$2020.$$

Rule.—I. *Multiply the face of the draft by the rate of premium or discount, and the result will be the premium or discount.*

II. *The sum of the face and the premium, or the difference between the face and the discount, will be the cost of the draft.*

WRITTEN PROBLEMS.

570. *1.* What is the cost of a sight draft on St. Louis for $3650, at $\frac{3}{4}$% premium?

2. How much will be the cost of a sight draft on New Orleans for $8690, at $\frac{1}{2}$% discount?

3. The face of a sight draft is $4375.50, discount $1\frac{1}{2}$%; what is the cost?

4. Face of sight draft, $7268.40; premium, $1\frac{3}{8}$%; what is the cost?

5. How much will it cost me to pay a bill of $6375.80 in Denver, exchange being at $1\frac{1}{4}$% premium?

6. What must I pay for a draft on San Francisco for $4640, exchange being at $\frac{1}{2}$% discount?

571. To find the cost of a time draft.

I. Required to find the cost of the following draft at $1\frac{1}{4}\%$ premium, interest being 6%.

\$3600. *Brooklyn, N. Y., Jan. 11, 1886.*

Sixty days after sight, pay to James I. White, or order, three thousand six hundred dollars, value received, and charge the same to my account. *Horace Waters.*

To Pearce, Wagner & Co., Cincinnati, Ohio.

Explanation.—The interest on \$3600 for 63 days is \$37.80. Subtracting \$37.80 from the face of the draft gives \$3562.20, the present worth of \$3600 payable in 60 days with 3 days of grace. Increasing this sum by the premium, which is $1\frac{1}{4}\%$ of \$3600 or \$45, gives \$3607.20, the cost of the draft.

Process.

$$\$3600 - \$37.80 = \$3562.20$$
$$\$3600 \times .01\tfrac{1}{4} = \$45.00$$
$$\$3562.20 + \$45.00 = \$3607.20$$

Rule.—I. *Find, by the method of bank discount, the proceeds of the draft for the given time and rate.*

II. *Increase this result by the premium, computed on the face of the draft, or diminish it by the discount, and the result will be the cost of the draft.*

WRITTEN PROBLEMS.

572. *1.* Find the cost of a draft for \$5600, payable 30 days after sight, at a premium of $\frac{3}{4}\%$, money being worth 6%.

2. What is the cost of a draft for \$6000, payable in 90 days, discount $1\frac{1}{2}\%$, money being worth 8%?

3. When exchange is at $1\frac{1}{4}\%$ premium, and interest 6%, what must I pay for a draft for \$8320, payable in `0 days?

4. What is the cost of a draft on Denver for \$12620, payable in 90 days, premium being $1\frac{3}{4}\%$ and interest 6%?

5. The face of a draft payable 30 days after sight is \$7560: what is its cost, discount being $\frac{1}{2}\%$ and interest 6%?

573. To find the face of a draft.

I. What is the face of a sight draft which cost $5000, exchange being ¾% discount?

Explanation.—The discount subtracted from $1.00 gives $.9925, the cost of each dollar of the face. Dividing $5000, the money paid for the draft, by .9925, gives $5037.78, the face of the draft.

Process.

$$\$1.00 - \$.00\tfrac{3}{4} = \$.9925$$
$$\$5000 \div .9925 = \$5037.78$$

II. Required the face of a draft payable in 60 days, costing $5000, premium being 1% and interest 6%.

Explanation.—The proceeds of $1, discounted at 6% for 63 days is $.9895. This amount, increased by the premium on $1, gives $.9995, the cost of each $1 of the face of the draft. The cost of the draft, $5000, divided by the cost of $1 of the face, gives the face of the draft, $5002.50.

Process.

$$\$1.00 - \$.0105 = \$.9895$$
$$\$.9895 + \$.01 = \$.9995$$
$$\$5000 \div .9995 = \$5002.50$$

Rule.—*Divide the cost of the draft by the amount required to pay for $1 of its face.*

WRITTEN PROBLEMS.

574. *1.* A draft due 90 days after sight costs $1200; exchange being ¾% premium, and interest 6%, what is its face?

2. A draft due 60 days after sight, with which to pay a debt in Omaha, cost me $5630 when exchange was at ¾% discount and interest at 6%; what was the amount of the debt?

3. Find the face of a sight draft costing $329.75, when exchange was at 1¼% discount?

4. What is the face of a sight draft that can be purchased for $964.80, when exchange is at ½% premium?

5. If a draft payable 30 days after sight costs $3200 when exchange is at 1¾% discount and interest 8%, what is its face?

EQUATION OF PAYMENTS.

575. Equation of payments is a method of finding a time when several sums, due at different times, may be paid in one payment, without loss to either debtor or creditor.

The time for such payment is called the **equated or average time of payment.**

576. The **term of credit** of a debt is the time it has to run before coming due.

577. The **average term of credit** of several debts due at different times is the period at the end of which they may equitably be paid together.

578. To find the average term of credit and the equated time of several debts.

I. A owes B $800, of which $300 is due in 4 months and $500 in 6 months; in what time must A pay the whole at once, so that neither may lose?

Process.

$$\begin{array}{rcr} 4 \times 300 & = & 1200 \\ 6 \times \underline{500} & = & \underline{3000} \\ 800 &) & \underline{4200} \\ & & 5\frac{1}{4} \end{array}$$

Explanation.—If A makes the first payment immediately, he loses the use of $300 for 4 months, equal to $1200 for 1 month. To pay the $500 immediately, he loses the use of it for 6 months, equal to a loss of $3000 for 1 month. A loss of $1200 for 1 month and of $3000 for 1 month make together a loss of $4200 for 1 month. To avoid this loss, A must retain the $800 for as many months as 800 is contained times in 4200, or $5\frac{1}{4}$ months. The payment of the entire amount at this time is equivalent to the payment according to the agreement, and neither party loses.

II. Find the equated time for paying $300 due May 1, $400 due June 15, and $600 due July 1.

Explanation.—Assuming the three payments all to be made when the first payment is due, that is, on May 1, the loss to the debtor will be nothing on the first payment. On the second payment, he will lose the use of \$400 from May 1 to June 15, 45 days, equal to a loss of \$18000 for 1 day. On the third payment, he will lose the use of \$600 from May 1 to July 1, 61 days, equal to a loss of \$36600 for 1 day. The sum of these losses is equal to the loss of \$54600 for 1 day. The debtor is entitled, therefore, to the use of the sum of \$300, \$400, and \$600, or \$1300, as many days as \$1300 is contained times in \$54600, equal to 42 days. 42 days after May 1 is June 12, the equated time of payment.

Process.

$$\begin{array}{rcr} 0 \times 300 & = & 0 \\ 45 \times 400 & = & 18000 \\ 61 \times \underline{600} & = & \underline{36600} \\ 13\cancel{0}\cancel{0} &) & 546\cancel{0}\cancel{0} \\ & & \overline{42} \end{array}$$

42 days after May 1 is June 12.

Rule.—*Find the time that each debt has to run, counting from the earliest date when a debt is due, and multiply each debt by its term of credit thus found. Divide the sum of the products by the sum of the debts, and the quotient will express the time after the earliest date for the payment at once of the sum of the debts.*

NOTE 1.—The same result is obtained by finding the interest at any convenient rate on each debt for its term of credit as found by the rule, and dividing the sum of the interest by the interest on the sum of the debts for one day, if the terms of credit were expressed in days, or for one month, if in months.

NOTE 2.—When the equated time contains a fraction of a day, it is called 1 day, if equal to $\frac{1}{2}$ a day or more; if less, it is disregarded.

NOTE 3.—The equated time may be found by using the latest date instead of the earliest, and counting the time backward instead of forward.

WRITTEN PROBLEMS.

579. *1.* A owes B \$500 due in 4 months, \$800 due in 9 months; when may he equitably pay both debts at once?

2. John Smith bought, April 1, three bills of goods from a merchant of Boston; a bill of $800 on 3 months' credit, $1200 on 5 months' credit, and $1600 on 9 months' credit; what is the equated time of payment?

3. I bought goods for $600, $900, and $1500, due in 30. 60, and 90 days respectively from June 15, 1885; what is the equated time of payment?

4. On Jan. 1, 1886, A owes B $3000 due in 9 months, $5000 due in 12 months, and $8000 due in 15 months; what is the equated time of payment?

5. Bought goods as follows: Jan. 6, '86, for $875, due in 30 days; Feb. 10, '86, for $1250, due in 45 days; March 12, '86, for $1860, due in 60 days. What is the equated time of payment?

6. Required the equated time of the following bills, each on 90 days' credit: April 5, '86, $1650; April 24, '86, $2040; May 13, '86, $1865; May 27, '86, $1480.

7. What is the equated time of the following bills: Jan. 14, '86, $875 on 30 da. Jan. 22, '86, $930 on 60 da. March 22, '86, $1800 on 60 da. Apr. 20, '86, $3600 on 90 da.?

8. A buyer agreed to pay for a house $6750, as follows: $1000 at the time of purchase; $2000, 3 months after the first payment; $2500, 6 months after the second payment, and the remainder 1 year after the third payment. How long after the time agreed upon for the first payment could he equitably pay the whole amount?

9. What is the equated time of the following bill?

SAMUEL COLTON,

To PARK & TILFORD, *Dr.*

1886.			
Jan. 7	To Mdse. on 30 days' credit, ...	$275	
Feb. 15	" " " 60 " "	960	
Feb. 26	" " " 90 "	1375	
Mar. 19	" " " 60 " "	1060	

580. To find the equated time for the settlement of an account in which there are both debit and credit entries.

What is the equated time for the settlement of the following account?

(1.)

Dr. GEORGE M. SMITH. *Cr.*

1886.			1886.		
Jan. 19	To mdse. @ 60 da.	$500	Mar. 25	By cash, . . .	$400
Feb. 26	" " " 90 "	1200	Apr. 15	" " . . .	1000
Mar. 22	" " " 30 "	800	Apr. 29	" " . . .	300

Process by Products.

March 20,	$500 \times 0 =$	0	March 25,	$400 \times 5 =$	2000
May 27,	$1200 \times 68 =$	81600	April 15,	$1000 \times 26 =$	26000
April 21,	$800 \times 32 =$	25600	April 29,	$300 \times 40 =$	12000
	2500	107200		1700	40000
	1700	40000			
	800	) 67200			
		84			

Average term of credit is 84 *days.*

Equated time is 84 *days after March* 20, *or on June* 12.

Process by Interest.

Due.	*Prin.*	*Time.*	*Int.*	*Paid.*	*Prin.*	*Time.*	*Int.*
Mar. 20,	$500	68 da.	5.66\frac{2}{3}$	Mar. 25,	$400	63 da.	$4.20
May 27,	1200	0 "	0	Apr. 15,	1000	42 "	7.00
Apr. 21,	800	36 "	4.80	Apr. 29,	300	28 "	1.40
	$2500		10.46\frac{2}{3}$		$1700		$12.60
	1700						10.46$\frac{2}{3}$
	$800						2.13\frac{1}{3}$

Int. for 1 da. on $800 = $.13$\frac{1}{3}$; 2.13\frac{1}{3}$ ÷ $.13$\frac{1}{3}$ = 16.

16 *days after May* 27, *or June* 12, *is the equated time.*

Explanation.—In the Method by Products, either the earliest or the latest date at which a debt becomes due or a payment is made is selected as the *date of reference.* In this case, the earliest is taken, March 20, the time when the bill of Jan. 19 is due. The bill of Feb. 26 is due May 27. 68 days after the date of reference. That is, the purchaser is entitled to the use of $1200 worth of goods for 68 days from the date of reference, equal to the use of $81600 worth for 1 day. In like manner he is entitled to the use of $800 worth for 32 days, equal to $25600 worth for 1 da. These together amount to $107200 for 1 da.

On the other hand, the seller has lost the use of $400 for 5 days after the date of reference, equal to the use of $2000 for 1 day. In like manner, he has lost the use of $26000 for 1 day, and of $12000 for 1 day. These losses aggregate $40000 for 1 day. Deducting the loss of the seller from the claim of the buyer leaves a claim for the latter of $67200 for 1 day. Hence, he should retain the $800 yet due until this claim is canceled, that is, as many days after the date of reference as $800 is contained times in $67200, equal to 84 days.

In the Method by Interest, May 27 is selected as the *date of reference,* that being the latest date at which any debt becomes due. Interest is computed on each debt from the time when it is contracted until it is due. In like manner interest is computed on each payment from the time it is made to May 27. The rest of the process is obvious.

NOTE.—The cash balance of an account is the sum which paid on the day of reference will exactly balance the account. In the example above, the cash balance on May 27 is the true present worth of $800 payable June 12, or in 16 days. Reckoning at 6% it is $\$800 \div 1.002\frac{2}{3} =$ $797.87.

Find the equated time and the cash balance of the following accounts.

(*2.*)

Dr. JOHN R. DEAN. *Cr.*

1886.					
Jan. 5	To mdse. @ 3 mo.	$900	Feb. 1	By cash, . . .	$700
Jan. 20	" " " 4 "	1200	Feb. 25	" " . . .	1000
Feb. 27	" " " 6 "	1800	June 24	" " . . .	2200
Mar. 31	" " " 3 "	1200	July 1	" " . . .	500

(3.)

Dr. WILLIAM CORBIN. *Cr.*

1882.			1882.		
Apr. 1	To mdse. @ 90 da.	$800	May 20	By cash, . . .	$2000
May 16	" " " 60 "	1600	June 30	" " . . .	500
June 25	" " " 90 "	1500	Aug. 10	" " . . .	1200
Sept. 12	" " " 30 "	2000	Sept. 20	" " . . .	1400

(4.)

Dr. LUTHER SPECK. *Cr.*

1878.			1878.		
May 1	To mdse. @ 90 da.	$2000	June 1	By cash, . . .	$4000
June 21	" " " 60 "	3000	June 20	" " . . .	6000
July 15	" " " 60 "	4000	Aug. 30	" " . . .	1000
July 30	" " " 90 "	5000	Sept. 15	" " . . .	4000
Aug. 10	" " " 30 "	6000	Nov. 10	" " . . .	2000

(5.)

Dr. JOHN WHITE. *Cr.*

1880.			1880.		
Oct. 10	To mdse. @ 90 da.	$600	Nov. 1	By cash, . . .	$500
Nov. 1	" " " 60 "	900	Nov. 29	" " . . .	1000
Nov. 15	" " " 30 "	1500	Dec. 20	" " . . .	1000
Dec. 20	" " " 60 "	1200	1881.		
1881.			Jan. 20	" mdse. @ 30 da.	1400
Jan. 15	" " " 30 "	1800	Feb. 10	" cash, . . .	1200

(6.)

Dr. HARRISON OSWALD. *Cr.*

1884.			1884.		
July 10	To mdse. @ 30 da.	$860	July 20	By cash, . . .	$500
Aug. 15	" " " 60 "	1540	Aug. 25	* " note @ 60 da.	2000
Sept. 20	" " " 90 "	600	Nov. 30	" cash, . . .	500
Oct. 25	" " " 60 "	820	1885.		
Nov. 30	" " " 30 "	1240	Jan. 5	" " . . .	600
			Feb. 10	" " . . .	700

* Find the proceeds if discounted at a bank at 6%.

REVIEW QUESTIONS.

581. Percentage.—Define percentage. Rate. Base. Give the three cases of percentage. What is meant by a deduction of 40% and 10% ?

Define profit and loss. What terms in profit and loss correspond to those in percentage?

What is commission? What is an agent? Give the terms in commission equivalent to those in percentage.

What is insurance? The premium? The policy? Give the terms corresponding to those in percentage.

Define tax. Poll tax. In taxes, what is the base? The rate? The percentage?

What is a corporation? What is stock? What are bonds? Coupons? When is stock at par? Below par? At a premium? What is a stock broker? What is brokerage? On what is it calculated?

Define customs. What is a specific duty? An *ad valorem* duty? Tare? Leakage and breakage?

Define interest. The principal. The amount. The rate. What is usury? In interest, what is the base? The percentage? The rate? Give the rule for the method of interest by aliquot parts. The six per cent. method. The special method for days. Explain in what respect exact interest differs from ordinary interest. Give the rule for exact interest. What are the three problems of interest?

What is a promissory note? A negotiable note? Why are the words *value received* put in notes? What is meant by the maturity of a note? When is a note legally due? What is meant by *days of grace?*

What are partial payments? Indorsements? Give the United States Rule. The Merchants' Rule.

What is annual interest? Compound interest?

What is true discount? Define true present worth. Bank discount. Proceeds.

What is exchange? Define the two varieties. What is a sight draft? A time draft?

What is equation of payments? What is the date of reference? Term of credit? Equated time? What two methods of equation of payments are given? What date is convenient as the date of reference? Give the rule for equation of payments by the method of products.

RATIO.

582. Ratio is the relation of one number to another of the same kind, and is expressed by the quotient of the first number divided by the second.

Thus, the ratio of 10 to 5 is $10 \div 5 = 2$; of \$4 to \$12 is \$4 ÷ \$12 = $\frac{1}{3}$.

583. The ratio of one number to another is indicated by a colon; so used, the colon is a substitute for the sign of division.

Thus, the ratio of 6 to 3 is written 6 : 3; and it is read, *the ratio of* 6 *to* 3.

584. The **terms** of a ratio are the numbers that are compared. The first term is called the **antecedent** and the second the **consequent.**

585. The terms of a ratio together form a **couplet.**

586. A **simple ratio** is the ratio of two numbers.

Thus, 5 : 9, 8 yds. : 12 yds., 12 men : 3 men, are simple ratios.

587. A **compound ratio** is a ratio formed by the combination of two or more simple ratios. A compound ratio may be reduced to a simple ratio whose antecedent and consequent are respectively the products of the antecedents and the consequents of the compound ratio.

Thus, $\left.\begin{matrix} 2 : 4 \\ 5 : 9 \end{matrix}\right\}$ is a compound ratio. Reduced to an equivalent simple ratio it is $5 \times 2 : 4 \times 9$, or 10 : 36.

588. Since a ratio may be expressed as a fraction whose numerator is the antecedent, and its denominator the consequent, of the ratio, it follows that the principles applicable to fractions are applicable also to ratios.

Principles.—I. *Multiplying the antecedent or dividing the consequent of a ratio by any number multiplies the value of the ratio by that number.*

II. *Dividing the antecedent or multiplying the consequent of a ratio by any number divides the value of the ratio by that number.*

III. *Multiplying or dividing both terms of a ratio by any number does not change the value of the ratio.*

ORAL EXERCISES.

589. *1.* What is the value of 6 : 3? Of 10 : 2? Of 9 : 3? Of 12 : 4?

2. What is the value of 4 : 8? Of 3 : 12? Of 7 : 21? Of 10 : 50?

3. Find the value of 10 : 12. Of 12 : 10. Of 8 : 12. Of 15 : 9. Of 18 : 12. Of 100 : 60. Of 40 : 28.

4. What is the value of 2 ft. : 1 yd.? Of 40 min. : 1 hour? Of 1 T. : 15 cwt.? Of $1\frac{1}{2}$ bu. : 5 pks.?

5. What is the value of $2\frac{1}{2} : 5$? Of \$3 : \$1.50? Of $1\frac{1}{2}\% : 6\%$? Of $3\frac{1}{2}$ lb. : $4\frac{1}{2}$ lb.? Of ℥2 : ℥1 ʒ4?

6. Find the ratio of $\frac{8}{9} : \frac{2}{3}$. Of $\frac{2}{3} : 1\frac{1}{2}$. Of $\frac{4}{5} : \frac{3}{4}$. Of $\frac{3}{4} : \frac{2}{5}$.

7. Express as simple ratios and find the value:

Of $\left.\begin{matrix} 2 : 3 \\ 6 : 5 \end{matrix}\right\}$. Of $\left.\begin{matrix} \$5 : \$12 \\ 3 : 10 \end{matrix}\right\}$. Of $\left.\begin{matrix} 4\frac{1}{2} : 8\frac{2}{3} \\ 6 : 3 \end{matrix}\right\}$. Of $\left.\begin{matrix} 2\frac{1}{2} : 5 \\ 4 : 8 \end{matrix}\right\}$.

8. What is the value of a compound ratio formed of the simple ratios, 2 : 4, 6 : 3 and 8 : 4? Of 2 : 3, 5 : 9 and 12 : 4?

9. The value of a ratio is 5, and the antecedent 10; what is the consequent?

10. The value of a ratio is 4, and the consequent is 3; what is the antecedent?

11. What ratio will express the comparative value of 2 pounds of coffee and 5 pounds of coffee of the same quality?

12. What ratio will express the comparative amount of work a man can do in 12 days and the amount of work he can do in 3 days?

13. Write three ratios, each equivalent to 2 : 3. Three, each equivalent to 5 : 3.

14. Write three ratios, each having a value of $\frac{2}{5}$. A value of $1\frac{1}{2}$. Of $1\frac{2}{3}$. Of .5.

PROPORTION.

590. Proportion is an equality of ratios.

Thus, 2 : 4 = 3 : 6 is a proportion. It is read, 2 *is to* 4 *as* 3 *is to* 6, or, *The ratio of* 2 *to* 4 *is equal to the ratio of* 3 *to* 6.

591. The equality of ratios is usually indicated by a double colon.

Thus, 3 : 9 :: 2 : 6, and 3 : 9 = 2 : 6, are equivalent expressions.

592. The **antecedents** and the **consequents** of a proportion are respectively the antecedents and the consequents of the ratios composing the proportion.

Thus, 3 and 2 are the antecedents, and 9 and 6 the consequents of the preceding proportion, and of the ratios forming it.

593. The **extremes** of a proportion are the first and fourth terms, and the **means** are the second and third terms.

Thus, in the proportion 2 : 5 :: 4 : 10, the extremes are 2 and 10, and the means are 5 and 4.

594. Three numbers are in proportion, when the first is to the second as the second is to the third. In such case, the second is the **mean proportional** between the other two.

Thus, in the proportion 2 : 4 :: 4 : 8, 4 is a mean proportional between 2 and 8.

595. Since $\frac{4}{3} = 4 : 3$, and $\frac{8}{6} = 8 : 6$; and since $\frac{4}{3} = \frac{8}{6}$,

$$4 : 3 :: 8 : 6.$$

That is, *The equality of two fractions may be indicated by a proportion.*

An inspection of the foregoing proportion will show the correctness of the following

Principles.—I. *In any proportion, the product of the means is equal to the product of the extremes.*

II. *Either extreme is equal to the product of the means divided by the other extreme.*

III. *Either mean is equal to the product of the extremes divided by the other mean.*

596. Proportion may be **simple** or **compound**.

SIMPLE PROPORTION.

597. A **simple proportion** is a proportion composed of two simple ratios.

Thus, 3 : 5 :: 9 : 15 is a simple proportion.

598. To complete a proportion when one term is missing.

I. Required to find the term indicated by x in the following proportion: $25 : 35 :: 45 : x$.

Process.

$$x = \frac{\overset{7}{\cancel{35}} \times \overset{9}{\cancel{45}}}{\cancel{25}} = 63.$$

Explanation.—Since either extreme is equal to the product of the means divided by the other extreme, the extreme whose place is filled by x is found by dividing the product of the means, 35 and 45, by the other extreme, 25, giving 63 as the extreme required.

II. Required to find the missing term in the proportion, 175 : x :: 125 : 150.

Explanation.—Since either mean is equal to the product of the extremes divided by the other mean, the mean whose place is filled by x is found by dividing the product of the extremes, 175 and 150, by the other mean, 125, giving 210 as the mean required.

Process.

$$x = \frac{\overset{7}{\cancel{175}} \times \overset{30}{\cancel{150}}}{\underset{5}{\cancel{125}}} = 210.$$

599. Complete the following proportions:

1. 6 : 14 :: 9 : x.
2. 15 : 100 :: 12 : x.
3. x : 35 :: 14 : 70.
4. x : 154 :: 6 : 84.
5. 15 : 195 :: x : 65.
6. 5.2 : 3.64 :: x : 44.1.
7. 4.45 : x :: 1.15 : 1.38.
8. \$17 : \$$x$ :: 44 : 55.
9. x : 39 :: \$5.26 : \$7.89.
10. $1\frac{1}{2} : 4\frac{1}{4} :: \frac{3}{4} : x$.
11. $\frac{2}{3} : x :: 55 : 49\frac{1}{2}$.
12. $x : 8.37 :: \frac{2}{9} : 4.65$.
13. $5\frac{1}{3}$ yds. : x yds. :: \$4 : \$9.
14. 85 : 120 :: x : 144.
15. \$9.20 : \$$x$ :: 161 : 217.
16. $\frac{2}{3} : \frac{3}{4} :: x : \frac{9}{10}$.
17. 5.5 : x :: 6.5 : 14.3.
18. \$7.50 : \$9.50 :: \$$x$: \$5.70.
19. $28\frac{4}{5} : 22\frac{2}{7} :: x : 27\frac{6}{7}$.
20. $46\frac{2}{3} : 22.4 :: x : 16.8$.

I. A man in 15 days can earn \$41.25; in how many days can he earn \$68.75 at the same wages?

Explanation.—Since only like numbers can be compared, the two sums of money will form one couplet of the proportion, and the two periods of time (one given and the other indicated by x) will form the other couplet. The money earned in 15 days is made the antecedent in one couplet, and the time required to earn it the antecedent in the other. If \$41.25 were made a consequent, then would 15 days also be made a consequent.

Process.

\$41.25 : \$68.75 :: 15 da. : x da.

$$\frac{\$\overset{25}{\cancel{68.75}} \times \cancel{15}}{\$\underset{275}{\cancel{41.25}}} = 25 \text{ da.}$$

After the proportion is correctly written, the operation consists in multiplying the means together and dividing by the given extreme.

Rule.—I. *Write for the third term the number that is of the same kind as the answer.*

II. *If the answer is to be greater than the third term, write the less of the two remaining numbers as the first term, and as the second term if the answer is to be smaller than the third term.*

III. *Multiply the means together, and divide the product by the given extreme, and the result will be the answer.*

WRITTEN PROBLEMS.

600. *1.* If 5 barrels of flour cost $36, what will be the cost of 18 barrels?

2. In how many days can 12 men dig a trench, if 15 men take 32 days to dig it?

3. If $45 will pay for 18 barrels of apples, how many barrels will $75 pay for?

4. An express train runs 315 miles in 7 hours; how far, at the same rate, will it run in 9½ hours?

5. I gain $987.30 on a certain investment in 1 year 3 months; how much should I gain on it in 5 years 5 months?

6. A pedestrian travels 264 miles in 8 days; at this rate, how far could he travel in 15½ days?

7. The interest on a certain sum of money for 19 months was $589; how much would it be for 12 months?

8. How far can a man walk in 23 days if he can walk 253 miles in 11 days?

9. If 75 pounds of sugar cost $6, what will be the cost of 95 pounds?

10. If my horses eat 9 tons of hay in 24 weeks, how long will 15 tons last them?

11. How long will it take 12 men to do a piece of work that can be done by 18 men in 8 days?

12. How many days will 196 pounds of flour last a family that consumes 100 pounds in 25 days?

13. How long will provisions sufficient for 165 men 35 days last 105 men?

14. If I paid \$31.25 for 25 yards of carpet, what should I pay for 41 yards of the same kind of carpet?

15. In 12 years I saved \$5730; at that rate, how much could I save in 27 years?

16. A steeple 120 feet high, cast, at a certain time of day, a shadow 96 feet long; what would be the length of the shadow if the steeple were 150 feet high?

17. If 15 men can do as much work as 25 boys, how many boys would it take to do as much work as 36 men?

18. If 25 bushels of wheat cost as much as 40 bushels of rye, how many bushels of rye will cost as much as 60 bushels of wheat?

19. How many acres of land can be bought for \$5232. if 145 acres are worth \$7902.50?

COMPOUND PROPORTION.

601. **A compound proportion** is an expression of the equality of two ratios, one or both of them compound.

Thus, $\left.\begin{matrix}2:3\\4:5\end{matrix}\right\} :: 24 : 45$, and $\left.\begin{matrix}3:4\\4:5\end{matrix}\right\} :: \left\{\begin{matrix}2:3\\9:10\end{matrix}\right.$, are compound proportions.

I. If 5 men in 18 days can earn \$275, how much can 12 men earn in 15 days?

Explanation.—Since the answer is to be dollars, \$275 is made the third term. Since 12 men will earn more than 5 men, the smaller number, 5, is made the first term of the couplet expressing men; and since less money can be earned in 15 days than in 18 days, 18 is made the first term of the couplet expressing days. That is, the terms of each simple ratio are arranged as if the answer depended on it alone.

Process.

$$\left.\begin{matrix}5 : 12\\18 : 15\end{matrix}\right\} :: \$275 : \$x$$

$$\frac{\overset{2}{\cancel{12}} \times \overset{5}{\cancel{15}} \times 275}{\cancel{5} \times \underset{6}{\cancel{18}}} = \$550.$$

Rule.—I. *Make the number like the answer the third term.*

II. *Of each pair of like numbers remaining, form a ratio as if the answer depended on that ratio alone.*

III. *Multiply the means together, and divide the product by the product of the given extremes, and the quotient will be the answer.*

WRITTEN PROBLEMS.

602. *1.* If 25 men can earn $150 in 3 days, how many men can earn $320 in 4 days?

2. If 24 5-cent loaves of bread weigh 7.5 pounds, how much should 50 8-cent loaves weigh?

3. If the interest on $500 for 15 months at 6% is $37.50, what is the interest on $800 for 18 months at 8%?

4. A pile of wood 12 feet long 4 feet wide and 3 feet high being worth $12, what is the value of a pile of wood 15 feet long, 5 feet wide and 6 feet high?

5. If 12 horses in 5 days eat $25 worth of oats, how much will it cost to feed 20 horses on oats for 8 days?

6. If the freight on 150 cattle averaging 900 pounds is $250 for 100 miles, what should be the freight on 275 cattle averaging 1200 pounds, for 150 miles?

7. If $75 is paid for painting the front of a building 160 feet long and 60 feet high, what should be paid for painting the front of a building 90 feet long and 40 feet high?

8. If it costs $35 to carry 1875 pounds of freight 160 miles, how much should be paid for carrying 2500 pounds 250 miles?

9. If a party of 15 persons pay $690 for 8 weeks' board, how much should be paid by a party of 12 persons for 16 weeks' board?

10. A block of marble 2 feet long, 1 foot 6 in. wide, and 1 foot thick, weighs 465 pounds; what is the weight of a block 6 feet long, $4\frac{1}{2}$ feet wide and 2 feet thick?

CAUSE AND EFFECT.

603. The solution of every example in proportion proceeds on the assumption that effects are in the same ratio as the causes that produce them. Every proportion is the comparison of two causes and two effects. In the method known as Cause and Effect, the causes form one ratio, and the effects the other. The first cause and first effect are antecedents; the second cause and second effect are consequents.

I. If 15 men earn $1900, how much will 24 men earn in the same time?

Explanation.—15 men working for a certain time is a cause that produces an effect,—the earning of $1900. These two elements are the first cause and first effect.

Again, the work of 24 men is a second cause that produces an unknown effect. The solution of the example consists in determining what this effect will be, on the assumption that effects are in the same ratio as their respective causes.

Process.

1st cause.		2d cause.		1st effect.		2d effect.
15	:	24	::	\$1900	:	\$x.

$$\frac{\$\overset{380}{\cancel{1900}} \times \overset{8}{\cancel{24}}}{\$x \times \underset{1}{\cancel{15}}}.$$

Hence, $x = \$3040$.

II. If 18 men in 10 days can cut 540 cords of wood, in how many days can 15 men cut 630 cords?

Explanation.—The first cause is 18 men laboring for 10 days; the first effect is the cutting of 540 cords of wood. The second cause is 15 men laboring for an unknown number of days; the second effect is the cutting of 630 cords of wood. Arranging these elements as in the margin and canceling gives 14 days as the missing element of the second cause.

Process.

1st cause.		2d cause.		1st effect.		2d effect.
18 10	:	15 x	::	540	:	630.

$$\frac{\cancel{18} \times \cancel{10} \times \overset{14}{\cancel{630}}}{\cancel{15} \times x \times \underset{1}{\cancel{540}}}.$$

Hence, $x = 14$ days.

WRITTEN PROBLEMS.

604. *1.* If 5 men can earn $165 in a week, how much can 12 men earn in the same time?

2. If 18 bushels of wheat make 4 barrels of flour, how many bushels will it take to make 15 barrels of flour?

3. A merchant gained $1313.40 on goods amounting to $8756; at that rate, what would he gain on goods amounting to $12375?

4. If $4\frac{3}{4}$ tons of hay cost $59.85, what cost $9\frac{4}{5}$ tons?

5. When $562.50 is paid for 12.5 cubic yards of granite, what should be paid for 11.75 cubic yards?

6. If 15 masons in $8\frac{1}{2}$ days can lay 38.75 perches of wall, how many perches should 24 masons lay in the same time?

7. If $5750 invested in business gains $875, how much, at the same rate, would $11875 gain in the same time?

8. If A can walk $480\frac{3}{8}$ miles in $15\frac{3}{4}$ days, in what time should he walk 610 miles?

9. How many men working 10 hours a day will be required to build a wall 160 feet long, 40 feet high and 3 feet thick, in 15 days, if 75 men in 12 days of 9 hours each can build a wall 120 feet long, 30 feet high and 2 feet thick?

10. If 10 horses in 14 weeks eat 5.65 tons of hay, how long will 11.3 tons last 7 horses?

11. If 12 men working 8 hours a day can saw 340 cords of wood in 10 days, how long will it take 16 men working 9 hours a day to saw 204 cords?

12. When $280 is paid for 175 barrels of apples containing 2 bushels each, how much should be paid for 125 barrels of apples containing $2\frac{1}{2}$ bushels each?

13. If it takes 15 men 12 days, working 10 hours a day, to make a concrete pavement 160 feet long and 10 feet wide, how long would it take 18 men, working 9 hours a day, to make a similar pavement 180 feet long and 12 feet wide?

PARTNERSHIP.

605. Partnership is the association of several persons for the transaction of business with joint capital.

606. The **capital** or **stock** is the money or other property invested in the business.

607. The **assets, effects,** or **resources** of a firm are the property it owns, together with the debts due to it.

608. The **liabilities** of a firm are the debts it owes.

609. To apportion the gain or loss of each partner, when the shares are invested for equal times.

I. A, B, and C engage in trade; A puts in \$5000, B \$7000, and C \$8000; apportion a gain of \$7500.

BY PROPORTION.

Explanation.—The entire capital is \$20000, and the gain on this amount is \$7500. The ratio of the entire capital to A's share of it equals the ratio of the entire gain to his share of the gain. The same is true with respect to B and C.

Process.

$$\$5000 + \$7000 + \$8000 = \$20000.$$

$$\$20000 : \$5000 :: \$7500 : \text{A's share.}$$
$$\$20000 : \$7000 :: \$7500 : \text{B's share.}$$
$$\$20000 : \$8000 :: \$7500 : \text{C's share.}$$

$$\text{A's} = \$1875; \quad \text{B's} = \$2625; \quad \text{C's} = \$3000.$$

BY FRACTIONS.

Explanation.—A's investment is $\frac{5000}{20000}$ or $\frac{1}{4}$ of the capital; he is entitled, therefore, to $\frac{1}{4}$ of the gain. In like manner, B is entitled to $\frac{7}{20}$ of the gain, and C to $\frac{2}{5}$ of the gain.

Process.

$$\tfrac{5000}{20000} \text{ of } \$7500 = \$1875, \text{ A's.}$$
$$\tfrac{7000}{20000} \text{ of } \$7500 = \$2625, \text{ B's.}$$
$$\tfrac{8000}{20000} \text{ of } \$7500 = \$3000, \text{ C's.}$$

WRITTEN PROBLEMS.

610. *1.* Two men, A and B, formed a partnership, A putting in \$8000 and B \$9000. They gained \$3400. What was each man's share?

2. Three men, A, B, and C, engage in business. A puts in \$15000, B \$18000, and C \$25000. They gain \$11600. What is each man's share?

3. A, B and C speculated in land, investing \$100000. A put in \$23000, B \$37000, and C the rest. What was each man's share of a loss amounting to \$18700?

4. Three men hired a pasture for \$204; the first pastured 375 sheep, the second 425 sheep, and the third 475 sheep; how much should each one pay?

5. A man dying, leaves a property worth \$6000. He owes A \$8000, B \$10000, and C \$12000. What is each man's equitable share of the property?

611. To apportion the gain or loss of each partner when the shares are invested for different times.

I. A, B and C engage in business. A puts in \$5000 for 6 months, B \$3000 for 12 months, and C \$2000 for 17 months. How shall a gain of \$4750 be apportioned?

Process.

$$
\begin{aligned}
\$5000 \times 6 &= \$30000 \\
\$3000 \times 12 &= \$36000 \\
\$2000 \times 17 &= \underline{\$34000} \\
& \quad \$100000
\end{aligned}
$$

$$
\left.\begin{array}{l}
\frac{30000}{100000}, \text{ or } \frac{30}{100} \\
\frac{36000}{100000}, \text{ or } \frac{36}{100} \\
\frac{34000}{100000}, \text{ or } \frac{34}{100}
\end{array}\right\} \text{of } \$4750 = \left\{\begin{array}{l}
\$1425, \text{ A's.} \\
\$1710, \text{ B's.} \\
\$1615, \text{ C's.}
\end{array}\right.
$$

Explanation.—The use of \$5000 for 6 months is equivalent to the use of \$30000 for 1 month; the use of \$3000 for 12 months is equivalent to the use of \$36000 for 1 month; the use of \$2000 for 17 months is equivalent to the use of \$34000 for 1 month. The three investments, therefore, are equivalent to \$100000 for 1 month, and from them a gain of \$4750 results. The respective shares of A, B and C in this gain are $\frac{30}{100}$, $\frac{36}{100}$, and $\frac{34}{100}$, since these fractions represent each man's investment respectively.

II. Three persons engage in business, and at the end of a year their gain is \$15000. A puts in \$25000, and at the end of 5 months draws out \$15000; B puts in \$10000, and at the end of 6 months puts in \$10000 more; C puts in \$30000, and after 8 months draws out \$20000. Apportion the gain.

Explanation.—A's capital for 5 months is \$25000, and for the remainder of the year, 7 months, it is \$10000; his entire capital is therefore equivalent to \$195000 for 1 month. B's entire capital is equivalent to \$180000 for 1 month; and C's capital to \$280000 for 1 month. The entire investment is equivalent to \$655000 for 1 month, the gain on which is \$15000. The process of apportioning the gain is the same as in Example I.

Process.

$$\left.\begin{array}{l}\$25000 \times 5\\ \$10000 \times 7\end{array}\right\} = \$195000, \text{ A's.}$$

$$\left.\begin{array}{l}\$10000 \times 6\\ \$20000 \times 6\end{array}\right\} = \$180000, \text{ B's.}$$

$$\left.\begin{array}{l}\$30000 \times 8\\ \$10000 \times 4\end{array}\right\} = \$280000, \text{ C's.}$$

$$\overline{\$655000}$$

WRITTEN PROBLEMS.

612. *1.* How shall a gain of \$9500 be apportioned among three partners who invest respectively, \$6000 for 6 months, \$8000 for 8 months, and \$9000 for 10 months?

2. A, B and C go into business and lose \$18400. A's investment is \$20000 for 10 months, B's \$25000 for 1 year, and C's \$30000 for 14 months. What is each man's loss?

3. In a business of which the capital is \$60000, A furnishes 35% of it for 8 months, B 45% of it for 9 months, and C the remainder for 10 months. Apportion a gain of \$12000.

4. M, N and R are partners for a year. M puts in \$8000 for 4 mo., and then adds \$5000; N puts in \$10000 for 6 mo., and then withdraws \$5000; R puts in \$12000 for 8 mo., and then withdraws \$8000. Apportion a gain of \$6760.

5. Two men, A and B, hire a pasture for \$420. A puts in 300 sheep for 5 weeks and B 450 sheep for 6 weeks; what should each one pay?

INVOLUTION.

613. A **power** of a number is the product obtained by using the number two or more times as a factor. (See Arts. 274–'6.)

614. A **perfect power** is a number that can be resolved into two or more equal factors. A number that can be resolved into *two* equal factors is a **perfect square**; into three, a **perfect cube**, etc.

Thus, 64 is at the same time a *perfect square*, a *perfect cube*, and a *perfect sixth power*.

615. **Involution** is the process of finding any required power of a number.

616. To find any power of an integer or decimal.

I. Required to find the third power of 25; also of 2.5.

Explanation.—Multiplying 25 by itself, and the result again by 25, gives 15625, the third power or cube of 25.

In the second process the result is pointed off as in multiplication of decimals.

Process.

$$25^3 = 25 \times 25 \times 25 = 15625.$$

Process.

$$2.5^3 = 2.5 \times 2.5 \times 2.5 = 15.625.$$

617. To find any power of a common fraction.

I. Required to find the cube of $\frac{2}{3}$.

Explanation.—Treating the fraction the same as if it were an integer, it is taken as a factor three times, giving $\frac{8}{27}$ as its cube.

Process.

$$(\tfrac{2}{3})^3 = \tfrac{2}{3} \times \tfrac{2}{3} \times \tfrac{2}{3} = \tfrac{8}{27}.$$

618. To find any power of a given power of a number.

I. Required to find the cube of the square of 8.

Process.

$$(8^2)^3 = (8\times8)\times(8\times8)\times(8\times8) = 8^6.$$
$$8^6 = 8\times8\times8\times8\times8\times8 = 262144.$$

Explanation.—The square of 8 requires the use of 8 twice as a factor, and the cube of the square of 8 requires the use of 8×8 three times, or of 8 six times as a factor. Performing the multiplication gives 262144 as the power required.

619. To find the product of two powers of the same number.

I. Required to find the product of 3^2 by 3^3.

Explanation.—3^2 is equal to 3 taken twice as a factor, and 3^3 is equal to 3 taken three times as a factor. The product of 3^2 by 3^3 is, therefore, equal to 3 taken as a factor $2+3$ or 5 times.

Process.

$$3^2\times3^3 = (3\times3)\times(3\times3\times3)$$
$$= 3^{2+3} = 3^5 = 243.$$

WRITTEN PROBLEMS.

620. *1.* Find the squares of 1, 2, 3, etc., to 12.

2. Square 25, 24, 65, 75, 64, 125, 200.

3. Find the squares of 4.5, 7.5, 8.35, 9.75, 37.5.

4. Square $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{12}{25}$, $\frac{9}{8}$, $\frac{8}{13}$, $\frac{7}{12}$, $1\frac{3}{4}$, $1\frac{4}{5}$, .06.

5. What are the cubes of 1, 2, 3, etc., to 12?

6. Find the cubes of 20, 25, 30, 3.5, .75, .05, .005.

7. Find the cube and the fourth power of 7.5. Of 1.01.

8. What are the fourth powers of $\frac{3}{4}$, $\frac{4}{5}$, $1\frac{1}{2}$, and $3\frac{3}{4}$?

9. What is the cube of 7^3? The fourth power of 3^2?

10. What is the product of 5^2 by 5^3? Of 3^5 by 3^4? Of 7^4 by 7^3? Of 4^4 by 4^5?

11. Find the product of 5^3 by 4^2. Of 6^3 by 5^4. Of 8^3 by 3^5.

EVOLUTION.

621. A **root** of a number is one of the two or more equal factors that will, if multiplied together, produce the number.

622. The **square root** of a number is one of *two* equal factors whose product equals the number; **the cube** or **third root,** one of *three* equal factors, etc.

623. Evolution is the process of finding any required root of a number.

624. The **radical sign,** $\sqrt{}$, is placed before and over a number to indicate evolution.

625. The **index** of a root is a small figure placed before the radical sign to indicate the root required.

Thus, $\sqrt[3]{64}$ indicates the cube root of 64.
The square root is meant when no index is written.

The root required may be indicated by a fractional exponent.

Thus, $81^{\frac{1}{2}}$ and $64^{\frac{2}{3}}$ have the same meaning as $\sqrt[2]{81}$ and $\sqrt[3]{64^2}$.
The numerator of the fraction indicates the power, and the denominator shows what root of that power is to be found.

SQUARE ROOT.

626. The square of a number contains twice as many figures, or one figure less than twice as many, as the number itself. Thus,

Numbers:	1,	5,	10,	25,	7.5,	.99,	100,	1.25.
Squares:	1,	25,	100,	625,	56.25,	.9801,	10000,	1.5625.

Hence, *If the square of a number be separated into periods of two figures each, beginning at the right, there will be as many periods as there are figures in the square root of the number.*

NOTE 1.—The left-hand period may contain only one figure.

NOTE 2.—Decimals are separated into periods by beginning at the decimal point. If the right-hand period is not full, a cipher is added. Thus, 1.56,25; .15,62,50; 15.62,50

627. To analyze the square of a number.

I. Required to show the composition of the square of 48.

Explanation.—48 may be written $40+8$. Hence, $48^2=(40+8)^2$. Writing the factors as in the margin, $40+8$ is first multiplied by 40, and then by 8, and the partial products are added.

Process.

$$\begin{array}{l} 40+8 \\ \underline{40+8} \\ 40^2+(40\times 8) \\ \underline{\qquad\quad (40\times 8)+8^2} \\ 40^2+2\times(40\times 8)+8^2= \end{array}$$

$$40^2+[(2\times 40)+8]\times 8=2304.$$

Hence, *The square of any number composed of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

NOTE.—Twice the product of the tens by the units, plus the square of the units, is the same as the sum of twice the tens and the units, multiplied by the units.

WRITTEN EXERCISES.

628. *1.* Express in terms of the tens and units the square of 25. Of 36. Of 45. Of 64. Of 93. Of 98.

2. In like manner, find the square of 38. Of 49. Of 69. Of 78. Of 89. Of 98.

629. To find the square root of a number.

I. Find the square root of 2304.

Process.

	23'04	40 + 8 = 48
	16 00	
80 + 8 = 88	7 04	
	7 04	

Explanation.—Separating 2304 into periods of two figures, beginning at the right, gives two periods. Hence, there are two figures in the square root. Whatever the tens' figure of the root is, the significant figures of its square are included in the left-hand period. The largest square in the left-hand period is 1600, and its square root is 40. Writing 40 as the tens of the root, and subtracting 1600 from 2304, leaves 704. The remainder consists of twice the tens plus the units, multiplied by the units. The units being very small as compared with twice the tens, 704 may be regarded as a product composed of twice the tens multiplied by the units. One of these factors, twice the tens, or 80 is known; the other, which is the unit's figure of the root, may be found by dividing 704 by 80, giving 8 as the unit's figure sought. Multiplying 80, or twice the tens, by 8, the unit's figure, and the unit's figure by itself; or, what is the same, multiplying 88 by 8, gives 704. This product, being equal to the remainder of the square, shows that 2304 is a perfect square, of which 48 is the square root.

GEOMETRICAL ILLUSTRATION.

Fig. 1.

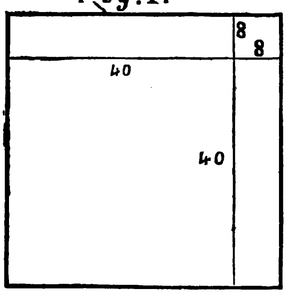
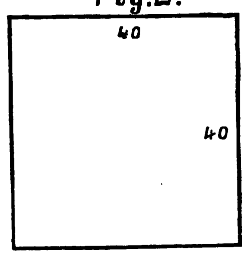


Fig. 2.



Let Fig. 1 be a square whose area is 2304. Since the area of a square is the product of two equal factors, the square root of the area will be the side of the square. If the side were 40, the area would be 1600; and it

50, the area would be 2500. The area being between 1600 and 2500, the side must be between 40 and 50. Removing from Fig. 1 a square 40 on a side, the remainder of Fig. 1 must have an area of 2304−1600, equal to 704. This remainder consists of a small square, and two rectangles whose united length is 40 × 2, or 80, and whose width added to the side of the square removed will give the side of Fig. 1. These two rectangles form the greater part of the remaining area, 704. Dividing 704 by 80, the united length of the two rectangles, gives 8 as the probable width of the rectangles. Since 704 is somewhat greater than the united areas of the two rectangles, 8 may be greater than their width. Two rectangles, each 40 long and 8 wide, and a square 8 on a side, are together equal to $2 \times (40 \times 8) + 8^2$, or 704. This being equal to the remainder of the square, 8 is the width of the rectangles. Adding 8 to the side of the square whose area is 1600 gives 48 as the side of the square whose area is 2304.

In practice, unnecessary figures are omitted.

II. Required to find the square root of 258.2449.

Explanation.—The greatest square in the left-hand period is 1 and its root is 1. Writing 1 in the root place, subtracting its square from 2, and annexing the next period to the remainder, gives 158. Dividing 15, the first two figures of 158, by twice the root already found, and diminishing the quotient as may be made necessary by the subsequent increase of the divisor, gives 6 as the next root figure. Annexing 6 to the trial divisor gives 26, and this multiplied by 6 gives 156. Subtracting and annexing the next period to the remainder gives 224. Multiplying 16, the root already found, by 2 gives 32 as the next trial divisor. This is not contained in the first two figures of 224, and to show this a cipher is written in the root; a cipher is also annexed to the trial divisor and the next period brought down. Dividing 2244 by 320 gives 7 as the last figure of the root, after which the operation is completed as before explained.

Process.

```
         2'58.24'49 | 16.07
         1
         ----
   26 |  1 58
         1 56
         ---------
  3207 |    22449
            22449
            -----
```

Rule.—I. *Begin at the unit's place, and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if any.*

II. *Find the greatest square in the left-hand period, write its root in the root place, and subtract the square from the left-hand period. To the remainder annex the next period.*

III. *Double the root already found for a trial divisor. Divide the remainder by the trial divisor, remembering that the latter is to be increased by annexing another figure to it. Annex the quotient to the trial divisor, and multiply the complete divisor by this last root figure; subtract, and bring down the next period as before.*

IV. *So continue to the last period, after which, if additional places are required in the root, continue the operation by bringing down cipher periods.*

V. *Should the trial divisor at any time not be contained in the remainder, write a cipher in the root place, annex a cipher to the trial divisor, and bring down another period.*

NOTE 1.—The decimal places in the root should be as many as the decimal periods.

NOTE 2.—The square root of a fraction is found by extracting the square root of its numerator and of its denominator, and writing them as the numerator and denominator, respectively, of the root.

WRITTEN PROBLEMS.

630. Find the square root of the following numbers:

1.	2025.	*9.*	15625.	*17.*	4260.854.
2.	4096.	*10.*	11664.	*18.*	30.62562.
3.	7396.	*11.*	182329.	*19.*	8.6925865.
4.	2809.	*12.*	319225.	*20.*	429.68475.
5.	9025.	*13.*	259081.	*21.*	.004225.
6.	7569.	*14.*	16224784.	*22.*	.015625.
7.	4489.	*15.*	35892081.	*23.*	.042025.
8.	5929.	*16.*	3714.9025.	*24.*	162.2025.

25. $\frac{1225}{2304}$, $\frac{5625}{4489}$, $\frac{27225}{130321}$, $\frac{90601}{164025}$, $\frac{5184}{8649}$.

CUBE ROOT.

631. The cube of a number contains three times as many figures as the number itself, three times as many figures less one, or three times as many figures less two. Thus,

Numbers:	1,	3,	5,	10,	99,	.05,	2.5.
Cubes,	1,	27,	125,	1000,	970299,	.000125,	15.625

Hence, *If the cube of any number be separated into periods of three figures each, beginning at the right, there will be as many periods as there are figures in the cube root.*

NOTE 1.—The left-hand period may contain one, two, or three figures

NOTE 2.—Decimals are separated into periods by beginning at the decimal point. The right-hand period must contain three figures, ciphers being annexed if necessary.

632. To analyze the cube of a number.

I. Required to show the composition of the cube of 48.

Process.

$$
\begin{array}{rl}
48^2 = & 40^2 + 2 \times (40 \times 8) + 8^2 \\
& \underline{40 \quad + 8 \qquad\qquad\qquad\qquad\qquad} \\
& 40^3 + 2 \times (40^2 \times 8) + \qquad (40 \times 8^2) \\
& \underline{\qquad\qquad (40^2 \times 8) + 2 \times (40 \times 8^2) + 8^3} \\
& 40^3 + 3 \times (40^2 \times 8) + 3 \times (40 \times 8^2) + 8^3 = \\
& 40^3 + [(3 \times 40^2) + (3 \times 40 \times 8) + 8^2] \times 8.
\end{array}
$$

Explanation.—It has been shown that $48^2 = 40^2 + 2 \times (4 \times 8) + 8^2$. Multiplying this by $40 + 8$ will give the cube of $40 + 8$.

Hence, *The cube of any number composed of tens and units is equal to the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

NOTE 1.—That the last two lines of the process are equal is shown thus : $3\times(40^2\times8)=(3\times40^2)\times8$; $3\times(40\times8^2)=(3\times40\times8)\times8$; $8^3=(8^2)\times8$ Hence, their sum equals $[(3\times40^2)+(3\times40\times8)+8^2]\times8$.

NOTE 2.—A decimal may be cubed in a similar manner. Thus, $(4.8)^3=4.0^3+3\times(4.0^2\times.8)+3\times(4.0\times.8^2)+.8^3$.

WRITTEN EXERCISES.

633. *1.* Express in terms of the tens and units the cube of 23. Of 32. Of 43. Of 51. Of 62. Of 74.

2. Express the cube of 29. Of 68. Of 89. Of 9.5. Of 8.1.

634. To find the cube root of a number.

I. Required to find the cube root of 110592.

Process.

$$\begin{array}{rr|l} & 110'592 & \underline{40+8} \\ & \underline{64\ 000} & \\ 3\times40^2 = & 4800 & 46\ 592 \\ 3\times40\times8 = & 960 & \\ 8^2 = & \underline{64} & \\ & 5824 & 46\ 592 \end{array}$$

Explanation. — Separating 110592 into periods of three figures each, beginning at the right, gives two periods. Hence, there are two figures in the cube root. Whatever the tens' figure of the root is, the significant figures of its cube are included in the left-hand period. The largest cube in the left-hand period is 64000, and its cube root is 40. Writing 40 as the tens of the root, and subtracting 64000 from 110592, leaves 46592. This remainder consists of two factors, the first of which is 3 times the square of the tens, plus 3 times the product of the tens by the units, plus the square of the units. The second factor is the units' figure, as yet unknown. In the first factor, 3 times the square of the tens is alone almost equal to the entire factor. Taking 3 times the square of the tens, or 4800, as one of two factors whose product equals the remainder, 46592, the other factor is equal to $46592\div4800$, or 9. But since 4800 is too small, 9 is likely to be too large. Taking 8 as more probably the correct units' figure, and completing the trial divisor by adding to it 3 times the product of the tens by the units, and the square of the units gives 5824 as the complete divisor. Multiplying 5824 by 8 gives a product equal to the remainder 46592, showing that 110592 is a perfect cube, whose cube root is 48.

GEOMETRICAL ILLUSTRATION.

FIG. 1.

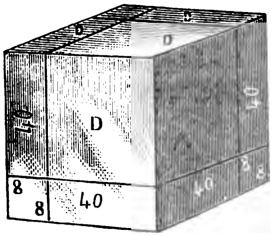
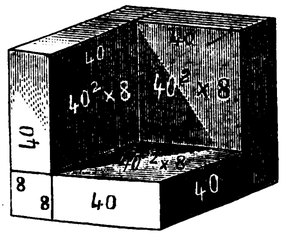


FIG. 2.



Let Fig. 1 be a cube whose volume is 110592. Since the volume of a cube is the product of three equal factors, the cube root of the volume will be one of the factors, that is, the edge of the cube. If the edge of the cube were 40, its volume would be 64000, and if it were 50, the volume would be 125000. The volume being between 64000 and 125000, the edge must be between 40 and 50. Removing a cube whose edge is 40, the remainder of Fig. 1, shown by Fig. 2, has a volume equal to $110592 - 64000$, or 46592. This remainder is made up of three rectangular solids each 40 by 40 on the face, of three others each 40 long with a width and depth equal to the excess of the edge of Fig. 1 above 40, and a cube whose edge is equal to the same excess. The depth of all these being the same, their united volumes, or the remainder 46592, is equal to $(3 \times 40^2 + 3 \times 40 \times depth + depth^2) \times depth$. Neither of these two factors is entirely known, but 3×40^2 is alone almost equal to the first factor. Dividing 46592 by 3×40^2, or 4800, gives 9 as the depth of these facing solids.

But since 3×40^2 is somewhat smaller than the first of the two factors, 9 is probably larger than the second factor. Taking 8 as more likely to be the other factor, the volume of the solids composing Fig. 2 may be calculated as follows: $[(3 \times 40^2) + 3 \times (40 \times 8) + 8^2] \times 8 = 46592$. This being equal to the remainder of the cube whose volume is 110592, 8 is the addition to be made to the edge of the cube whose volume is 64000 to give the edge of the cube whose volume is 110592. Adding 8 to 40 gives 48, the edge required.

The extraction of cube root may be abridged by omitting unnecessary figures. To show this, let it be

II. Required to find the cube root of 390457.604629.

Explanation.—Beginning at the decimal point, the number is separated into periods of three figures each, the integral part from right to left, and the decimal part from left to right.

Process.

	390'457.604'629	73.09
	343	
14700	47 457	
630		
9		
15339	46 017	
1598700	1 440 604	
159870000	1 440 604 629	
197100		
81		
160067181	1 440 604 629	

The greatest cube in the left-hand period is 343, and its cube root is 7. The first figure of the root is 7, and the remainder, after subtracting the cube of 7 and bringing down the second period, is 47457. Dividing this by 3 times the square of 7 regarded as tens, that is, by 14700, gives 3 as the second figure of the root. Adding to 14700, the trial divisor 3 times 70×3, or 630, and 3^2, gives 15339, the complete divisor. The complete divisor multiplied by the second root figure, and the product subtracted from 47457, gives 1440, to which the next period is annexed.

Taking 3 times the square of 73, regarded as tens, for the next trial divisor, it is not contained in 1440604. A cipher is, therefore, written in the root, and the last period is brought down.

Taking 3 times the square of 730, regarded as tens, as the next trial divisor, or what is the same, annexing two ciphers to the previous trial divisor, and dividing by it as before, gives 9 as the final root figure. Completing the trial divisor by adding to it 3 times 730×9, and 9^2, and multiplying the divisor so completed by 9, gives a product equal to the remainder of the number whose cube is to be found. Hence, 73.09 is the cube root required.

Rule.—I. *Separate the numbers into periods of three figures each, beginning at the unit's place.*

II. *The first figure of the root will be the cube root of the greatest cube contained in the left-hand period. Subtract this cube from the left-hand period, and to the remainder annex the next period to form a dividend.*

III. *For a trial divisor take three times the square of*

the root figure regarded as tens, and divide the dividend by it to obtain the second figure of the root.

IV. *For a complete divisor add to the trial divisor three times the second figure of the root multiplied by the first figure considered as tens, and the square of the second figure of the root.*

V. *From the dividend subtract the product of the complete divisor by the second figure of the root.*

VI. *Bring down the next period, if any remain, and proceed as before to find the trial divisor, treating the part of the root already found as if it were a single figure expressing tens.*

NOTE 1.—If the trial divisor is not contained in the dividend, annex a cipher in the root, two ciphers to the trial divisor, and another period to the dividend. Then divide the dividend by the trial divisor for the next figure of the root.

NOTE 2.—When the number whose cube root is to be found is not a perfect cube, decimal places may be obtained in the root by annexing cipher periods to each successive remainder.

WRITTEN PROBLEMS.

635. Find the cube root of the following numbers:

1.	5832.	*8.*	50653.	*15.*	4492.125.
2.	6859.	*9.*	97336.	*16.*	8.615125.
3.	9261.	*10.*	74088.	*17.*	907039232.
4.	29791.	*11.*	205.379.	*18.*	890277128.
5.	13824.	*12.*	614.125.	*19.*	521.660125.
6.	17576.	*13.*	.373248.	*20.*	1758.416743.
7.	32768.	*14.*	9938375.	*21.*	9.234565192.

22. Find the cube root of 2 to three decimal places.

23. Find the cube root of 3 to three decimal places.

24. What is the cube root of $\frac{512}{2744}$? Of $\frac{2197}{4096}$?

25. Find the sum of the cube root of 12167 and 35937.

26. How much greater is the cube root of 857.375 than the cube root of 274.625?

PRACTICAL APPLICATIONS OF SQUARE AND CUBE ROOTS.

636. *1.* A square lawn contains 15625 square yards; what will it cost to fence it at $1.75 a yard?

2. How many links in the side of a square field whose area is 124.25625 acres?

3. A square bin 10 feet deep has a capacity of 40960 cubic feet; what is its length?

4. The united area of the faces of a cube is 3750 square feet; what is the side of the cube?

5. The surface of a ceiling whose length is twice its width is 4050 square feet; what are the dimensions of the ceiling?

6. A cubical block of granite contains 13824 cubic feet; what is the length of its side?

7. What must be the depth of a cubical box that shall contain 5832 cubic feet?

8. A cubical cistern having a capacity of 2744 cubic feet is lined on the sides and bottom with copper, at 75 cents per square foot; what is the cost of the lining?

9. A room whose width and height are equal has a length 3 times as great; what are the dimensions of the room, if its volume is 46875 cubic feet?

10. A square timber 28 feet long contains 112 cubic feet; how many inches square is it?

11. How much more will it cost, at $1.25 a rod, to fence a field in the form of a rectangle 108 rods long and 48 rods wide than to fence a field of equal area in the form of a square?

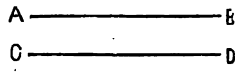
12. What is the height of a cubical box of the same capacity as a box 32 in. long, 24 in. wide, and 18 in. deep?

13. A cube immersed in a rectangular reservoir 36 inches long and 16 inches wide raises the water 3 inches; what is the edge of the cube?

MENSURATION.

637. Mensuration treats of the measurement of *lines angles*, *surfaces*, and *volumes* or *solids*.

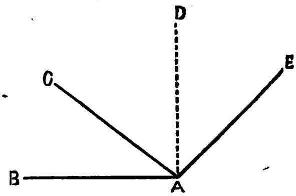
638. Parallel lines are lines lying in the same plane and extending in the same direction.



639. An **acute angle** is an angle less than a right angle.

A **right angle** is an angle of 90°. See page 192.

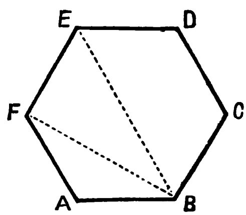
640. An **obtuse angle** is an angle greater than a right angle.



641. A **polygon** is a plane figure or flat surface bounded by straight lines.

A **regular polygon** is one having all its sides equal and all its angles equal.

The **perimeter** of a polygon is the distance around it; a **diagonal** is a straight line joining any two angles not adjacent.

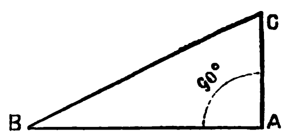


642. A polygon of **three** sides is called **a triangle,** of **four** sides a **quadrilateral,** of **five** sides a **pentagon,** of **six** sides a **hexagon,** of **seven** sides a **heptagon,** etc.

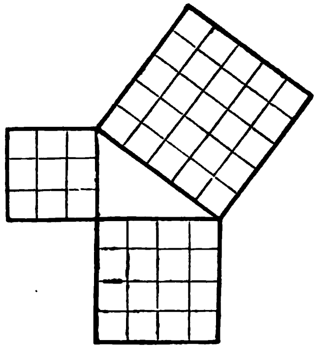
MENSURATION OF SURFACES.

643. A right-angled triangle is a triangle having a right angle.

The **base** of any triangle is the side on which it is supposed to stand, as BA in the figure. BC is the **hypothenuse,** and AC the **perpendicular.**



644. It is proved in Geometry



I. That the square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

II. That the square on either of the sides forming the right angle is equal to the square on the hypothenuse diminished by the square on the other side.

645. To find any side of a right-angled triangle, the other two sides being given.

Rule.—I. *Add the squares of the sides forming the right angle, extract the square root of the sum, and the result will be the hypothenuse.*

II. *From the square of the hypothenuse subtract the square of either of the other two sides, and the square root of the remainder will be the third side.*

WRITTEN PROBLEMS.

646. *1.* The sides forming the right angle of a right-angled triangle are 36 feet and 48 feet; what is the hypothenuse?

Solution.—$\sqrt{36^2+48^2} = \sqrt{3600} = 60$, the hypothenuse.

2. The hypothenuse of a right-angled triangle is 105, and the perpendicular 63; what is the base?

3. What is the length of the diagonal of a city lot 25 feet by 100 feet?

4. The foot of a ladder 35 feet long is placed 12 feet from a building; how high will the top of the ladder reach on the side of the building?

5. How far is it from a village situated 20 miles north of Mr. Evans' house to a village 18 miles east of his house?

6. A flag-pole broke off 65 feet from the ground, and while the foot of the broken piece remained on the stump, its top struck the ground 45 feet from the foot of the pole; how long was the pole?

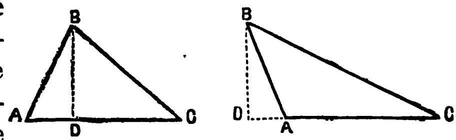
7. The rafters of a house are 20 feet long and their ends are 30 feet apart; what is the height of the gable?

8. How long is the diagonal of a rectangular room 30 feet long and 25 feet wide?

9. How much does a man save by crossing diagonally a field 28 rods long and 23 rods wide, instead of going along the end and side?

10. A tree 140 feet high throws a shadow 90 feet long; how far is it from the end of the shadow to the top of the tree?

647. The **altitude** of a triangle is the perpendicular distance from the angle opposite the base to the base, or the base extended, as BD.



The altitude of a right-angled triangle is the perpendicular.

648. To find the area of a triangle, its base and altitude being given.

Rule.—*Multiply the base by the altitude, and take half the product. The result will be the area.*

WRITTEN PROBLEMS.

649. *1.* The base of a triangle is 24 feet, and its altitude 15 feet; what is the area of the triangle?

Solution.—$\frac{24 \times 15}{2} = 180$, the area in square feet.

2. What is the area of a triangle whose base is 60 rods, and altitude 35 rods?

3. How many acres in a triangular field, the base of which is 90 rods and the altitude 48 rods?

4. How many acres in a triangular field whose base is 95 chains and altitude 60 chains?

5. How many square feet of inch lumber will it take to cover the gable of a barn 56 feet wide, if its height above the level of the eaves is $14\frac{2}{3}$ feet?

6. The base of a right-angled triangle is 60 rods, and its perpendicular 45 rods; what is its area?

650. To find the area of a triangle when the three sides are given.

Rule.—*From half the sum of the three sides subtract each side; find the continued product of the half sum and the three remainders; the square root of the result will be the area.*

WRITTEN PROBLEMS.

651. *1.* What is the area of a triangular field whose three sides are 15 rods, 20 rods, and 25 rods?

Solution.—$(15+20+25) \div 2 = 30$; $\quad 30 - \begin{cases} 15 = 15, \\ 20 = 10, \\ 25 = 5; \end{cases}$

$$\sqrt{30 \times 15 \times 10 \times 5} = \sqrt{22500} = 150, \text{ the area in sq. rds.}$$

2. What is the area of a triangle whose sides are 24, 32, and 40 rods?

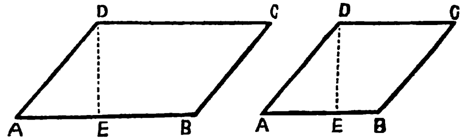
3. How many square rods in a triangular field whose sides are 60, 80, and 100 rods?

4. A man sold a triangular field whose sides were 65, 75, and 85 rods, at $160 an acre; what did he get for it?

5. How many acres of land in a field whose sides are 30, 40, and 50 chains?

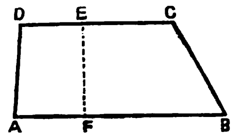
652. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

653. A **rhomboid** is a parallelogram whose angles are not right angles.



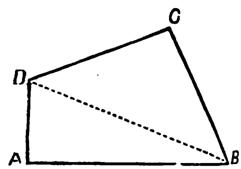
654. A **rhombus** is a rhomboid whose sides are equal.

655. A **trapezoid** is a quadrilateral having two sides parallel.



The **altitude** of a parallelogram or trapezoid is the perpendicular distance between its parallel sides.

656. A **trapezium** is a quadrilateral of which no two sides are parallel.



657. To find the area of a parallelogram.

Rule.—*Multiply the base by the altitude, and the product will be the area.*

WRITTEN PROBLEMS.

658. *1.* What is the area of a parallelogram whose base and altitude are respectively 50 and 20 rods?

Solution.—$50 \times 20 = 1000$, the area in sq. rds.

2. The altitude of a rhombus is 60 feet, and each of its sides 90 feet; what is its area?

3. Find the area of a field in the form of a rhomboid whose base is 60.8 rods and altitude 18.5 rods.

4. How many acres of land in a field in the form of a parallelogram whose base is 87.5 chains and altitude 37.5 chains?

5. A rectangular lawn 200 by 160 feet has on it a house 40 by 60 feet; find the surface of the lawn exclusive of the space occupied by the house.

6. A field 66 rods wide, in the form of a rectangle, has a diagonal of 110 rods; what is it worth at $85 an acre?

659. To find the area of a trapezoid.

Rule.—*Multiply half the sum of the parallel sides by the altitude, and the product will be the area.*

WRITTEN PROBLEMS.

660. *1.* The parallel sides of a trapezoid are 40 rods and 50 rods, and its altitude 20 rods; what is its area?

Solution.—$(40+50)\div 2 = 45$; $45 \times 20 = 900$, the area in sq. rds.

2. How many acres in a field in the form of a trapezoid, its parallel sides being 65 rods and 40.8 rods, and its altitude 20.5 rods?

3. What is the area of a trapezoid whose parallel sides are 30.5 chains and 45.5 chains, and its altitude 18.5 chains?

4. What is the value, at $160 an acre, of a field in the form of a trapezoid, whose parallel sides are 22.4 rods and 35.2 rods, and its altitude 6.4 rods?

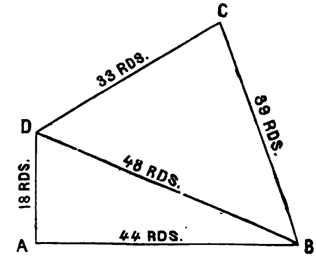
5. A rectangular field 18 rods wide is cut into two trapezoids by a fence running obliquely across it. The parallel sides of the trapezoids are respectively 36 and 18 rods, 31 and 13 rods; how many acres in the field?

661. To find the area of a trapezium, when its four sides and a diagonal are given.

Rule.—*Find the areas of the two triangles separately, and their sum will be the area of the trapezium.*

WRITTEN PROBLEMS.

662. *1.* Required to find the area of a trapezium whose dimensions are, AB 44 rds., DA 18 rds., DC 33 rds., BC 39 rds., and DB 48 rds.



Solution.—$(48+39+33)\div 2 = 60$;

$$60 - \begin{cases} 48 = 12; \\ 39 = 21; \\ 33 = 27; \end{cases}$$

$$\sqrt{60 \times 12 \times 21 \times 27} = 638.936+$$

$$(48+44+18)\div 2 = 55; \quad 55 - \begin{cases} 48 = 7; \\ 44 = 11; \\ 18 = 37; \end{cases}$$

$$\sqrt{55 \times 7 \times 11 \times 37} = \underline{395.847+}$$

$$1034.783+ \text{ sq. rds}$$

NOTE.—The area of any irregular figure whose sides are straight lines may be found by dividing it into triangles, or into triangles and rectangles, finding their areas separately, and taking their sum.

2. How many acres in a field in the form of a trapezium whose dimensions are as follows: AB = 55 rds., BC = 65 rds., CD = 80 rds., DA = 30 rds., AC = 90 rds.?

3. In a trapezium ABCD, AB is 40 chains, BC 30 chains, CD 35 chains, DA 38 chains, and the diagonal BD 42 chains; what is its area in acres?

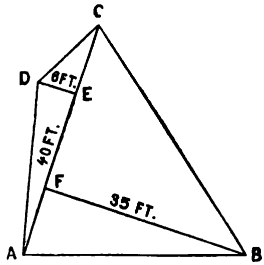
4. What is the value, at $75 an acre, of a plot of ground in the form of a trapezium ABCD, of which AB is 36 chains, BC 60 chains, CD 65 chains, DA 38 chains, and the diagonal BD 68 chains?

663. To find the area of a trapezium, a diagonal being given that forms the base of two triangles whose altitudes are also given.

Rule.—*Multiply the sum of the altitudes by half the diagonal, and the product will be the area of the trapezium.*

WRITTEN PROBLEMS.

664. *1.* Required to find the area of a trapezium whose dimensions are as those of the figure in the margin.



Process.

$(35 + 6) \times 20 = 820$ sq. ft.

Explanation.—The entire trapezium is divided into two triangles by the diagonal AC. This diagonal is the common base of the triangles and their altitudes are 35 ft. and 6 ft. The area of the larger triangle is 35×20, and that of the smaller is 6×20. The sum of these areas is $(35 \times 20) + (6 \times 20)$, or $(35 + 6) \times 20 = 820$ sq. ft.

2. Find the area of a trapezium divided by a diagonal of 60 rods into triangles whose altitudes are 20 rods and 32 rods.

3. How many acres in a field having the form of a trapezium, and divided by a diagonal of 80 rods into two triangles whose altitudes are 48 rods and 30 rods?

665. To find the circumference of a circle.

Rule.—*Multiply the diameter by* 3.1416, *and the product will be the circumference.*

NOTE.—Conversely: The diameter is equal to the circumference divided by 3.1416.

WRITTEN PROBLEMS.

666. *1.* What is the circumference of a circle whose diameter is 100 feet?

Solution.—100 feet $\times$ 3.1416 = 314.16 ft.

2. What is the diameter of a circle whose circumference is 628.32 rods?

3. How far is it around a circular race-course 180 rods in diameter?

4. What is the diameter of a circular reservoir 1500 feet in circumference?

5. A horse is attached to a post by a rope 60 feet long; find the circumference of the circle in which he may graze.

667. To find the area of a circle.

Rule.—*Multiply the square of the radius (half the diameter) by* 3.1416, *or the square of the diameter by* .7854.

NOTE.—Conversely: The radius is found by dividing the area by 3.1416, and extracting the square root of the quotient.

WRITTEN PROBLEMS.

668. *1*. What is the area of a circle whose radius is 50 feet?

Solution.—$50^2 \times 3.1416 = 7854$ sq. ft.

2. What is the radius of a circle whose area is 4071.5136 square feet?

Solution.—$4071.5136 \div 3.1416 = 1296$; $\sqrt{1296} = 36$ feet.

3. What is the area of a circular pond 160 feet in diameter?

4. Which is larger, and how much, a circle 120 rods in diameter, or a square 120 rods on a side?

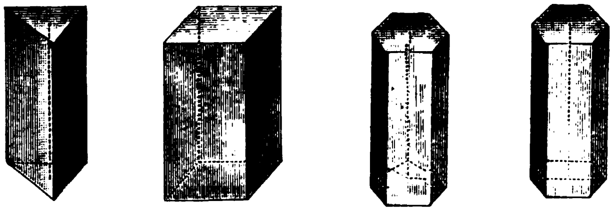
5. A horse is fastened by a rope long enough to allow him to graze on one acre; how long is the rope?

6. How many square yards of concrete will cover the bottom of a reservoir 90 yards in diameter?

7. What is the diameter of a circle whose area is 16 acres?

8. Find the diameter of a circular window that will admit the same amount of light as a rectangular window 7 ft. 6 in. high and 3 ft. 8 in. wide.

9. How much more will it cost, at $1.50 a rod, to fence 10 acres in the form of a square than in the form of a circle?



669. A **prism** is a solid or volume whose upper and lower bases are equal and parallel polygons, and whose sides, or lateral faces, are parallelograms.

A **triangular prism** is one whose bases are triangles; a **quadrangular prism** is one whose bases are quadrilaterals; a **pentagonal prism** is one whose bases are pentagons; etc.

670. The **altitude** of a prism is the perpendicular distance between its upper and lower bases.

671. A **right prism** is one whose lateral faces are all rectangles.

672. The **convex surface** of a prism is the area of its lateral faces.

673. To find the convex surface or the entire surface of a right prism.

Rule.—I. *Multiply the perimeter of the base by the altitude, and the product will be the convex surface.*

II. *Add the area of the upper and lower bases to the convex surface, and the sum will be the entire surface.*

WRITTEN PROBLEMS.

674. *1.* What is the convex and the entire surface of a right prism whose altitude is 12 feet, and the base of which is an equilateral triangle 10 feet on a side?

Solution.— 10 ft. × 3 = 30 ft. = perimeter of base.
30 × 12 = 360 sq. ft. = convex surface.
Area of base = 43.3 sq. ft. (Art. 650.)
43.3 sq. ft. × 2 = 86.6 sq. ft. = area both bases.
86.6 + 360 = 446.6 sq. ft. = entire surface.

2. What is the convex surface of a right prism whose altitude is 20 feet, and whose bases are squares 16 feet on a side?

3. What is the convex and the entire surface of a square shaft of marble 30 feet long, whose bases are 8 feet on a side?

4. What will it cost at $.25 a square yard to plaster the sides of a room 30 feet square, and 18 feet high?

675. To find the volume of a right prism.

Rule.—*Multiply the area of the base by the altitude and the product will be the volume.*

WRITTEN PROBLEMS.

676. *1.* What is the volume of a right prism, the altitude being 36 inches and the base a square 20 inches on a side?

Solution.—(20 × 20) × 36 = 14400 cu. in. = volume.

2. How many cubic inches in a prism the base of which is a triangle 20 inches on each side, and its altitude 30 inches?

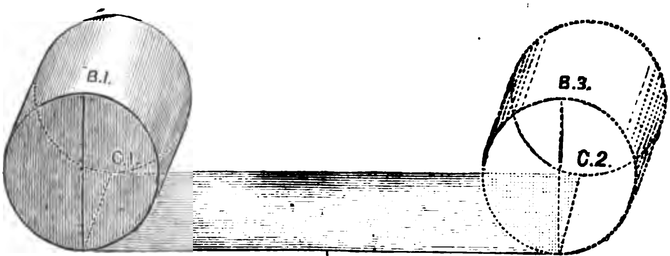
3. At $2.75 per cubic foot, what is the value of a square granite shaft 18 feet long whose base is 4 ft. 6 in. on a side?

4. Find the weight of a prism of granite 10 feet high, whose base is a rectangle 3 feet by 2 feet 6 inches, a cubic foot of granite weighing 168.75 lb.

5. Which is greater, and how much; the number of square feet in the entire surface of a cube 8 feet high, or the number of cubic feet in its volume?

677. A **cylinder** is the volume that is formed by turning a rectangle around one of its sides as an axis.

678. To find the convex or the entire surface of a cylinder.



The cylinder above is represented as lying on a plane and as touching it along the line A.1 C.1. Conceive it now to be rolled to the right. When B.1 reaches the plane at B.2, A.1 is exactly above, and the path described is a rectangle equal to half the convex surface of the cylinder. When A.1 reaches the plane at A.2, B.3 represents the position of B.1, and the path described from A.1 to A.2 is a rectangle equal to the entire convex surface of the cylinder.

Rule.—I. *Multiply the circumference of the base by the altitude, and the result will be the convex surface.*

II. *Add the areas of the upper and lower bases to the convex surface, and the sum will be the entire surface.*

WRITTEN PROBLEMS.

679. *1.* What is the convex and the entire surface of a cylinder 12 feet high, the diameter of the base being 6 feet?

Solution.— $6 \times 3.1416 \times 12 = 226.1952$ sq. ft. $=$ convex surface.
$(6^2 \times .7854 \times 2) + 226.1952 = 282.744$ sq. ft. $=$ entire surface.

2. Find the convex and entire surfaces of a cylinder 35 inches in altitude, the radius of the base being $6\frac{1}{2}$ inches.

3. How many square feet in the convex surface of a cylinder 4.5 feet long and 1.5 feet in diameter?

4. Find the entire surface of a cylinder 12 feet long, the diameter of the base being 1 foot 3 in.

680. To find the volume of a cylinder.

Rule.—*Multiply the area of the base by the altitude, and the product will be the volume.*

WRITTEN PROBLEMS.

681. *1.* What is the volume of a cylinder 9 feet long, the diameter of its base being 18 inches?

Solution.—18 in. $= 1\frac{1}{2}$ ft. $(1\frac{1}{2})^2 \times .7854 \times 9 = 15.904+$ cu. ft.

2. Find the capacity in gallons of a cistern 12 feet deep and 10 feet in diameter.

3. How many cubic feet in a log 20 feet long and 18 inches in diameter?

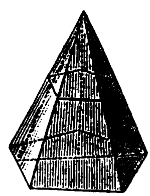
4. A bushel measure is a cylinder $18\frac{1}{2}$ inches in diameter and 8 inches deep; what is its capacity in cubic inches?

5. How much more or less than one bushel is the capacity of a cylinder 14 inches deep and 14 inches in diameter?

6. What is the weight of a cylindrical shaft of granite 20 inches in diameter and 30 feet long, if the weight of a cubic foot of granite is 169 pounds?

7. How many gallons liquid measure will a cylindrical vessel hold that is 10 inches in diameter and 12 inches deep?

682. A **pyramid** is a solid or volume whose base is a polygon, and whose convex surface is composed of triangles.



The slant height is the altitude of the triangles that form the sides.

683. To find the convex or lateral surface of a pyramid.

Rule.—*Multiply the perimeter of the base by one-half the slant height, and the product will be the convex surface.*

NOTE.—The convex surface and the area of the base added together give the entire surface.

WRITTEN PROBLEMS.

684. *1.* What is the convex surface of a pyramid the slant height of which is 30 inches and the base a pentagon each side of which is 20 inches?

Solution.—$\overline{20 \times 5} \times 15 = 1500$ sq. in. = convex surface.

2. The base of a pyramid is a square 20 inches on a side and the slant height 50 inches; what is the entire surface?

3. The base of a pyramid is a triangle 36 feet on a side, and the slant height is 60 feet; what is its entire surface?

4. The base of a pyramid is a hexagon 12 ft. 6 in. on a side, and the slant height is 30 feet; what is its lateral surface?

5. What is the difference between the convex surface of a square pyramid and a triangular pyramid each 8 feet 6 inches on a side, the slant height of each being 9 feet 8 inches?

685. To find the volume of a pyramid.

Rule.—*Multiply the area of the base by one-third of the altitude, and the product will be the volume.*

WRITTEN PROBLEMS.

686. *1.* The base of a pyramid is a square 60 feet on a side, and its altitude is 90 feet; what is its volume?

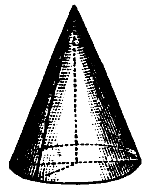
Solution.—$\overline{60 \times 60} \times 30 = 108000$ cu. ft. = volume.

2. What is the volume of a pyramid the base of which is a square 30 ft. 6 in. on a side and the altitude 75 feet?

3. The base of a triangular pyramid is 20 yards on a side and the altitude is 30 yards; what is its volume?

4. What is the volume of a triangular pyramid whose altitude is 20 feet and whose sides are respectively 6, 7 and 8 feet?

5. If a cubic foot of marble weighs 175 pounds, what is the weight of a marble pyramid 60 feet high, whose base is a square 20 feet on each side?



687. **A cone** is the solid that is formed by turning a right-angled triangle around one of the sides that include the right angle.

The **altitude** of a cone is the distance from its top or vertex to the center of the circle that forms its base.

The **slant height** of a cone is the distance from its vertex to the circumference of its base.

688. To find the convex or the entire surface of a cone.

Rule.—I. *Multiply the circumference of the base by half the slant height, and the product will be the convex surface.*

II. *To the convex surface add the area of the base, and the sum will be the entire surface.*

NOTE.—The slant height may be found by adding the square of the altitude to the square of the radius of the base, and extracting the square root of the sum.

WRITTEN PROBLEMS.

689. *1.* What is the convex surface and what the entire surface of a cone whose slant height is 24 inches and the diameter of whose base is 20 inches?

Solution.— $20 \times 3.1416 =$ circ. of base.

$20 \times 3.1416 \times 12 = 753.984$ sq. in. = convex surface.

$10^2 \times 3.1416 = 314.16$ sq. in. = area of base.

753.984 sq. in. $+ 314.16$ sq. in. $= 1068.144$ sq. in.
= entire surface.

2. Find the convex surface of a cone the diameter of whose base is 36 inches and the slant height 40 inches.

3. What is the entire surface of a cone formed by turning a right-angled triangle around its perpendicular as an axis, the perpendicular of the triangle being 20 inches and its base 15 in.?

4. What is the convex surface of a cone having a slant height of 12 ft. 6 in. and a base the diameter of which is 8 ft. 4 in.?

690. To find the volume of a cone.

Rule.—*Multiply the area of the base by one-third of the altitude, and the product will be the volume.*

WRITTEN PROBLEMS.

691. *1.* What is the volume of a cone whose altitude is 48 inches, the diameter of the base being 20 inches?

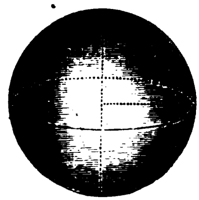
Solution.—$10^2 \times 3.1416 \times 16 = 5026.56$ cubic inches = volume.

2. The altitude of a cone is 30 inches and the diameter of its base is 24 inches; what is its volume?

3. The altitude of a cone is 60 inches and the diameter of its base is 72 inches; what is its volume?

4. The altitude of a cone is $8\frac{1}{4}$ feet, and the radius of the base 5 feet; what is its volume?

692. A sphere is a solid bounded by a surface every point of which is equally distant from a point within called the center.



693. To find the surface of a sphere.

Rule.—*Multiply the square of the diameter by* 3.1416, *and the product will be the surface.*

WRITTEN PROBLEMS.

694. *1.* What is the surface of a sphere whose radius is 5 inches?

Solution.— $5 \times 2 =$ diameter.
$10^2 \times 3.1416 = 314.16$ sq. in. = surface.

2. What is the surface of a sphere 20 feet in diameter?

3. What will it cost, at 60 cents a square foot, to gild a spherical ball 3 ft. 6 in. in diameter?

695. To find the volume of a sphere.

Rule.—*Multiply its surface by one-third of the radius, and the product will be the volume.*

WRITTEN PROBLEMS.

696. *1.* What is the volume of a sphere whose radius is 30 inches?

Solution.— $60^2 \times 3.1416 =$ surface.

$60^2 \times 3.1416 \times 10 = 113097.6$ cu. in. $= 65.45$ cu. ft.

2. What is the volume of a sphere whose radius is 2 feet?

3. Find the weight of a sphere of iron 1 ft. 8 in. in diameter, the weight of a cubic foot of iron being 487.5 lb.

GAUGING.

697. Gauging is the process of measuring the capacity of barrels and casks.

Rule.—I. *Divide the sum of the head diameter and the bung diameter by 2 for the mean diameter.*

II. *Multiply the square of the mean diameter in inches by the length in inches, and this product by* .0034. *The result is the capacity in wine gallons.*

WRITTEN PROBLEMS.

698. *1.* What is the capacity in gallons of a cask of which the head diameter is 24 inches, the bung diameter 26 inches, and the length 36 inches?

2. What is the capacity of a cask whose head diameter is 28 inches, its bung diameter 32 inches, and its length 40 inches?

3. The bung diameter of a hogshead is 46 inches, its head diameter 42 inches and its length 60 inches; what is its capacity?

SIMILAR SURFACES.

699. Similar surfaces are surfaces having the same *form* and corresponding dimensions proportional.

Two polygons are similar when the angles of the first are successively equal to the angles of the second taken in the same order, and the ratios of the dimensions of the first to the corresponding dimensions of the second are equal.

NOTE.—All circles are similar figures; so also are all equiangular polygons. Their corresponding dimensions are any lines similarly situated, as radii, diameters, circumferences, sides, diagonals, etc.

Principles.—I. *The areas of similar surfaces are to each other as the squares of their corresponding dimensions.*

II. *The corresponding dimensions of similar surfaces are to each other as the square roots of their areas.*

NOTE.—Articles whose values are determined by their quantity of surface vary in value as their areas vary.

WRITTEN PROBLEMS.

700. *1.* A triangular field, whose altitude is 8 chains, has an area of 8 acres; what is the area of a similar field whose altitude is 15 chains?

Solution.—$8^2 : 15^2 :: 8$ A. : x A. $= 28\frac{1}{8}$ acres.

2. A 2-inch pipe will empty a cistern in 12 hours; in what time will a 3-inch pipe empty it?

Solution.—$3^2 : 2^2 :: 12$ hrs. : x hrs. $= 5\frac{1}{3}$ hours.

3. If it costs $42.30 to carpet a room 18 feet square, what is the side of a square room that for $75.20 can be covered with carpet of the same quality?

Solution.—In this case, the ratio of the cost is equal to the ratio of the areas. Hence,

$$\$42.30 : \$75.20 :: 18^2 : x^2; \quad x = \sqrt{\frac{324 \times 75.20}{42.30}} = 24 \text{ feet.}$$

4. I bought two Smyrna rugs of the same quality, the corresponding dimensions of which were as 3 to 5; if I paid for the smaller \$15.30, what should I have paid for the larger?

5. The altitude of a field containing $2\frac{1}{4}$ acres is 13 rods; what is the area of a similar field whose altitude is $19\frac{1}{2}$ rods?

6. A sphere 9 inches in diameter was gilded for \$3; what would it cost to gild a sphere 15 inches in diameter?

7. A certain side of a field of $2\frac{1}{4}$ acres was 9 rods, what was the corresponding side of a similar field of $6\frac{1}{4}$ acres?

SIMILAR SOLIDS.

701. Similar solids are such as have the same form, and their corresponding dimensions proportional.

Principles.—I. *The volumes of similar solids are proportional to the cubes of their corresponding dimensions.*

II. *The dimensions of similar solids are proportional to the cube roots of their volumes.*

NOTE.—Similar solids vary in weight, value, and many other respects, in the same proportion as they vary in volume.

WRITTEN PROBLEMS.

1. If a cube of marble 2 feet on each side weighs 7 tons, what is the weight of a cube of marble 3 feet on each side?

Solution.—$2^3 : 3^3 :: 7\text{ T.} : x\text{ T.} = 23\frac{5}{8}$ tons.

2. The Winchester bushel is a cylinder $18\frac{1}{2}$ inches in diameter and 8 inches deep; what must be the diameter and the depth of a similar cylinder that will hold $3\frac{3}{8}$ bushels?

Solution.— $\sqrt[3]{1} : \sqrt[3]{3\frac{3}{8}} :: 18\frac{1}{2} : x$,

or $1 : \frac{3}{2} :: 18\frac{1}{2} : x = 27\frac{3}{4}$ in., the diameter.

Again, $\sqrt[3]{1} : \sqrt[3]{3\frac{3}{8}} :: 8 : x$,

or $1 : \frac{3}{2} :: 8 : x = 12$ in., the depth.

3. If a man 5 feet 6 inches high weighs 160 pounds, what is the weight of a man of similar build 6 feet high?

4. If a grindstone 2 feet in diameter is worth \$$3\frac{3}{4}$, what is the value of another grindstone 4 feet in diameter, and thick in proportion?

5. If a log 16 feet long is worth \$3.20, at that rate what is the value of a log 20 feet long and thick in proportion?

6. A stack of hay 12 feet high is worth \$54; what is a stack 20 feet high worth, the stacks having their corresponding dimensions proportional, and the hay being of the same quality?

7. Two steamers built after the same model were 90 and 120 feet long respectively; the former could carry 540 tons of freight; how many tons could the latter carry?

8. What will be the weight of a cube 8 inches high, if a cube 3 inches high, of the same density, weighs $6\frac{3}{4}$ pounds?

MISCELLANEOUS PROBLEMS IN MENSURATION.

702. *1.* The diagonal of a room is 35 feet and its width 21 feet; what is the area of the room?

2. A rectangular field 120 rods long contains 90 acres; what is the length of its diagonal?

3. The foot of a ladder 50 feet long is placed 15 feet from the base of a building; how high will it reach?

4. A cylindrical cistern is 12 feet deep and 8 feet in diameter; how many hogsheads of 63 gallons each will it hold?

5. Mr. Brown lives $4\frac{1}{2}$ miles due west from a certain town, and Mr. Felton $5\frac{1}{2}$ miles due south from the same town; how far apart, in a straight line, do they live?

6. Find the surface of a sphere $3\frac{1}{2}$ feet in diameter.

7. Find the volume of a cone whose altitude is 10 feet and the radius of its base 3 feet.

8. The altitude of a pyramid is 12 feet, and its base is 8 feet square; what is its volume?

9. The parallel sides of a field in the form of a trapezoid are 35 and 45 rods, and its altitude 30 rods; what is it worth at $175 an acre?

10. How many acres in a circular island 160 rods in diameter?

11. A man converted a plot of ground 200 feet square into city lots 25×100 feet; how many were there?

12. How many spheres 5 inches in diameter are equal in volume to a sphere 5 feet in diameter?

13. How many bushels in the capacity of a bin 6 feet deep, 8 feet wide and 12 feet long?

14. How many cubic feet in a log 18 inches in diameter and 20 feet long?

15. A trapezium ABCD has the following dimensions: AB 45 feet, BC 10 feet, CD 48 feet, DA 40 feet and BD 50 feet; what is its area?

16. How many bricks 4 by 8 inches will it take to pave a walk 12 feet wide and 200 feet long?

17. How many times will the driving-wheel of a locomotive turn in a trip of 48 miles, the wheel being 6 feet in diameter?

18. Around the outside of a rectangular garden 100 feet long and 60 feet wide is a walk 12 feet wide; what is its area?

19. Around the inside of an enclosure 8 rods square is a drive of uniform width whose area is equal to $\frac{7}{16}$ of the area of the enclosure; what is the width of the drive?

20. How long must a fence be to enclose a square field of 40 acres?

21. The entire surface of a cube is 1176 square feet; what is the volume of the cube?

22. A room is 18 by 24 feet, and 12 feet high; how far from one corner below to the corner diagonally opposite above?

23. If oranges 2 inches in diameter are worth 75 cents a dozen, how much are oranges $2\frac{1}{2}$ inches in diameter worth?

24. How many cubic feet in the walls of a building 60 feet long, 40 feet wide and 30 feet high, the walls being 16 inches thick and there being no allowance for doors or windows?

REVIEW QUESTIONS.

703. What is ratio? How is it illustrated? What are the terms? What is the antecedent? The consequent? What is a couplet? A simple ratio? A compound ratio? Give the principles.

Define proportion. Antecedents. Consequents. Means. Extremes. What is a mean proportional? Give the principles.

What is a simple proportion? Give the rule for finding a missing term.

What is a compound proportion? Give the rule. Explain the cause and effect method of solving examples in proportion.

What is partnership? Capital or stock? Assets? Liabilities? Illustrate the method of apportioning gains and losses.

Define a power. A perfect power. A perfect square. A perfect cube. What is involution? Give the principle.

What is a root? The square root of a number? Cube root? Define evolution. Radical sign. Index. How many figures in the square root of a number? Select a number expressing tens and units, and analyze its square. Give the rule for square root. How many figures in the cube root of a number? Analyze the cube of a number expressing tens and units. Give the rule for cube root.

Define mensuration. Parallel lines. Acute angle. Obtuse angle. What is a polygon? A regular polygon? The perimeter of a polygon? Diagonal of a polygon? Give names of particular polygons. What is a right-angled triangle? Give the rule for finding any side of a right-angled triangle. What is the altitude of a triangle? Give rule for the area of a triangle when the base and altitude are given. When the three sides are given.

Define the various quadrilaterals. What is the rule for finding the area of a parallelogram? Of a trapezoid? Of a trapezium whose sides and a diagonal are given? Give the rule for the other case.

How is the circumference of a circle found? The area?

What is a prism? Mention particular kinds. Give the rule for the convex and the entire surface of a prism. For the volume. Define a cylinder. Give the rule for finding the convex surface of a prism. For finding the volume.

What is a pyramid? The slant height? The altitude? What is the rule for finding the convex surface? The volume?

Define a cone. Give rule for finding the convex surface. The volume.

What is a sphere? How is its surface found? Its volume?

What is gauging? Give rule. Define similar surfaces. Give the principles. What are similar solids? Give the principles.

APPENDIX.

704. The names of periods beyond Quadrillions are: 7. Quintillions; 8. Sextillions; 9. Septillions; 10. Octillions; 11. Nonillions; 12. Decillions; 13. Undecillions; 14. Duodecillions; 15. Tredecillions; 16. Quatuordecillions; 17. Quindecillions; 18. Sexdecillions; 19. Septendecillions; 20. Octodecillions; 21. Novemdecillions; 22. Vigintillions; etc.

705. ADDITION TABLE.

1+1= 2	1+2= 3	1+3= 4	1+4= 5	1+ 5= 6
2+1= 3	2+2= 4	2+3= 5	2+4= 6	2+ 5= 7
3+1= 4	3+2= 5	3+3= 6	3+4= 7	3+ 5= 8
4+1= 5	4+2= 6	4+3= 7	4+4= 8	4+ 5= 9
5+1= 6	5+2= 7	5+3= 8	5+4= 9	5+ 5=10
6+1= 7	6+2= 8	6+3= 9	6+4=10	6+ 5=11
7+1= 8	7+2= 9	7+3=10	7+4=11	7+ 5=12
8+1= 9	8+2=10	8+3=11	8+4=12	8+ 5=13
9+1=10	9+2=11	9+3=12	9+4=13	9+ 5=14
1+6= 7	1+7= 8	1+8= 9	1+9=10	1+10=11
2+6= 8	2+7= 9	2+8=10	2+9=11	2+10=12
3+6= 9	3+7=10	3+8=11	3+9=12	3+10=13
4+6=10	4+7=11	4+8=12	4+9=13	4+10=14
5+6=11	5+7=12	5+8=13	5+9=14	5+10=15
6+6=12	6+7=13	6+8=14	6+9=15	6+10=16
7+6=13	7+7=14	7+8=15	7+9=16	7+10=17
8+6=14	8+7=15	8+8=16	8+9=17	8+10=18
9+6=15	9+7=16	9+8=17	9+9=18	9+10=19

706. SUBTRACTION TABLE.

1−1= 0	2−2= 0	3−3= 0	4−4= 0	5− 5= 0
2−1= 1	3−2= 1	4−3= 1	5−4= 1	6− 5= 1
3−1= 2	4−2= 2	5−3= 2	6−4= 2	7− 5= 2
4−1= 3	5−2= 3	6−3= 3	7−4= 3	8− 5= 3
5−1= 4	6−2= 4	7−3= 4	8−4= 4	9− 5= 4
6−1= 5	7−2= 5	8−3= 5	9−4= 5	10− 5= 5
7−1= 6	8−2= 6	9−3= 6	10−4= 6	11− 5= 6
8−1= 7	9−2= 7	10−3= 7	11−4= 7	12− 5= 7
9−1= 8	10−2= 8	11−3= 8	12−4= 8	13− 5= 8
10−1= 9	11−2= 9	12−3= 9	13−4= 9	14− 5= 9
11−1=10	12−2=10	13−3=10	14−4=10	15− 5=10

6−6= 0	7−7= 0	8−8= 0	9−9= 0	10−10= 0
7−6= 1	8−7= 1	9−8= 1	10−9= 1	11−10= 1
8−6= 2	9−7= 2	10−8= 2	11−9= 2	12−10= 2
9−6= 3	10−7= 3	11−8= 3	12−9= 3	13−10= 3
10−6= 4	11−7= 4	12−8= 4	13−9= 4	14−10= 4
11−6= 5	12−7= 5	13−8= 5	14−9= 5	15−10= 5
12−6= 6	13−7= 6	14−8= 6	15−9= 6	16−10= 6
13−6= 7	14−7= 7	15−8= 7	16−9= 7	17−10= 7
14−6= 8	15−7= 8	16−8= 8	17−9= 8	18−10= 8
15−6= 9	16−7= 9	17−8= 9	18−9= 9	19−10= 9
16−6=10	17−7=10	18−8=10	19−9=10	20−10=10

707. Many teachers prefer the method of subtraction in which III. of the rule reads:

III. *When any figure of the minuend is less than that of the same order in the subtrahend, add* 10 *to it before subtracting; and then consider the next figure of the subtrahend as* **increased** *by* 1.

The principle upon which the explanation of the preceding

rule depends is: *If the same number be added to both minuend and subtrahend, the result is not changed.*

Thus, in subtracting 56 from 183, the operation of adding 10 may be shown as follows:

$$\left.\begin{array}{r}183\\ \underline{56}\end{array}\right\} = \left\{\begin{array}{r}(180+3)+10\\ (50+6)+10\end{array}\right\} = \left\{\begin{array}{r}180+13\\ \underline{60+\ \ 6}\end{array}\right.$$
$$\begin{array}{r}127\end{array} \qquad\qquad\qquad\qquad 120+\ \ 7 = 127.$$

Adding 10 units to the minuend makes the remainder greater by 10 than it would otherwise be; adding 1 ten to the subtrahend corrects the error.

708. MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50
6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9	10 × 1 = 10
6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18	10 × 2 = 20
6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27	10 × 3 = 30
6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36	10 × 4 = 40
6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45	10 × 5 = 50
6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54	10 × 6 = 60
6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63	10 × 7 = 70
6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72	10 × 8 = 80
6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81	10 × 9 = 90
6 × 10 = 60	7 × 10 = 70	8 × 10 = 80	9 × 10 = 90	10 × 10 = 100

709. The word *and,* except when it takes the place of the decimal point, is omitted in reading numbers, in order to prevent ambiguity in dictating mixed decimal numbers.

Thus, were *and* used indiscriminately, *five hundred and five thousandths* might mean .505, or 500.005. When it is understood that *and* always takes the place of the decimal point, no such ambiguity can occur.

GREATEST COMMON DIVISOR BY DIVISION.

710. Principles.—I. *A common divisor of two numbers is also a divisor of their sum, of their difference, and of any multiple of either of the numbers.*

II. *A common divisor of two numbers is also a divisor of the sum of any multiples of the numbers.*

Thus, 6 being a divisor of 30 and 18, is a divisor of $30+18$, $30-18$, 30×2, 18×3, $30+(18\times2)$, $(30\times3)+18$, $(30\times3)+(18\times5)$, etc.

711. I. To find the greatest common divisor of 552 and 744.

Process.

```
552 ) 744 ( 1
      552
      ---
      192 ) 552 ( 2
            384
            ---
            168 ) 192 ( 1
                  168
                  ---
                   24 ) 168 ( 7
                        168
                        ---
```

Explanation.—The greatest common divisor of 552 and 744, whatever it is, must divide their difference, 192. It will divide also 2 times 192, or 384. Since it will divide 552 and 384, it will divide their difference, 168. Since it will divide 192 and 168, it will divide their difference, or 24. Hence, *it cannot be greater than* 24. But 24 is a divisor of 24 and 168; hence, the greatest common divisor of 552 and 744 *cannot be less than* 24. Since it cannot be *greater* than 24 or *less* than 24, it must be *equal to* 24.

1. Find the greatest common divisor of 288 and 648. Of 384 and 1296. Of 1778 and 1722. Of 3672 and 7992.

GREATEST COMMON DIVISOR OF FRACTIONS.

712. The **greatest common divisor** of two or more fractions or mixed numbers is the greatest divisor that will divide each of them and give a whole number for a quotient.

I. Required to find the greatest common divisor of $\frac{5}{8}$, $3\frac{1}{3}$ and $3\frac{3}{4}$.

Process.

$$3\tfrac{1}{3} = \tfrac{10}{3};\quad 3\tfrac{3}{4} = \tfrac{15}{4}.$$

$$\text{G. C. D. of } \tfrac{5}{8}, \tfrac{10}{3}, \tfrac{15}{4} = \tfrac{5}{24}.$$

$$\frac{\text{G. C. D. of } 5,\ 10,\ 15 = 5.}{\text{L. C. M. of } 8,\ 3,\ 4 = 24.}$$

Analysis.

$$\tfrac{5}{8}, \tfrac{10}{3}, \tfrac{15}{4} = \tfrac{15}{24}, \tfrac{80}{24}, \tfrac{90}{24}.$$

$$\text{G. C. D. of } \frac{15, 80, 90}{24} = \frac{5}{24}.$$

Explanation. — The mixed numbers are first reduced to improper fractions, and then all the fractions are reduced to equivalent fractions having the least common denominator. The greatest common divisor of these fractions is evidently a fraction whose denominator is 24, and whose numerator is the greatest common divisor of 15, 80 and 90. This divisor is 5, which written over 24 gives $\frac{5}{24}$ as the greatest common divisor of $\frac{5}{8}$, $3\frac{1}{3}$ and $3\frac{3}{4}$.

The same result is obtained by writing the greatest common divisor of 5, 10 and 15, the numerators, over the least common multiple of 8, 3 and 4, the denominators.

713. Rule.—I. *Reduce mixed numbers to improper fractions.*

II. *Find the greatest common divisor of the numerators, and write it over the least common multiple of the denominators. The result will be the greatest common divisor of the fractions.*

714. ***Find the greatest common divisor of the following:***

1. $\frac{3}{4}, \frac{5}{6}$.	*5.* $2\frac{4}{5}, 5\frac{1}{4}$.	*9.* $\frac{3}{4}, \frac{5}{6}, \frac{2}{3}$.	*13.* $3\frac{3}{4}, 2\frac{4}{7}, 13\frac{1}{2}$.
2. $\frac{8}{9}, \frac{6}{7}$.	*6.* $3\frac{3}{4}, 4\frac{1}{6}$.	*10.* $\frac{8}{9}, \frac{4}{5}, \frac{2}{15}$.	*14.* $2\frac{4}{5}, 10\frac{1}{2}, 12\frac{1}{4}$
3. $\frac{4}{5}, \frac{6}{7}$.	*7.* $\frac{8}{9}, 2\frac{2}{5}$.	*11.* $\frac{4}{5}, \frac{2}{3}, \frac{7}{15}$.	*15.* $7\frac{1}{3}, 4\frac{5}{7}, 3\frac{13}{14}$.
4. $\frac{3}{5}, \frac{9}{10}$.	*8.* $\frac{7}{8}, 5\frac{5}{6}$.	*12.* $\frac{5}{6}, \frac{5}{9}, \frac{10}{27}$.	*16.* $2\frac{1}{4}, 3\frac{3}{8}, 4\frac{1}{2}$.

715. *1.* I wish to order demijohns of the same size that will exactly contain, without mixing, two barrels of wine, one containing $46\frac{3}{8}$ gal., the other $39\frac{3}{8}$ gal. How many of the largest demijohns that will answer the requirement shall I order?

2. A baker at one time made a certain number of loaves of equal weight, using $28\frac{11}{16}$ lb. of flour, and at another time he made a different number of loaves like the first in weight, and used $46\frac{1}{8}$ lb. of flour. How much flour was used for each loaf, the loaves being the largest possible?

3. A merchant wishes to cut, without waste, two pieces of silk, containing $281\frac{1}{4}$ yards and $412\frac{1}{2}$ yards respectively, into the longest equal dress patterns possible; how many yards must each dress pattern contain?

4. My grocer sold me what seemed, as weighed by his balances, a certain whole number of pounds of sugar, and a certain whole number of pounds of coffee. On reaching home, I weighed them on balances known to be correct, and found that the sugar weighed $32\frac{13}{16}$ lb. and the coffee $43\frac{1}{8}$ lb. Find the deficiency of weight on each pound, supposing my loss to be the least possible.

5. What is the least number of kegs of equal size in which I may put, without mixing, three hogsheads of different kinds of syrup, the first hogshead containing $68\frac{1}{4}$ gallons, the second $78\frac{3}{4}$ gallons, and the third $120\frac{3}{4}$ gallons?

6. I have a vessel which when filled with water a certain number of times and emptied into a cistern having a capacity of $95\frac{5}{8}$ gallons will exactly fill it; in like manner it will exactly fill a cistern holding $123\frac{3}{4}$ gallons, or one holding $292\frac{1}{2}$ gallons. What is the largest possible size of the vessel to answer the conditions?

7. How much more than \$3 per day must a man receive in order that he may pay a bill of $\$42\frac{9}{10}$, of $\$70\frac{1}{5}$, or of $\$97\frac{1}{2}$, and use in each case the entire pay for a whole number of days?

LEAST COMMON MULTIPLE OF FRACTIONS.

716. The **least common multiple** of two or more fractions or mixed numbers is the least number that will contain each of them a whole number of times.

I. Required the least common multiple of $\frac{5}{8}$, $3\frac{1}{3}$ and $3\frac{3}{4}$.

Process.

$$3\tfrac{1}{3} = \tfrac{10}{3};\quad 3\tfrac{3}{4} = \tfrac{15}{4}.$$

$$\text{L. C. M. of } \tfrac{5}{8}, \tfrac{10}{3}, \tfrac{15}{4} = 30.$$

$$\frac{\text{L. C. M. of } 5,\ 10,\ 15 = 30}{\text{G. C. D. of } 8,\ 3,\ 4 = 1} = 30.$$

Analysis.

$$\tfrac{5}{8}, \tfrac{10}{3}, \tfrac{15}{4} = \tfrac{15}{24}, \tfrac{80}{24}, \tfrac{90}{24}.$$

$$\text{L. C. M. of } \frac{15, 80, 90}{24} = \frac{720}{24} = 30$$

Explanation. — In the analysis the mixed numbers are first reduced to improper fractions, and then all the fractions are reduced to equivalent fractions having the least common denominator. The least common multiple of these fractions is evidently a fraction whose denominator is 24, and whose numerator is the least common multiple of 15, 80 and 90. This multiple is 720, which written over 24 gives $\frac{720}{24}$, equal to 30, for the least common multiple.

The same result is obtained by writing the least common multiple of 5, 10 and 15, the numerators, over the greatest common divisor of 8, 3 and 4, the denominators.

717. Rule.—I. *Reduce mixed numbers to improper fractions.*

II. *Find the least common multiple of the numerators, and write it over the greatest common divisor of the denominators. The result will be the least common multiple of the fractions.*

718. ***Find the least common multiple of the following:***

1. $\frac{3}{5}, \frac{9}{10}$.	*5.* $4\frac{1}{2}, 3\frac{3}{5}$.	*9.* $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$.	*13.* $1\frac{1}{7}, 2\frac{2}{5}, 6\frac{2}{3}$.
2. $\frac{2}{3}, \frac{4}{5}$.	*6.* $5\frac{5}{6}, 8\frac{1}{3}$.	*10.* $\frac{5}{9}, \frac{7}{12}, \frac{14}{15}$.	*14.* $2\frac{3}{4}, 3\frac{5}{8}, 5\frac{7}{12}$.
3. $\frac{8}{9}, \frac{3}{4}$.	*7.* $2\frac{5}{8}, 9\frac{4}{5}$.	*11.* $\frac{5}{6}, \frac{7}{8}, \frac{2}{3}$.	*15.* $5\frac{4}{5}, 8\frac{9}{10}, 9\frac{7}{15}$.
4. $\frac{7}{8}, \frac{9}{10}$.	*8.* $4\frac{3}{8}, 6\frac{5}{12}$.	*12.* $\frac{1}{2}, \frac{8}{15}, \frac{2}{3}$.	*16.* $4\frac{4}{7}, 5\frac{11}{14}, 6\frac{17}{21}$.

719. *1.* A can walk $3\frac{2}{3}$ miles per hour, and B $3\frac{4}{5}$ miles per hour; what is the least distance that will require either a whole number of hours to walk it?

2. I pay \$$5\frac{5}{8}$ per week for board, and my brother pays \$$6\frac{3}{4}$; what is the smallest sum of money that will pay the board of either of us for a whole number of weeks?

3. I can buy pears at $1\frac{3}{4}$ cents each and oranges at $2\frac{2}{3}$ cents each; what is the least number of either that I can buy with the same sum of money?

4. A tailor buys three pieces of cloth of equal length, but the shortest possible, so that, without waste, he may make from the first suits of $5\frac{1}{2}$ yds., from the second, suits of $6\frac{7}{8}$ yds., and from the third, suits of $7\frac{1}{3}$ yds. What is their length?

5. A farmer finds that bags having a capacity of $2\frac{1}{8}$ bu., $2\frac{1}{2}$ bu. or $3\frac{3}{4}$ bu. will exactly hold the wheat in one of his bins; find its least contents to answer the requirement.

CIRCULATING DECIMALS.

720. A **circulating** or **repeating** decimal is a decimal in which a figure or set of figures is constantly repeated in the same order.

Thus, .333+, .3636+, are circulating decimals.

721. A **repetend** is the figure or set of figures repeated.

A repetend is written but once; when it consists of one figure, a dot is placed above it to show that it is a repetend; when it consists of two or more figures, a dot is placed above the first and the last figure.

Thus, $.\dot{3}$, $.\dot{5}7142\dot{8}$, $.2\dot{7}\dot{3}$, etc.

722. A **pure circulating decimal** is a decimal that consists only of a repetend.

Thus, $.\dot{4}\dot{5}$, $.\dot{1}3\dot{5}$, are pure circulating decimals.

723. **A mixed circulating decimal** is a decimal that consists of an unrepeating or *finite* part and a repetend.

Thus, $.2\dot{7}\dot{3}$, $.54\dot{1}7\dot{3}$, are mixed circulating decimals.

724. **Similar repetends** are repetends that begin at the same decimal place.

Thus, $.\dot{5}$ and $.\dot{1}\dot{7}$; $.8\dot{2}\dot{3}$ and $.5\dot{6}5\dot{3}$, are similar repetends.

725. **Conterminous repetends** are repetends that end at the same decimal place.

Thus, $.51\dot{7}$ and $.\dot{3}1\dot{3}$, $.\dot{3}28\dot{7}$ and $.51\dot{6}\dot{7}$, are conterminous repetends.

726. To reduce a pure circulating decimal to a common fraction.

I. Required to reduce $.\dot{2}\dot{7}$ to a common fraction.

Explanation.—Since $.\dot{2}\dot{7} = .\dot{2}\dot{7}\dot{2}\dot{7}$, 100 times $\dot{2}\dot{7}$ is found by moving the decimal point two places to the right, giving $27.\dot{2}\dot{7}$. Subtracting $.\dot{2}\dot{7}$ from $27.\dot{2}\dot{7}$ leaves 27, which is 99 times $.\dot{2}\dot{7}$. The value of the repetend, $.\dot{2}\dot{7}$, is, therefore, equal to $\frac{1}{99}$ of 27, or $\frac{27}{99}$, equal to $\frac{3}{11}$.

Process.

$$
\begin{aligned}
.\dot{2}\dot{7} \times 100 &= 27.\dot{2}\dot{7} \\
.\dot{2}\dot{7} \times 1 &= .\dot{2}\dot{7} \\
\hline
.\dot{2}\dot{7} \times 99 &= 27
\end{aligned}
$$

$$.\dot{2}\dot{7} = \tfrac{27}{99} = \tfrac{3}{11}.$$

Rule.—*Write the repetend, with the dots and decimal point omitted, as the numerator of a fraction whose denominator consists of as many 9's as there are figures in the repetend. Reduce the result to simplest form.*

727. *1.* Express as common fractions: $.\dot{6}$, $.\dot{3}$, $.\dot{2}7\dot{9}$, $.\dot{5}7142\dot{8}$.

2. Reduce to common fractions in their simplest form, $.\dot{3}1\dot{5}$, $.\dot{1}72\dot{8}$, $.\dot{4}5\dot{9}$, $.\dot{1}8\dot{7}$, $.\dot{4}9\dot{5}$.

3. What common fractions are equal to $.\dot{3}\dot{9}$, $.\dot{8}\dot{1}$, $.\dot{3}5\dot{1}$, $.\dot{4}18\dot{5}$ and $.\dot{3}1418\dot{1}$?

728. To reduce a mixed circulating decimal to a common fraction.

I. Required to reduce $.2\dot{4}\dot{5}$ to a common fraction.

Explanation.—Multiplying the decimal by 1000 gives $245.\dot{4}\dot{5}$, and by 10 gives $2.\dot{4}\dot{5}$. Subtracting the latter product from the former gives 990 times the decimal, equal to 243. Hence the decimal equals $\frac{1}{990}$ of 243, or $\frac{243}{990}$, equal to $\frac{27}{110}$.

Process.

$$.2\dot{4}\dot{5} \times 1000 = 245.\dot{4}\dot{5}$$
$$.2\dot{4}\dot{5} \times \quad 10 = \quad 2.\dot{4}\dot{5}$$
$$.2\dot{4}\dot{5} \times \quad 990 = 243$$
$$.2\dot{4}\dot{5} = \tfrac{243}{990} = \tfrac{27}{110}.$$

Rule.—*Subtract the finite part from the entire decimal; write the remainder as the numerator of a fraction whose denominator consists of as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part. Reduce the resulting fraction to its simplest form.*

729. *1.* Reduce to common fractions in their simplest form: $.3\dot{3}\dot{6}$, $.45\dot{4}\dot{5}$, $.6\dot{3}\dot{6}$, $.83\dot{8}$, $.125\dot{2}2\dot{5}$, $.7\dot{4}\dot{7}$.

2. Reduce to improper fractions: $2.2\dot{7}$, $4.2\dot{3}$, $1.8\dot{1}$, $1.2\dot{3}\dot{5}$.

3. Express as common fractions, $2.3\dot{3}\dot{9}$, $5.7\dot{4}$, $8.12\dot{6}\dot{3}$, $.32\dot{6}\dot{3}$ and $.18\dot{9}$.

730. To make two or more repetends similar and conterminous.

I. Required to make $.\dot{5}$, $.\dot{3}4\dot{3}$ and $.42\dot{5}3\dot{2}$ similar and conterminous.

Explanation.—In the given decimals, the repetends begin at different decimal places. To be similar they must be made to begin at the same decimal place. The repetend of the last begins at the thousandths' place. The repetends of the other two are made to begin at the thousandths' place by moving the point to the right.

Process.

$$.\dot{5} = .55\dot{5}555\dot{5}$$
$$.\dot{3}4\dot{3} = .34\dot{3}434\dot{4}$$
$$.42\dot{5}3\dot{2} = .42\dot{5}3253\dot{2}$$

They are made conterminous by writing the repetend $\dot{5}3\dot{2}$ twice, $\dot{3}\dot{4}$ three times, and $.\dot{5}$ six times, and placing the second point over the sixth figure beyond the first point.

The correctness of the process depends upon the fact that $.\dot{5}$ extended

indefinitely gives the same result as $\dot{5}5555\dot{5}$ extended indefinitely, $.8\dot{4}\dot{3}$ extended indefinitely gives the same result as $.34\dot{3}4343\dot{4}$ extended indefinitely ; etc.

Rule.—I. *Extend the repetends, and make the repetends all begin at the same decimal place.*

II. *Make the repetends each contain as many figures as the least common multiple of the number of places in the several given repetends.*

731. *1.* Make $.\dot{5}\dot{3}$, $.7\dot{5}\dot{9}$, $.\dot{5}6\dot{8}$ and $.4\dot{8}1\dot{9}$ similar and conterminous.

2. Make $.\dot{5}6\dot{1}$, $.8\dot{1}\dot{9}$ and $.\dot{7}\dot{3}$ similar and conterminous.

3. Make $.\dot{3}$, $.\dot{5}$, $.\dot{1}\dot{9}$ and $.5\dot{3}21\dot{7}$ similar and conterminous.

4. Make $.3\dot{1}7\dot{9}$, $.43\dot{8}1246\dot{7}$ and $.1032\dot{4}\dot{7}$ similar and conterminous.

ADDITION AND SUBTRACTION.

732. Circulating decimals may be added and subtracted by making them similar and conterminous, and then proceeding as in ordinary decimals.

I. Required to find the sum of $.\dot{3}\dot{4}$, $.4\dot{6}\dot{7}$, $5.\dot{1}3\dot{7}$ and $25.12\dot{8}\dot{3}$.

Explanation.—The decimals are first made similar and conterminous, and written with like decimal orders in the same columns. If the decimals were extended farther, the next column to the right would be the same as the thousandths' column, and its sum would be 26. Hence, there is 2 to carry to the right-hand column, making its sum 18.

Process.

$$
\begin{array}{rcr}
.\dot{3}\dot{4} & = & .34\dot{3}4343\dot{4} \\
.4\dot{6}\dot{7} & = & .46\dot{7}6767\dot{6} \\
5.\dot{1}3\dot{7} & = & 5.13\dot{7}1371\dot{3} \\
25.12\dot{8}\dot{3} & = & 25.12\dot{8}3838\dot{3} \\
\hline
 & & 31.07\dot{6}6320\dot{8}
\end{array}
$$

The repetend in the result is similar and conterminous with the repetends added.

II. Required to find the difference between $5.1\dot{3}\dot{2}$ and $3.\dot{4}57\dot{3}$

Explanation. — If the decimals were extended farther, the next column to the right would be the same as the hundredths' column. Hence, there is 1 to carry to the right-hand column, after which the subtraction is performed as in finite decimals.

Process.

$$
\begin{array}{rcr}
5.1\dot{3}\dot{2} & = & 5.1\dot{3}23\dot{2} \\
3.\dot{4}57\dot{3} & = & 3.\dot{4}573\dot{4} \\
\hline
 & & 1.6\dot{7}49\dot{7}
\end{array}
$$

733. *1.* Find the sum of $75.\dot{7}\dot{5}$, $83.\dot{8}3\dot{1}$, $7.\dot{0}1\dot{5}$ and $32.\dot{7}$.

2. From the sum of $38.\dot{3}1\dot{9}$ and $47.4\dot{7}\dot{3}$ take the sum of $19.\dot{1}\dot{9}$ and $23.\dot{2}3\dot{1}$.

3. Find the value of $3.\dot{8}19\dot{3}+47.\dot{4}\dot{7}-16.\dot{7}+8.0\dot{1}\dot{3}$.

4. Find the value of $.\dot{4}7\dot{3}+.\dot{6}18\dot{1}+8.\dot{2}1631\dot{7}-5.\dot{1}\dot{7}+11.\dot{1}$.

5. Find the value of $5.5\dot{1}+7.7\dot{3}\dot{1}-1.\dot{8}\dot{3}-.\dot{1}7\dot{3}$.

734. Circulating decimals are multiplied and divided by first reducing them to common fractions, performing the operations required, and then reducing the result to decimal form.

Thus, $.\dot{5}\times.3\dot{5}=\frac{5}{9}\times\frac{32}{90}=\frac{16}{81}=.\dot{1}9753086\dot{4}$.

Also, $.\dot{5}\div.\dot{1}\dot{3}=\frac{5}{9}\div\frac{13}{99}=\frac{5}{\cancel{9}}\times\frac{\overset{11}{\cancel{99}}}{13}=\frac{55}{13}=4\frac{3}{13}=4.\dot{2}3076\dot{9}$.

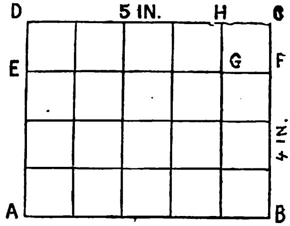
ANALYSIS OF REDUCTION.

735. Strictly, it is not correct to say that 15 *bushels* multiplied by 4 gives 60 *pecks*. The logical explanation is: *Since in* 1 *bushel there are* 4 *pecks, in* 15 *bushels there are* 15 *times* 4 *pecks, equal to* 60 *pecks.*

Good usage, however, permits the explanation given in this book. The correct analysis should be given by the teacher in explaining the rule, and also by the pupils when required.

AREAS OF RECTANGLES.

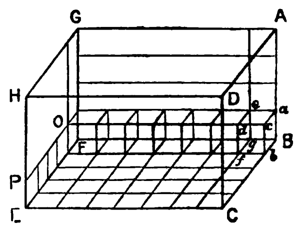
736. The rule for finding the area of a surface whose length and width are given is likely to mislead, unless carefully explained. The pupil is directed to multiply the length by the width. Strictly, however, feet cannot be multiplied by feet, or inches by inches, any more than dollars can be multiplied by dollars, or pounds by pounds. In every product of two factors, one of them, at least, must be abstract. The rule is developed as follows:



The rectangle ABCD is represented as 5 in. long and 4 in. wide. Lines drawn as in the figure will divide it into equal squares like FGHC, each equal to 1 square inch. In the row CDEF there are 5 such squares, equal to 5 square inches, and the figure consists of 4 such rows. Hence, the area of the entire figure is equal to *5 sq. in* × *4*. (not by 4 inches), equal to *20 sq. in.*

VOLUMES OF RECTANGULAR SOLIDS.

737. The figure in the margin represents a solid 4 inches high, 7 inches long and 5 inches wide. It may, therefore, be separated into 4 layers, each 1 inch deep, 7 inches long and 5 inches wide. Each one of these layers may be divided into 5 rows, each consisting of 7 cubes 1 inch each way.



In one layer there will be *7 cu. in.* × *5*, or *35 cu. in.;* and in the 4 layers, or what is the same, in the entire solid, there will be *7 cu. in.* × *5* × *4*, equal to *140 cu. in.*

MERIDIAN AND STANDARD TIME.

738. A watch that shows correct time at any given point, is 4 minutes slow at a point one degree east, and 4 minutes fast at a point one degree west.

To remedy this difficulty, the principal railroads of the United States and Canada, in 1883, adopted the time of four different meridians, 15° apart, as the standard time for four sections embracing the entire width of both countries. The most eastern of these is the meridian of 75° west of Greenwich; the others being the meridians of 90°, 105° and 120°.

All places between the Atlantic coast and a line extending from Detroit through Pittsburg, Wheeling, Parkersburg, Augusta and Charleston have the time of the 75th meridian, called Eastern Time.

All places between the western boundary of the Eastern Time section and a line from Bismarck through North Platte, Dodge City, etc., to the City of Mexico have the time of the 90th meridian, called Central Time, one hour earlier than Eastern Time.

Roughly, the Rocky Mountain region has the time of the 105th meridian, 2 hours earlier than Eastern Time, and called Mountain Time; and the Pacific slope has the time of the 120th meridian, 3 hours earlier than Eastern Time, and called Pacific Time.

GOVERNMENT LANDS.

739. In the survey of government land, a line called the **principal meridian** is run north and south, and another line called the **base line** is run east and west.

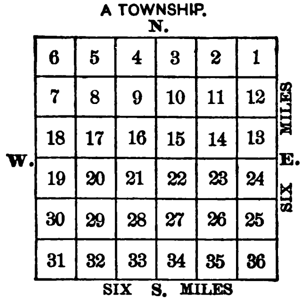
740. Parallel to these two lines, other lines are run so as to form squares six miles on a side. These squares are

called **townships.** Each township contains 36 square miles, or 23040 acres.

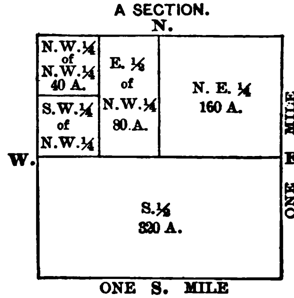
741. Each township is divided into 36 equal squares, called **sections.** A section is 1 mile square, and contains 640 acres. The sections of a township are designated as shown in the diagram, Section No. 1 being in the north-eastern corner of the township, Section No. 31 being in the south-western corner, etc.

742. The sections are themselves divided and subdivided as shown in the diagram.

A TOWNSHIP.



A SECTION.



743. *1.* How many farms of 72 acres each in a township of government land?

2. What will be the cost of the S. E. ¼ of a section of land at $2.25 an acre?

3. I bought the W. ½ of the N. W. ¼ of a section of land, and the S. ½ of the W. ½ of the same section at $1.75 an acre; what did it cost me?

4. What will it cost at $.75 a rod to fence the N. W. ¼ of the N. W. ¼ of a section of land?

5. Locate by diagram the E. ½ of the S. E. ¼ of a section of land, and find its cost at $1.25 an acre.

6. A man bought the E. ½ of the S. ½ of a section of land at $2.25 an acre and sold it at $5.75 an acre; what did he gain?

DUODECIMALS.

744. Duodecimals is a system of numbers whose scale is 12. Its principal application is by artificers in estimating areas and volumes.

745. The foot is the principal unit. It is divided and sub divided according to the following

TABLE.

12 fourths ($''''$)	=	1 third,	written	$1'''$.
12 thirds	=	1 second,	"	$1''$.
12 seconds	=	1 inch, or prime,	"	$1'$.
12 primes	=	1 foot,	"	1 ft.

MULTIPLICATION OF DUODECIMALS.

746. *1.* What is the area of a floor 15 ft. $8'$ by 13 ft. $5'$?

Explanation.—Beginning at the right, $8'$ multiplied by $5'$ equals $40''$, or $3'$ $4''$. The $4''$ is written one place to the right and the $3'$ added to the product of 15 ft. by $5'$, giving $78'$, equal to 6 sq. ft. $6'$, which is written as in the margin.

The product by the other term of the multiplier is 203 sq. ft. $8'$. The addition of the partial products is obvious.

Process.

15 ft.	$8'$	
13 ft.	$5'$	
6 sq. ft.	$6'$	$4''$
203 sq. ft.	$8'$	
210 sq. ft.	$2'$	$4''$

2. Multiply 9 ft. $6'$ by 3 ft. $5'$.

3. A rectangular bin is 9 ft. $6'$ long, 4 ft. $8'$ wide, and 3 ft $4'$ deep; find its solid contents.

4. A block of granite is 5 ft. $10'$ square, and 4 ft. $8'$ thick; what is its volume?

5. How many square feet in a mahogany board 14 ft. 8 long by 1 ft. $2'$ wide?

MISCELLANEOUS TABLES.

747. LINEAR MEASURES.

3 barley-corns, or sizes	= 1 inch.	Used by shoemakers.
4 inches	= 1 hand.	Used to measure the height of horses at the shoulder.
21.888 inches	= 1 sacred cubit.	
6 feet	= 1 fathom.	Used to measure depths at sea.
3 feet	= 1 pace.	
1.152$\frac{2}{3}$ common miles	= 1 geog. mi.	Used to measure distances at sea.
3 geographic miles	= 1 league.	

748. Linear Equivalents.

1 inch	=	2.54	centimeters.
1 foot	=	.3048	meter.
1 yard	=	.9144	meter.
1 rod	=	5.029	meters.
1 mile	=	1.6093	kilometers.

Equivalents of Volume.

1 cu. in.	=	16.387	cu. centim.
1 cu. ft.	=	28.317	cu. decim.
1 cu. yd.	=	.7645	cu. meter.
1 cord	=	3.624	steres.

Equivalents of Surface.

1 sq. in.	=	6.4528	sq. centim.
1 sq. ft.	=	.0929	sq. meter.
1 sq. yd.	=	.8361	sq. meter.
1 sq. rd.	=	25.293	centares.
1 acre	=	40.47	ares.
1 sq. mi.	=	259	hektares.

Equivalents of Capacity.

1 fl. ounce	=	.02958	liter.
1 liq. qt.	=	.9463	liter.
1 gallon	=	3.785	liters.
1 dry qt.	=	1.101	liters.
1 bushel	=	.35243	hektoliter.

Equivalents of Weight.

1 gr. Troy	=	.0648	gram.
1 oz. Troy	=	31.103	grams.
1 lb. Troy	=	.37324	kilogram.
1 oz. av.	=	28.35	grams.
1 lb. av.	=	.4536	kilogram.
1 ton	=	.907	tonne.

BOOK MEASURE.

A sheet folded into	*The book is*	*A sheet of paper makes*
2 leaves	a folio,	4 pp. (pages).
4 "	a quarto or 4to,	8 "
8 "	an octavo or 8vo,	16 "
12 "	a duodecimo or 12mo,	24 "
16 "	a 16mo,	32 "
18 "	an 18mo,	36 "

749. The **weight of a bushel** of certain articles is as follows:

Barley, . .	48 lb.	Corn in ear, .	68 lb.	Peas, . . .	60 lb
Beans, . . .	60 "	Clover seed, .	60 "	Potatoes, . .	60 "
Buckwheat, .	42 "	Flax seed, .	56 "	Rye, . . .	56 "
Bran, . . .	20 "	Hemp seed, .	44 "	Salt, . . .	56 "
Corn, . . .	56 "	Oats, . . .	32 "	Timothy seed,	45 "
Corn meal, .	50 "	Onions, . .	60 "	Wheat, . . .	60 "

The following are in common use:

100 lb.	of grain or flour	=	1 cental.
100 "	dry fish	=	1 quintal.
100 "	nails	=	1 keg.
196 "	flour	=	1 barrel.
200 "	beef or pork	=	1 barrel.
240 "	lime	=	1 cask.
280 "	salt at N. Y. Salt Works	=	1 barrel.

MISCELLANEOUS EQUIVALENTS.

231 cu. in.	=	1 gal. liquid measure.
$268\frac{4}{5}$ "	=	1 gal. dry measure.
277.274 "	=	1 imperial gal. of Great Britain.
2150.42 "	=	1 bushel of U. S.
2216.192 "	=	1 " " Great Britain.
2747.7 "	=	1 heaped bushel.

1000 oz. or $62\frac{1}{2}$ lb.	=	1 cu. ft. of pure water.
$8\frac{1}{3}$ lb. pure water	=	1 gallon.
5760 grains	=	1 lb. Troy or apothecaries'.
7000 "	=	1 " avoirdupois.
24.75 cu. ft.	=	1 perch of masonry.
36 to 45 cu. ft.	=	1 ton anthracite coal.

750. *1.* Webster's Unabridged Dictionary is a quarto of 1928 pages; how many sheets of paper are used in each copy?

2. How many 12mo books of 482 pages can be made from the paper in one copy of Webster's Unabridged Dictionary?

3. How many bushels in 6000 pounds of potatoes?

4. How many bushels of corn in the ear in $8\frac{1}{2}$ tons?

5. How many liquid gallons will a vessel 44 inches long, 28 inches wide and 1 foot deep hold?

6. A cylindrical vessel 6 feet deep and 3 feet in diameter is filled with water; what is the weight of the water?

7. There are 360° of 60 geographic miles each in the circumference of the earth; how many common miles in the circumference of the earth?

8. How many pounds of water will make 63 gallons?

9. How many lb. Troy are equal to 144 lb. avoirdupois?

10. Counting 40 cu. ft. of anthracite coal to a ton, how many tons will a bin 12 feet long, 10 feet wide and 6 feet deep hold?

11. How many bushels in Great Britain are equal to 1000 bushels in the United States?

12. What is the height in inches of a horse $16\frac{1}{2}$ hands high?

13. How many perches of masonry in a wall 225 feet long, 22 feet high and $1\frac{1}{2}$ feet thick?

14. How many bushels will a bin 12 feet long 8 feet wide and 6 feet deep hold?

DIFFERENCE BETWEEN DATES.

751. There are three methods of finding the time between two dates. To illustrate them, let it be

1. Required to find the time from Jan. 29, 1882, to August 15, 1884.

First Method.

Explanation.—The years, months and days are written and subtracted the same as in denominate numbers, giving 2 yrs. 6 mo. 16 da. for the result.

Process.

yr.	mo.	da.
1884	8	15
1882	1	29
2	6	16

Second Method.

Explanation.—The number of whole years is first found, giving 2 years. The number of whole months is next found, giving 6 months. Finally, the number of days from July 29 to August 15 is found, giving 17 days.

Process.

Jan. 29, '82, to Jan. 29, '84 = *2 yrs.*
Jan. 29 to July 29 = *6 mo.*
July 29 to August 15 = *17 da.*

Third Method.

Explanation.—The number of whole years is first found, and then the number of days; giving 2 years 199 days.

Process.

Jan. 29, '82, to Jan. 29, '84 = *2 yrs.*

Jan.	*Feb.*	*Mar.*	*Apr.*	*May.*	*June.*	*July.*	*Aug.*	
2 +	29 +	31 +	30 +	31 +	30 +	31 +	15	= *199 da.*

752. *1.* Find by each method the time from Nov. 1, 1879, to Mar. 16, 1880.

2. Find by each method the time from June 18, 1863, to Mar. 1, 1868.

3. What is the time by each method from Jan. 3, 1884, to Dec. 15, 1884?

4. Find by each method the time from Dec. 29, 1883, to Apr. 14, 1884.

5. Find by each method the time from August 29, 1881, to March 3, 1884.

INTEREST TABLE.

753. The interest on any sum of money may readily be found by means of a table showing the interest on $1 at various rates and times.

	5%	6%	7%	8%		5%	6%	7%	8%
Yr.					**Yr.**				
1	.05	.06	.07	.08	4	.20	.24	.28	.32
2	.10	.12	.14	.16	5	.25	.30	.35	.40
3	.15	.18	.21	.24	6	.30	.36	.42	.48
Mo.					**Mo.**				
1	.00416	.005	.00583	.00666	7	.02916	.035	.04083	.04666
2	.00833	.01	.01166	.01333	8	.03333	.04	.04666	.05333
3	.01250	.015	.01750	.02000	9	.03750	.045	.05250	.06000
4	.01666	.02	.02333	.02666	10	.04166	.05	.05833	.06666
5	.02083	.025	.02916	.03333	11	.04583	.055	.06416	.07333
6	.02500	.03	.03500	.04000					
Da.					**Da.**				
1	.00013	.00016	.00019	.00022	16	.00222	.00266	.00311	.00355
2	.00027	.00033	.00038	.00044	17	.00236	.00283	.00330	.00377
3	.00041	.00050	.00058	.00066	18	.00250	.00300	.00350	.00400
4	.00055	.00066	.00077	.00088	19	.00263	.00316	.00369	.00422
5	.00069	.00083	.00097	.00111	20	.00277	.00333	.00388	.00444
6	.00083	.00100	.00116	.00133	21	.00291	.00350	.00408	.00466
7	.00097	.00116	.00136	.00155	22	.00305	.00366	.00427	.00488
8	.00111	.00133	.00155	.00177	23	.00319	.00383	.00447	.00511
9	.00125	.00150	.00175	.00200	24	.00333	.00400	.00466	.00533
10	.00138	.00166	.00194	.00222	25	.00347	.00416	.00486	.00555
11	.00152	.00183	.00213	.00244	26	.00361	.00433	.00505	.00577
12	.00166	.00200	.00233	.00266	27	.00375	.00450	.00525	.00600
13	.00180	.00216	.00252	.00288	28	.00388	.00466	.00544	.00622
14	.00194	.00233	.00272	.00311	29	.00402	.00483	.00563	.00644
15	.00208	.00250	.00291	.00333					

I. What is the interest on $875.50 for 5 yrs. 7 mo. 29 da. at 5%?

Explanation.—From the column showing the interest at 5% are taken the interest on $1 for 5 years, the interest for 7 months, and the interest for 29 days. The sum of these gives the part of the principal that equals the interest for 5 yrs. 7 mo. 29 da. Multiplying $875.50 by this decimal gives $247.92, the interest.

Process.

.25	= Int. on $1 for	5 yrs.	
.02916 =	"	"	7 mo.
.00402 =	"	"	29 da.
.28318 =	"	"	5 yrs. 7 mo. 29 da.

$875.50 × .28318 = $247.92.

754. Find by the table the interest on:

1. $890 for 5 yrs. 3 mo. 24 da., at 6%.

2. $1875 for 3 yrs. 5 mo. 18 da., at 6%.

3. $3600 for 7 yrs. 1 mo. 23 da., at 5%.

4. $1275 for 8 mo. 27 da., at 8%.

5. $123.75 for 1 yr. 11 mo. 28 da., at 7%.

6. $235.50 for 3 yrs. 5 mo. 26 da., at 7%.

7. $1323.60 for 5 yrs. 5 mo. 25 da., at 6%.

8. $2387.50 for 4 yrs. 4 mo. 14 da., at 5%.

9. Find by the table the amount of $750 for 5 yrs. 9 mo. 25 da. at 8%.

10. What is the amount of $3800 for 3 yrs. 4 mo. 29 da. at 5%?

11. What is the interest on a note for $8600 from June 23, 1880, to Mar. 4, 1884, at 6%?

12. Find the amount at 5% of $3600 for 6 yrs. 4 mo. 23 da.

13. A note at 5% for $3570 was dated June 13, 1881, and was paid March 1, 1884; what was the amount due?

14. Find the interest at 6% on $18600 from July 25, 1883, to Apr. 4, 1884, ascertaining the time by the second method.

15. Find the interest at 7% on $63000 from Aug. 31, 1881, to Feb. 29, 1884, employing the first method of finding the difference between dates.

16. What is the interest at 5% on a note for $5000 from Jan. 10, 1882, to Dec. 5, 1884?

755. THE VERMONT RULE FOR PARTIAL PAYMENTS.

"On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time, WITH INTEREST, *when payments are made, such payments shall be applied: first, to liquidate the interest that has accrued at the time of such payments; and, secondly, to the extinguishment of the principal.*

"On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time, WITH INTEREST ANNUALLY, *the annual interests that remain unpaid shall be subject to simple interest, from the time they become due to the time of final settlement; but if in any year, reckoning from the time such annual interest began to accrue, payments have been made, the amount of such payments at the end of such year, with interest thereon from the date of payment, shall be applied: first, to liquidate the simple interest that has accrued upon the unpaid annual interests; secondly, to liquidate the annual interests that have become due; and thirdly, to the extinguishment of the principal."*

756. THE NEW HAMPSHIRE RULE

is the same as the Vermont Rule, with the following provision:

"If at the time of any payment, no interest is due, except what is accruing for the year, and the payment or payments are less than the interest due at the end of the year, then no interest is allowed on such payment or payments."

NOTE.—Solve by each of the above rules the examples on p. 247.

757. INTEREST LAWS

Laws of each State and Territory regarding Rates of Interest and Penalties for Usury, with the Law or Custom as to Days of Grace on Notes and Drafts. Compiled from the latest State and Territorial Statutes.

STATES AND TERRITORIES.	Legal Rate of Interest.	Rate Allowed by Contract.	Penalties for Usury.	Grace or No Grace.
	per cent.	per cent.		
Alabama	8	8	Forfeiture of entire interest.	Grace.
Arizona	10	Any rate.	None.	Grace.
Arkansas	6	10	Forfeiture of principal and int.	No statute.
California	7	Any rate.	None.	No grace.
Colorado	10	Any rate.	None.	Grace.
Connecticut	6	6	None.	Grace.
Dakota	7	12	Forfeiture of excess.	Grace.
Delaware	6	6	Forfeiture of principal.	Grace.
Dist. of Columbia	6	10	Forfeiture of entire interest.	Grace.
Florida	8	Any rate.	None.	No statute.
Georgia	7	8	Forfeiture of excess.	Grace.
Idaho	10	18	Forf. of 3 times excess of int.	No grace.
Illinois	6	8	Forfeiture of entire interest.	Grace.
Indiana	6	8	Forfeiture of excess of interest.	Grace.
Iowa	6	10	Forf. of 10% per year on amt.	Grace.
Kansas	7	12	Forfeiture of excess of interest.	Grace.
Kentucky	6	10	Forfeiture of excess over 10%.	Grace.
Louisiana	5	8	Forfeiture of entire interest.	Grace.
Maine	6	Any rate.	None.	Grace.
Maryland	6	6	Forfeiture of excess of int.	Grace.
Massachusetts	6	Any rate.	None.	Grace.
Michigan	7	10	Forfeiture of excess of int.	Grace.
Minnesota	7	10	Forfeiture of excess over 10%.	Grace.
Mississippi	6	10	Forfeiture of excess of int.	Grace.
Missouri	6	10	Forfeiture of entire interest.	Grace.
Montana	10	Any rate.	None.	No grace.
Nebraska	7	10	Forfeiture of interest and cost.	Grace.
Nevada	10	Any rate.	None.	Grace.
New Hampshire	6	6	Forfeiture of thrice the excess.	Grace.
New Jersey	6	6	Forfeiture of entire interest.	Grace.
New Mexico	6	12	None.	No grace.
New York *	6	6	Forfeiture of principal and int.	Grace.
North Carolina	6	8	Forfeiture of entire interest.	Grace.
Ohio	6	8	Forfeiture of excess above 6%.	Grace.
Oregon	8	10	Forfeiture of principal and int.	Grace.
Pennsylvania	6	6	Forfeiture of excess of int.	Grace.
Rhode Island	6	Any rate.	None.	Grace.
South Carolina	7	Any rate.	None.	Grace.
Tennessee	6	10	Forf. of exc. int. & $100 fine.	Grace.
Texas	8	12	Forfeiture of entire interest.	Grace.
Utah	10	Any rate.	None.	Grace.
Vermont	6	6	Forfeiture of excess of int.	Grace.
Virginia	6	8	Forfeiture of excess over 6%.	Grace.
Washington Ter.	10	Any rate.	None.	......
West Virginia	6	6	Forfeiture of excess of int.	Grace.
Wisconsin	7	10	Forfeiture of entire interest.	Grace.
Wyoming	12	Any rate.	None.	Grace.

* Upon call loans of $5,000 or upward, on collateral security, any rate of int. is legal.

COMPOUND INTEREST TABLE,

Showing the amt. of $1, at various rates, compound int. from 1 to 20 years

Yrs.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yrs.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.269712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973823
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

Rule.—I. *Multiply the given principal by the amount of $1 for the given time at the given rate, and the product will be the amount required. The difference between this amount and the original principal will be the compound interest.*

II. *If there is a final partial period, add to the amount for entire periods the simple interest on that amount.*

NOTE.—When the interest is compounded semi-annually, the tabular amount at half the annual rate is taken for twice the number of years. When it is compounded quarterly, the tabular amount at $\frac{1}{4}$ the rate is taken for four times the number of years; etc.

758. *1.* What is the compound interest of $520 for 3 years, at 5%?

2. What is the amount at compound interest of $960 for 9 years at $3\frac{1}{2}$%, interest compounded annually?

3. What is the compound interest of $340 for 2 years, interest being payable semi-annually, at 6%?

4. What is the compound interest of $600 for 20 years at 4%, interest compounded annually?

5. What is the amount of $500 at 8% for 2 years, interest compounded semi-annually?

6. What is the interest for 5 years at 4% on $8360, interest compounded annually?

7. How much will $4600 amount to in 4 yrs. 3 mo. 18 da., deposited in a savings-bank that pays 10%, interest compounded quarterly?

8. Find the amount at 6% compound interest of $3760 for 3 yrs. 11 mo. 24 da., interest compounded semi-annually

9. A man deposits for his son, $100 in a savings-bank at $3\frac{1}{2}$% compound interest. If the deposit is made when the son is 1 year old, how much will it be worth when he is 21, interest compounded annually?

10. What is the compound interest at 4% on $1800 from June 19, 1863, to Apr. 12, 1881?

FOREIGN EXCHANGE.

759. Bills of **foreign exchange** are those drawn in one country and payable in another.

Bills of exchange are usually drawn in sets of two or three, and sent by different mails. When one is paid, the others are valueless.

760. Exchange with Europe is usually made through London, Paris, Berlin, Antwerp, Hamburg, Frankfort, or Amsterdam.

The unit of exchange on London is the *pound* sterling; on Paris and Antwerp it is the *franc;* on Berlin, Hamburg and Frankfort it is *four marks,* and on Amsterdam the *guilder.*

761. Quotations of foreign exchange are given in the New York papers as *long* and *short,* that is, for bills payable 60 days after sight, and 3 days after sight, respectively. Such a report, of recent date, gave the rates of exchange as follows:

£1	= $4.86½	*long;*	$4.88½ *short.*
$1	= 5.15⅛ francs	" ;	5.17⅛ "
4 marks	= $.95½	" ;	$.96 "
1 guilder	= $.40	" ;	$.40¼ "

VALUES OF FOREIGN COINS.

762. The following table of coins and their values was promulgated by the Secretary of the Treasury of the United States, Jan. 1, 1886.

NOTE.—The "Standard" of a given country is indicated as follows, namely: *Double,* where its standard silver coins are unlimited legal tender, the same as its gold coins; *Single gold* or *Single silver,* as its standard coins of one or of the other metal are unlimited legal tender. The par of exchange of the monetary unit of a country with a single gold, or

a double standard, is fixed at the value of the gold unit as compared with the United States gold unit. In the case of a country with a single silver standard, the par of exchange is computed at the mean price of silver in the London market for the three months ending December 24, 1885, as per daily cable dispatches to the Bureau of the Mint.

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Argentine Repub.	Peso	Double	$0.96,5
Austria	Florin	Single silver	.37,1
Belgium	Franc	Double	.19,3
Bolivia	Boliviano	Single silver	.75,1
Brazil	Milreis of 1000 reis	Single gold.	.54,6
British America	Dollar	Single gold.	$1.00
Chili	Peso	Double	.91,2
Cuba	Peso	Double	.93,2
Denmark	Crown	Single gold.	.26,8
Ecuador	Peso	Single silver	.75,1
Egypt	Piaster	Single gold.	.04,9
France	Franc	Double	.19,3
German Empire	Mark	Single gold.	.23,8
Great Britain	Pound sterling	Single gold.	4.86,6½
Greece	Drachma	Double	.19,3
Hayti	Gourde	Double	.96,5
India	Rupee of 16 annas	Single silver	.35,7
Italy	Lira	Double	.19,3
Japan	Yen	Single silver	.81,0
Liberia	Dollar	Single gold.	1.00
Mexico	Dollar	Single silver	.81,6
Netherlands	Florin	Double	.40,2
Norway	Crown	Single gold.	.26,8
Peru	Sol	Single silver	.75,1
Portugal	Milreis of 1000 reis	Single gold.	1.08
Russia	Rouble of 100 copecks	Single silver	.60,1
Spain	Peseta of 100 centimes	Double	.19,3
Sweden	Crown	Single gold.	.26,8
Switzerland	Franc	Double	.19,3
Tripoli	Mahbub of 20 piasters	Single silver	.67,7
Turkey	Piaster	Single gold.	.04,4
U. S. of Colombia	Peso	Single silver	.75,1
Venezuela	Bolivar	Double	.19,3

763. To find the cost of a bill of exchange.

1. What is the cost, in New York, of a bill of exchange on London for £180 7s. 6d., exchange being quoted at $4.88, and gold being at 1.05?

Solution.—£180 7s. 6d. = £180.375.

$\$4.88 \times 180.375 = \$880.23$ = cost in gold.

$\$880.23 \times 1.05 = \$924.245$ = cost in currency.

2. Desiring to pay a bill of 18270 francs in France, I bought in New York a bill of exchange on Paris. What did it cost me in currency, exchange on Paris being 5.22, and gold being at 1.02?

Solution.—$18270 \div 5.22 = \$3500$ = cost in gold.

$\$3500 \times 1.02 = \$3570$ = cost in currency.

3. What will be the cost of a bill on Berlin for 3600 marks, when the exchange value of four marks is $.956, brokerage being $\frac{1}{2}\%$, and gold at par?

Solution.—$\frac{\$.956}{4} \times 3600 = \$860.40$ = cost less brokerage.

$\$860.40 \times .005 = \$4.30$; $\$860.40 + \$4.30 = \$864.70$ = cost.

4. What will be the cost of a bill on Amsterdam for 12600 guilders, exchange being 41?

5. Wishing to pay a bill of £860 15s. in Liverpool, I buy a bill of exchange at 60 days' sight on London; what does it cost, exchange being at 4.87 and brokerage $\frac{1}{2}\%$?

6. Find the cost of a bill of exchange on Geneva, Switzerland, for 25600 francs, exchange being 5.18 francs to the dollar, and brokerage $\frac{1}{2}\%$.

7. What is the cost of a bill at 3 days' sight on Hamburg for 8800 marks, exchange being at $96\frac{1}{2}$, and brokerage $\frac{1}{2}\%$?

8. What is the cost of a bill at 60 days' sight on Geneva, Switzerland, for 20000 francs, exchange being 5.24 francs to the dollar, and brokerage $\frac{1}{2}\%$?

ARBITRATION OF EXCHANGE.

764. Arbitration of exchange is the process of finding the cost of exchange between two places through one or more intermediate places. Exchange through intermediate places is called *indirect* or *circuitous exchange.*

NOTE.—Merchants often find it cheaper to remit by circuitous exchange than by direct exchange.

765. To find the cost of circuitous exchange.

1. I owe 7740 francs in Paris. Which is better, and how much; to remit directly, or through London, exchange between New York and London being 4.86, between London and Paris 24.85 francs to the pound, and between New York and Paris 5.16, and London brokerage being $\frac{1}{4}\%$?

Process.

$$7740 \div 5.16 = \$1500$$
$$7740 \div 24.85 = £311.469$$
$$£311.469 + \tfrac{1}{4}\% = £312.248$$
$$\$4.86 \times 312.248 = \$1517.53$$
$$\$1517.53 - 1500 = \$17.53$$

Explanation.—By direct exchange, since $1 will buy 5.16 francs it will take as many dollars to buy 7740 francs as 5.16 is contained times in 7740, equal to $1500.

By exchange through London, since 24.85 francs cost £1, 7740 francs will cost £311.469, and with the brokerage, £312.248. In New York, £1 costs $4.86; hence, £312.248 will cost $1517.53, or $15.73 more than by direct exchange.

2. How much will it cost a New York merchant to remit 8000 marks to Berlin through London and Amsterdam, exchange on London being 4.86, with $\frac{1}{8}\%$ brokerage, on Amsterdam 11.85 guilders to the pound, and on Berlin 1.72 marks to the guilder?

Process.

$$\begin{aligned}
\$x &= 8000 \text{ marks.}\\
1.72 \text{ marks} &= 1 \text{ guilder.}\\
11.85 \text{ guilders} &= £1.\\
£1 \text{ with brokerage} &= £1.00\tfrac{1}{8}.\\
£1 &= \$4.86.\\
\$x &= \frac{8000 \times 1.00\frac{1}{8} \times 4.86}{1.72 \times 11.85} = \$1909.95.
\end{aligned}$$

Analysis.

$$\frac{8000}{1.72} = \begin{cases}\text{No. of}\\ \text{guilders.}\end{cases} \qquad \frac{8000}{1.72 \times 11.85} = \begin{cases}\text{No. of}\\ \text{pounds.}\end{cases}$$

$$\frac{8000 \times 1.00\frac{1}{8}}{1.72 \times 11.85} = \begin{cases}\text{No. of pounds,}\\ \text{with brokerage.}\end{cases} \qquad \frac{8000 \times 1.00\frac{1}{8} \times 4.86}{1.72 \times 11.85} = \begin{cases}\text{No. of}\\ \text{dollars}\end{cases}$$

3. A merchant of New York wishes to pay a bill of 1500 milreis in Lisbon; he remits to London at \$4.86 to the pound; thence to Hamburg at 20.45 marks to the pound; thence to Paris at 4 marks for 5 francs; thence to Lisbon at 5.6 francs for 1 milreis; what does it cost?

NOTE.—The same process is applicable to those cases where it is required to find the equivalent of a given quantity of one commodity in terms of another through two or more intermediate commodities.

4. How many bushels of wheat should be given for 1000 bushels of potatoes if 8 bushels of potatoes are worth 5 of corn, $12\frac{1}{2}$ bushels of corn are worth 7 of rye, and 5 bushels of rye are worth 3 of wheat?

5. Exchange at New York on London being at 4.875, and at London on Amsterdam at 12.5 guilders to the £, what must a person in New York remitting through London pay for exchange on Amsterdam for 1000 guilders?

6. Exchange between Paris and Amsterdam being at 2 francs 20 centimes to the guilder, between London and Paris at 25 francs 80 centimes to the £, and between New York and London at 4.88, what will be the cost of a remittance for 1000 guilders from New York to Amsterdam by exchange through London and Paris?

ALLIGATION.

766. Alligation is a method of computation relating to the mixing of ingredients of different values.

ALLIGATION MEDIAL.

767. Alligation medial is the process of finding the average value of the ingredients of a compound when the quantity and price of each ingredient are given.

768. To find the average value of a mixture.

1. A grocer mixed 8 pounds of coffee worth 25 cents a pound with 15 pounds worth 26 cents a pound and 17 pounds worth 30 cents; what was a pound of the mixture worth?

Explanation.—The value of 8 pounds of coffee at 25 cents a pound is \$2.00, of 15 pounds at 26 cents is \$3.90, and of 17 pounds at 30 cents is \$5.10. The entire mixture of 40 pounds is, therefore, worth \$11, and 1 pound is worth $\frac{1}{40}$ of \$11, or \$.27½.

Process.

$$\begin{array}{rcr} \$.25 \times 8 & = & \$2.00 \\ .26 \times 15 & = & 3.90 \\ .30 \times 17 & = & 5.10 \\ \hline 40 &) & 11.00 \\ \hline & & \$.27\frac{1}{2} \end{array}$$

2. A grocer mixed 5 lb. of tea at 60 cents a pound with 3 lb. at 50 cents, 45 lb. at 80 cents, and 15 lb. at 70 cents; what was the value of a pound of the mixture?

3. At what price per pound should a grocer sell a mixture consisting of 21 pounds of coffee at 40 cents, 42 pounds at 30 cents, and 12 pounds at 25 cents?

4. A mixture consists of 50 pounds of sugar at 6 cents, 150 pounds at 8 cents, 75 pounds at 11 cents, and 50 pounds at 12 cents; what is a pound of it worth?

5. A merchant sold 30 yards of silk at \$1¾ per yard, 60 yards at \$2¾, 30 yards at \$3½, and 60 yards at \$4½; what average price did he receive?

ALLIGATION ALTERNATE.

769. Alligation alternate is the process of determining the proportional quantities of the several ingredients in a mixture having a given average value.

770. To find the proportional quantities of a mixture.

1. Teas worth 40, 45, 60, and 75 cents a pound each, are combined in a mixture worth 55 cents a pound; what are the proportional weights of the different kinds?

Explanation. — The ingredients are linked so that one on which there is gain is joined with one on which there is loss. On one pound of tea worth \$.40 and sold at \$.55 there is a gain of \$.15; hence, to gain 1 cent there must be sold $\frac{1}{15}$ of a pound. To lose 1 cent there must be sold $\frac{1}{20}$ of a pound of tea at \$.75. Again, on $\frac{1}{10}$ of a pound of 45-cent tea there is a gain of 1 cent, and on $\frac{1}{5}$ of a pound of 60-cent tea there is a loss of 1 cent. On any multiple of these pairs of fractions of a pound the gain will equal the loss. Multiplying the first pair, $\frac{1}{15}$ and $\frac{1}{20}$, by 60, the least common multiple of their denominators, gives 4 pounds and 3 pounds. Multiplying the second pair by 10 gives 1 pound and 2 pounds.

In the second process the reasoning is the same, the linking only being different.

First Process.

55	40	$\frac{1}{15}$		4	= \$1.60
	45		$\frac{1}{10}$	1	= .45
	60		$\frac{1}{5}$	2	= 1.20
	75	$\frac{1}{20}$		3	= 2.25
				10	) \$5.50
					\$.55

Second Process.

55	40	$\frac{1}{15}$		1	= \$.40
	45		$\frac{1}{10}$	2	= .90
	60	$\frac{1}{5}$		3	= 1.80
	75		$\frac{1}{20}$	1	= .75
				7	) \$3.85
					\$.55

2. Dried fruits worth 8, 9, 15 and 18 cents a pound are combined in a mixture worth 13 cents a pound; what are the proportional weights of the different kinds?

3. In what proportional parts must gold respectively 14, 18 and 24 carats fine be combined to make a metal 20 carats fine?

4. Grades of wheat worth \$.95, \$1.00, \$1.10, and \$1.25 per bushel compose a mixture worth \$1.15; how many bushels of each grade in the mixture?

Process.

\$1.15	\$.95	$\frac{1}{20}$			1			1	=	\$.95
	1.00		$\frac{1}{15}$			2		2	=	2.00
	1.10			$\frac{1}{5}$			2	2	=	2.20
	1.25	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	2	3	1	6	=	7.50
								11	)	\$12.65
										\$1.15

Explanation.—Each price below the mean price is linked with \$1.25, the only one greater than the mean price. On a bushel of wheat at \$.95 sold at \$1.15 there is a gain of \$.20; hence, to gain 1 cent, $\frac{1}{20}$ of a bushel must be sold. To lose 1 cent, $\frac{1}{10}$ of a bushel at \$1.25 must be sold. Of the next pair, the fractions required to balance the gain and the loss are $\frac{1}{15}$ and $\frac{1}{10}$; and of the remaining pair, they are $\frac{1}{5}$ and $\frac{1}{10}$. Multiplying these three pairs of fractions by 20, 30 and 10 respectively, gives the proportional quantities.

5. A mixture composed of coffees at \$.25, \$.28, \$.36 and \$.40 a pound each, is sold at \$.32 a pound; what proportion of the several kinds in the mixture?

6. A confectioner mixed candies at 20, 25, 30 and 35 cents a pound, and sold the mixture at 28 cents a pound; in what proportion should the different grades have been mixed?

7. A farmer sold oats at \$.35 a bushel, wheat at \$1.10, rye at \$.90, and corn at \$.70; in what proportional amounts did he sell the grain if his average price was \$.80 a bushel?

8. A butcher bought sheep at \$3 each, hogs at \$10 each, sheep at \$5 each, and calves at \$14; in what proportional numbers did he buy them if he paid an average price of \$8?

9. A merchant sold cloths at \$1.50, \$1.75, \$2.00 and \$2.40 a yard each; what proportional amounts did he sell if the average price received was \$1.90 a yard?

10. A merchant sold 88 yards of different kinds of silk for \$198; how many yards of each kind did he sell if the prices of the different grades were \$1.75, \$1.90, \$2.50 and \$3.00?

ARITHMETICAL PROGRESSION.

771. An **arithmetical progression** or series is a succession of numbers, each of which is greater or less than the preceding number by a constant difference.

Thus, 1, 3, 5, 7, 9, etc., and 21, 17, 13, 9, etc., are arithmetical series. The first is an *increasing*, and the second a *decreasing* arithmetical series.

772. In every arithmetical series five elements occur: 1. *The first term,* a. 2. *The last term,* l. 3. *The common difference,* d. 4. *The number of terms,* n. 5. *The sum of the series,* S. The first and last terms are the **extremes**. If any three of these are given, the other two may be found.

773. To find the last term of an increasing or a decreasing arithmetical series when the first term, number of terms, and common difference are given.

I. Required the last term of an increasing arithmetical series of 10 terms, the first term being 3, and the common difference 5; also the last term of a decreasing series of 10 terms, the first term being 71, and the common difference 5.

First Process.

$$3 + (10 - 1)\,5 = 48.$$

Second Process.

$$71 - (10 - 1)\,5 = 26.$$

Explanation.—In the first series, 3, the first term, *increased* by *once* 5 gives the 2d term, by *twice* 5 gives the 3d term, by *three times* 5 gives the 4th term, etc. Hence, 3 increased by *nine times* 5 gives the 10th term.

In the second series, 71, the first term, *diminished* by *once* 5 gives the 2d term, by *twice* 5 gives the 3d term, by *three times* 5 gives the 4th term, etc. Hence, 71 diminished by *nine times* 5 gives the 10th term.

Rule.—*Multiply the common difference by a number less by* 1 *than the number of terms; add the product to the first term of an increasing series, subtract it from the first term of a decreasing series, and the result will be the last term.*

FORMULA. $\quad a + (n - 1) \times d = l.$

774. To find the first term of an increasing or a decreasing arithmetical series, when the last term, number of terms and common difference are given.

By reversing the terms of a series, the last term of an increasing series becomes the first term of a decreasing series, and the last term of a decreasing series becomes the first term of an increasing series. Hence,

Rule.—*Find the first term of an increasing series as if it were the last term of a decreasing series, and find the first term of a decreasing series as if it were the last term of an increasing series.*

775. *1.* The first term of an increasing arithmetical series of 15 terms is 8, and the common difference 4; what is the last term?

2. What is the last term of a decreasing arithmetical series of 20 terms, the first term being 100 and the common difference 3?

3. Find the first term of an increasing arithmetical series, the last term being 191, the number of terms 45, and the common difference 4.

4. The last term of a decreasing arithmetical series is 7 the number of terms 31 and the common difference 5; what is the first term?

5. A man deposited in a savings-bank $5 for his son when the boy was one year old, and increased the deposit by $10 a each subsequent birthday until his son was 21 years old; what was the last deposit?

6. A merchant bought 20 pieces of cloth, giving $1 for the first piece, $3 for the second, $5 for the third, and so on; what, at this rate, did the last piece cost?

7. A board, $2\frac{1}{2}$ inches wide at the narrow end, and 10 feet long, increases in width $1\frac{1}{2}$ inches for every foot in length. What is the width at the wide end?

776. To find the sum of an arithmetical series when one of the extremes, the number of terms, and the common difference are given.

I. Required to find the sum of an arithmetical series of which the first term is 1, the number of terms 7, and the common difference 5.

Explanation.—The last term, found by the method already explained, is 31. Half the sum of the extremes multiplied by the number of terms gives the sum of the series, equal to 112.

Process.

$$1+(7-1)\times 5 = 31.$$

$$\frac{1+31}{2}\times 7 = 112.$$

The reason for this process may be shown as follows:

Writing the series in full gives	1, 6, 11, 16, 21, 26, 31.
Writing the series in reverse order gives	31, 26, 21, 16, 11, 6, 1.
Adding, gives twice the sum of the series,	32, 32, 32, 32, 32, 32, 32.
Hence, the sum of the series is	$(32\div 2)\times 7 = 112.$

Rule.—*Multiply half the sum of the extremes by the number of terms, and the result will be the sum of the series.*

FORMULA. $$\left(\frac{a+l}{2}\right)\times n = S.$$

777. *1.* The first term of an increasing arithmetical series is 5, the number of terms 30, and the common difference 3; what is the sum of the series?

2. The last term of an increasing arithmetical series is 67, the number of terms 8, and the common difference 7; what is the sum of the series?

3. The first term of a decreasing arithmetical series is 165, the number of terms 37, and the common difference 4; what is the sum of the series?

4. A man gave his daughter \$1 the first week, and 25 cents more each week than the preceding week; if he continued this practice for a year, how much did the daughter receive?

GEOMETRICAL PROGRESSION.

778. A **geometrical progression** or **series** is a succession of numbers that increase or decrease by a constant multiplier, called the **ratio**.

Thus, 1, 3, 9, 27, 81, etc., and 64, 32, 16, 8, etc., are geometrical series.

When the **ratio** is greater than 1, the series is increasing; when less than 1, it is decreasing.

779. In every geometrical series five elements occur, three of which being given, the others may be found. They are, 1. *The first term, a.* 2. *The last term, l.* 3. *The number of terms, n.* 4. *The ratio, r.* 5. *The sum of the series, S.*

780. To find the last term of an increasing or a decreasing geometrical series, when the first term, the number of terms, and the ratio are given.

I. Required to find the last term of an increasing geometrical series of 5 terms, the ratio of which is 3 and the first term 1; also the last term of a decreasing geometrical series of 5 terms, the ratio of which is $\frac{1}{4}$, and the first term 256.

First Process.

$$1 \times 3^{5-1} = 81.$$

Second Process.

$$256 \times (\tfrac{1}{4})^{5-1} = 1.$$

Explanation.—In the first series, the first term multiplied by the ratio 3, taken *once* as a factor, gives the 2d term; multiplied by (3×3), or 3^2, gives the 3d term; multiplied by $(3 \times 3 \times 3)$, or 3^3, gives the 4th term, etc. Hence, the first term, 1, multiplied by 3 raised to a power whose exponent is less by 1 than the number of terms, that is, $1 \times 3^{5-1}$, or 1×3^4, gives the last term.

In the second series, the process is the same as in the first.

Rule.—*Multiply the first term by the ratio raised to a power whose exponent is less by* 1 *than the number of terms; the product will be the last term.*

FORMULA. $$a \times r^{n-1} = l.$$

781. To find the first term of an increasing or a decreasing geometrical series when the last term, the number of terms and the ratio are given.

By reversing the terms of a series, the last term of an increasing series becomes the first term of a decreasing series, and the last term of a decreasing series becomes the first term of an increasing series. In each case the ratio of the reversed series will be the reciprocal of the ratio of the original series. Hence,

Rule.—*Find the first term of an increasing series as if it were the last term of a decreasing series, and find the first term of a decreasing series as if it were the last term of an increasing series.*

782. *1.* The first term of an increasing geometrical series of 8 terms is 1, and the ratio is 2; what is the last term?

2. The first term of a decreasing geometrical series of 10 terms is 1536 and the ratio is $\frac{1}{2}$; what is the last term?

3. What is the first term of an increasing geometrical series of 11 terms, the last term being 118098 and the ratio 3?

4. What is the first term of a decreasing geometrical series of 15 terms, the last term being 5 and the ratio $\frac{1}{2}$?

783. To find the sum of a geometrical series when one of the extremes, the ratio, and the number of terms are given.

I. Required the sum of a geometrical series of 6 terms, the first term being 4 and ratio 3.

Explanation.—The last term, found as already explained, is 972. Multiplying the last term, 972, by the ratio 3, subtracting the first term from the product, and dividing the remainder by the ratio less 1 gives 1456, the sum of the series.

Process.

$$4 \times 3^5 = 972.$$

$$\frac{(972 \times 3) - 4}{3 - 1} = 1456.$$

The reason for this process is shown as follows:

The series written in full is, 4, 12, 36, 108, 324, 972.

Multiplying by the ratio, and reversing the terms of the resulting series, gives 3 times the series,

	2916, 972, 324, 108, 36, 12.
Subtracting the series from this product,	972, 324, 108, 36, 12, 4.
The remainder equals twice the series,	2916−4.

Hence, $2912 \div 2 = 1456$, the sum of the series.

Rule.—*Multiply the last term by the ratio, subtract from the product the first term, and divide the remainder by the ratio less* 1; *the result will be the sum of the series.*

FORMULAS.
$$\frac{rl - a}{r - 1} = S. \quad \text{(For increasing series.)}$$

$$\frac{a - rl}{1 - r} = S. \quad \text{(For decreasing series.)}$$

NOTE.—In the case of a decreasing series, conceive the series as reversed, and for the ratio take the reciprocal of the given ratio. The series is then an increasing series, and its sum is found by the foregoing rule. The sum may also be found by the second formula.

784. *1.* What is the sum of a geometrical series of 8 terms, the ratio being 2, and the first term 3?

2. The last term of a geometrical series is 3645, the ratio 3, and the number of terms 7; what is the sum of the series?

3. The first term of a geometrical series is 486, the number of terms 6, and the ratio $\frac{2}{3}$; what is the sum of the series?

4. The last term of a geometrical series is 162, the ratio $\frac{1}{2}$, and the number of terms 5; what is the sum of the series?

5. The first term of a decreasing series is 108, the last term 4, and the ratio $\frac{1}{3}$; required the sum of the series.

6. The first term of a decreasing series is 31250, the last term 10, and the ratio $\frac{1}{5}$; required the sum of the terms.

ANNUITIES.

785. An **annuity** is a sum of money to be paid regularly at stated periods.

786. A **certain annuity** is an annuity that continues for a specified time.

787. A **contingent annuity** is an annuity the payment of which is dependent upon some particular circumstance.

788. A **perpetual annuity** is an annuity that is to continue without specified limit.

An annuity is said to be *in possession* when there is a present claim upon it; *in reversion* or *deferred* when the claim upon it begins in the future.

789. The **present worth** of an annuity is the sum that in the given time and at the given rate amounts to the sum of all the payments with the interest on each from the time it is due until the annuity ceases.

790. The **amount** or **final value** of an annuity is the sum of all the payments, with interest on each until the annuity ceases.

791. To find the amount of an annuity at simple interest.

1. What is the amount of an annuity of \$800 for 6 years at 5%, simple interest?

Process.

$$\frac{\$800 + \$1000}{2} \times 6 = \$5400.$$

Explanation.—From the time when the first payment is due until the last payment is due is 5 years; that is, the 1st payment draws 5 years' interest. The 2d payment draws 4 years' interest, the 3d 3, the 4th 2, the 5th 1, and the 6th draws no interest. These amounts

in reverse order form an increasing arithmetical series of 6 terms, the first term of which is \$800, the common difference \$40 (the interest on \$800 for 1 year at 5%). The sum of the series is the amount or final value of the annuity.

2. What is the final value of an annuity of \$1200 for 10 years at 6%?

3. What will the rent of a hotel, at \$1500 a year in advance, amount to in 12 years, if the rent when received is invested at 5%?

4. If a person saves \$500 a year and invests it at 6% simple interest, what will he be worth in 20 years?

792. To find the amount of an annuity at compound interest.

1. What is the final value of an annuity of \$500 for 6 years at 4%, compound interest?

Process.

$$\frac{\$500 \times 1.04^6 - \$500}{.04} = \$3316.49$$

Explanation.—From the time when the first payment is due until the last payment is due is 5 years; that is, the first payment draws compound interest for 5 years. The 2d payment draws compound interest for 4 years, the 3d 3 years, the 4th 2 years, the 5th 1 year, and the 6th draws no interest. These amounts taken in reverse order, form an increasing geometrical series of 6 terms, the first of which is \$500, and the ratio 1.04. The sum is found as explained in geometrical progression.

2. What is the amount, at 6% compound interest, of an annuity of \$800 for 8 years?

3. A man deposited \$1000 a year for 5 years in a savings bank that paid 4% compound interest; what amount was due him?

4. If a person saves \$300 a year and invests it at 5% compound interest, what will he be worth in 20 years?

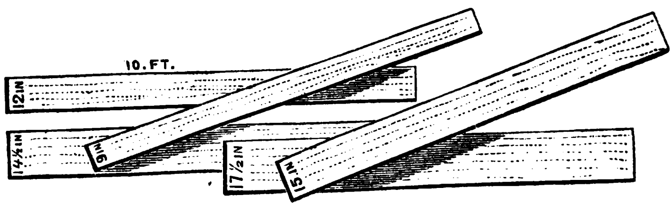
5. What is the amount, at 5% compound interest, of an annuity of \$1200 for 10 years?

MEASUREMENTS.

For tables of Surface and Solid Measures, see pages 177 and 181.

BOARDS AND TIMBER.

793. *1.* Required to find the cost of 5 boards, each 10 ft. long and 1 in. thick, their widths being 12, $14\frac{1}{2}$, 9, $17\frac{1}{2}$, and 15 in. respectively, at $18 per M.



Explanation.—The boards being of the same length, they measure the same as a single board whose width is the sum of the widths of the given boards, or $5\frac{2}{3}$ ft. The unit in board measure being a square foot, the width $5\frac{2}{3}$, multiplied by the length 10, gives $56\frac{2}{3}$ as the number of board feet. Multiplying $56\frac{2}{3}$ by .018, $18 per thousand being the same as $.018 per foot, gives $1.02.

Process.

$$(12+14\tfrac{1}{2}+9+17\tfrac{1}{2}+15)\div 12 = 5\tfrac{2}{3}$$
$$5\tfrac{2}{3}\times 10 = 56\tfrac{2}{3}$$
$$56\tfrac{2}{3}\times .018 = 1.02$$

NOTE 1.—See note on page 180.

NOTE 2.—The width of a tapering board is found by measuring the board in the middle, or by taking half the sum of both ends as the average width.

2. What is the cost of 1500 inch boards, 16 ft. long and 6 in. wide, at $20 per M.?

3. Find the contents of inch boards measuring 20 ft. × 13 in., 24 ft. × 15 in., 16 ft. × 18 in., 10 ft. × 26 in., 30 ft. × 14 in., and 6 ft. × 18 in.

4. Find the contents of inch boards measuring 18 ft. × 1 ft 4 in., 25 ft. × 1 ft. 6 in., 20 ft. × 2 ft., and 24 ft. × 1 ft. 3 in.

5. What is the cost of 25 planks 16 ft. long, 12 in. wide, and $2\frac{1}{2}$ in. thick, at \$2 per hundred feet board measure?

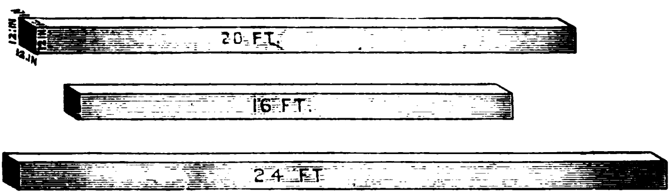
6. How many board feet in a board 20 ft. long, 21 in. wide at one end, and 15 in. wide at the other, and $1\frac{1}{4}$ in. thick?

7. How many board feet are there in a board 18 ft. long, 16 in. wide, and $\frac{1}{2}$ in. thick?

8. How many boards 16 ft. long and 12 in. wide will it take to build a close fence 4 ft. high and 8 rods long, and what will the boards cost at \$23.50 per M.?

9. How many boards 12 ft. long and 8 in. wide will be required to make a cubical box 4 ft. each way?

10. Required to find the cost, at \$20 per M., of 3 timbers, each 1 foot square and 20, 16, and 24 ft. in length.



Explanation. — The timbers being the same in width and thickness, they measure the same as a single timber whose length is 20 + 16 + 24 feet. The unit in board measure, for boards an inch or more in thickness, being a square foot one inch thick, each foot in length is equivalent to 12 board feet. Multiplying the sum of their lengths by 12 gives 720 board feet. 720 ft., at \$20 per M., or 2¢ per ft., will cost \$14.40.

Process.

$$(20 + 16 + 24) \times 12 = 720$$

$$720 \times .02 = 14.40$$

11. How many feet, board measure, in a stick of timber 10 in. by 12 in., and 30 ft. long?

12. Find the cost of 64 scantlings, each 2 in. by 4 in., and 12 ft. long, at \$17.50 per M.

13. How many board feet in a stick of timber 36 ft. long and 15 in. square?

14. At $25 per M., what will be the cost of 40 joists, each 16 ft. long, 8 in. wide, and 3 in. thick?

15. How many feet in a beam 30 ft. long, 12 in. wide, and 8 in. thick, board measure? Cubic measure?

16. Find the cost of the following bill of lumber: 300 inch boards, 14 ft. long, 8 in. wide, at $32.50 per M.; 50 joists, 4 in. by 3 in., and 12 ft. long, at $28 per M.; and 60 planks, 20 ft. long, 10 in. wide, and $2\frac{1}{2}$ in. thick, at $18.50 per M.

17. A piece of timber is 10 in. by 15 in. How long must it be to contain 200 board feet?

18. What is the value of 15 beams, 30 ft. long, 14 in. by 16 in., at 25¢ a cubic foot?

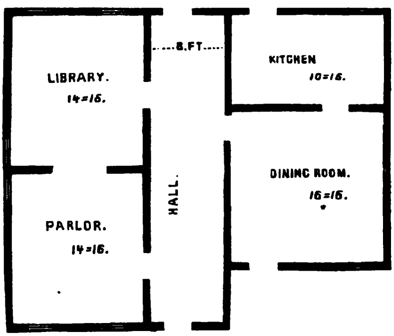
FLOORING, CARPETING, AND KALSOMINING.

794. *1.* Required to find the cost of flooring the rooms indicated in the plan, with lumber at $30 per M., no allowance being made for waste; the cost of laying the floor being $2 per square of 100 ft.

Process.

$$14 \times 32 = 448$$
$$16 \times 26 = 416$$
$$8 \times 32 = 256$$
$$1120$$

$$1.12 \times 30 = 33.60$$
$$11.2 \times 2 = 22.40$$
$$56.00$$



Explanation.—The surface of the library and parlor is 448 ft., of the kitchen and dining-room 416 ft., and of the hall 256 ft., making the entire surface 1120 ft. The lumber, at $30 per M., amounts to $33.60, and the cost of laying at $2 per square is $22.40, amounting in all to $56.

2. What will be the cost of kalsomining the ceilings of the rooms, including hall, in the foregoing plan, at 8¢ a square yard?

3. Required to find the cost of carpeting the parlor and library with Brussels carpet 27 in. wide, at $1.75 per yard, to be laid lengthwise, 6 in. being lost on each strip in matching.

Process.

$$(14 \times 12) \div 27 = 6\tfrac{2}{9}$$
$$(16 + 16 + 1) \div 3 = 11$$
$$11 \times 7 = 77$$
$$77 \times 1.75 = 134.75$$

Explanation.—Reducing 14 ft. to inches and dividing by 27 gives $6\frac{2}{9}$ as the number of breadths required. As 6 breadths will not cover the room, it will be necessary to buy 7. There being a loss of 6 inches on each strip, or 1 ft. on two strips, there will be needed 7 strips of 11 yards each, amounting to $134.75.

NOTE.—It will be seen from the explanation that the amount of carpet required to cover a given room depends not only upon the size of the room, but also upon the waste due to matching and upon the amount turned under where a full strip is not required.

4. How long must the strips of carpet $2\frac{1}{4}$ ft. wide be cut in order to carpet most economically a room 18 ft. long and 15 ft. wide, there being no waste for matching?

5. How many boards 16 ft. long and 4 in. wide are required to floor a room which is 48 ft. by 32 ft.?

6. Find the cost of carpeting two rooms each 18 ft. by 21 ft., and 3 rooms each 12 ft. by 16 ft., with ingrain carpet a yard wide, at 90¢ a yard, carpet to be laid lengthwise, and 3 yards allowed for waste in matching.

7. Find the cost of kalsomining the ceilings of the rooms, the dimensions of which are given in the preceding example, at 6¢ a square yard.

8. Find the cost of carpeting a room 18 ft. 4 in. by 14 ft. 8 in. with carpet 27 in. wide at $1.50 a yard and a border 22 in. wide at $1.25 a yard. The carpet is to be laid lengthwise, 4 in. being allowed on each strip for matching; the length of the border is the distance around the room.

PLASTERING, PAINTING, AND PAPERING.

795. *1.* Required to find the cost of plastering a room 30 ft. long, 20 ft. wide, and 10 ft. high, at 50¢ per square yard, no allowance being made for doors or windows.

Explanation.—The surface of the sides of the room, 600 ft., and of the ends, 400 ft., and of the ceiling, 600 ft., reduced to yards is $177\frac{7}{9}$, the plastering of which, at 50¢ a yard, amounts to $88.89.

NOTE.—In plastering, the usual custom is to make no allowance for doors and windows.

Process.

$$30 \times 10 \times 2 = 600$$
$$20 \times 10 \times 2 = 400$$
$$20 \times 30 = \underline{600}$$
$$1600$$
$$1600 \div 9 = 177\tfrac{7}{9}$$
$$177\tfrac{7}{9} \times .50 = 88.89$$

2. Required to find the cost of painting, with 3 coats, at 10¢ a square yard per coat, the woodwork of the same room, baseboards 12 in. wide, two doors each 3 ft. by 8 ft., and 8 windows each 3 ft. by 5 ft.

Explanation.—The distance around the room, less the space taken up by the doors, is the length of the baseboards, and, their width being 12 in., their surface is 94 ft. The surface of the doors is 48 ft., and of the windows 120 ft., making the entire surface 262 ft. Multiplying by 3, as there are to be 3 coats of paint, and dividing by 9 to reduce to yards —or dividing by 3, which amounts to the same—the surface to be painted is found to be $87\frac{1}{3}$ yd., amounting, at 10¢ per yd., to $8.73.

Process.

$$60 + 40 - 6 = 94$$
$$8 \times 3 \times 2 = 48$$
$$5 \times 3 \times 8 = \underline{120}$$
$$262$$
$$262 \div 3 = 87\tfrac{1}{3}$$
$$87\tfrac{1}{3} \times .10 = 8.73$$

NOTE.—In painting the woodwork of windows, it is customary to make no allowance for glass if there are more than two lights in the window.

3. Required to find the cost of papering the same room, each roll of paper being 8 yd. long and 18 in. wide, at 50¢ per roll, border 10¢ a yard.

Explanation.—The doors and windows measure in width 30 ft., which being deducted from 100 ft., the distance around the room, gives 70 ft. Each roll being 18 in. or $1\frac{1}{2}$ ft. wide, $46\frac{2}{3}$ strips will be required. The length of each roll being 8 yd. or 24 ft., two strips can be cut from a roll, thus requiring $23\frac{1}{3}$ rolls, or approximately 24 rolls, amounting at 50¢ a roll, to $12 ; $33\frac{1}{3}$ yd. of border at 10¢ amounts to $3.33, making a total cost of $15.33.

Process.

$$10 \times 3 = 30$$
$$60 + 40 - 30 = 70$$
$$70 \div 1\tfrac{1}{2} = 46\tfrac{2}{3}$$
$$8 \times 3 = 24$$
$$46\tfrac{2}{3} \div 2 = 23\tfrac{1}{3}$$
$$24 \times .50 = 12$$
$$(100 \div 3) \times .10 = 3.33$$
$$12 + 3.33 = 15.33$$

NOTE.—When there is much waste, as in the example, where it amounts to 4 ft. to each roll, it is assumed that this will cover the spaces under and over the doors and windows. Where there is little waste, an extra roll or two will be required.

4. Find the cost of lathing and plastering the rooms, including hall, shown in the plan on page 362, at 75¢ per square yard, if the height of the ceiling is 9 ft. 6 in. and no allowance is made for doors and windows.

5. How many rolls of paper will be required for papering the parlor and library in the same house, the doors and windows measuring 36 ft. in width?

6. What will it cost to paint the outside of a house, the entire surface of which is 20 ft. by 150 ft., at 18¢ per square yard?

7. Find the cost of lathing, plastering, and flooring a hall 30 ft. by 50 ft. and 12 ft. high, at 85¢ per square yard for lathing and plastering, and $1.75 per square of 100 ft. for flooring.

8. Find the cost of kalsomining the ceiling and papering the walls of the same hall, baseboards 10 in. wide, 8 windows 4 ft. by 10 ft., and 4 doors 4 ft. 3 in. by 9 ft.; the kalsomining at 7¢ a square yard, paper 60¢ a roll, border 15¢ a yard, two rolls of paper being allowed for spaces over doors and windows.

WOOD, STONE, AND BRICK.

796. *1.* Required to find the cost of a pile of wood 200 ft. long, 8 ft. high, and 4 ft. wide, at $6 a cord.

Explanation.—The pile being 8 ft. high and 4 ft. wide, 4 ft. in length make a cord; hence, dividing 200 by 4 gives 50, the number of cords, at $6 a cord amounting to $300.

Process.

$$200 \div 4 = 50$$
$$50 \times 6 = 300$$

2. Required to find the cost, at $2 a perch, of laying the walls of a cellar 28 ft. long and 22 ft. wide, outside measurement, the walls to be 8 ft. high and 18 in. thick, no allowance being made for openings.

Explanation.—The length of the wall, outside measurement, is 100 ft. Multiplying 100 by 8 and $1\frac{1}{2}$ gives 1200, the number of cu. ft. in the wall. Dividing 1200 by 25 gives 48 perches, amounting at $2 a perch to $96.

Process.

$$(28+22) \times 2 = 100$$
$$100 \times 8 \times 1\tfrac{1}{2} = 1200$$
$$1200 \div 25 = 48$$
$$48 \times 2 = 96$$

NOTE 1.—A perch is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, amounting to $24\frac{3}{4}$ cu. ft. It is customary, however, to call 25 cu. ft. a perch.

NOTE 2.—In finding the cost of material, the corners are measured but once and allowance is made for doors and windows, but in estimating mason-work the corners are counted twice.

3. Required to find the number of bricks it will take to build a house, the walls of which are 50 ft. long, 25 ft. wide, 40 ft. high, and 1 ft. thick, making no allowance for windows, doors, nor corners.

Explanation.—There are 6000 cubic feet of wall. Allowing 20 bricks for a cubic foot, it will take to build the house 120 thousand bricks.

Process.

$$(50+25) \times 2 = 150$$
$$150 \times 40 = 6000$$
$$6000 \times 20 = 120000$$

NOTE.—The average size of bricks is 8 in. long, 4 in. wide, and 2 in. thick. In estimating 20 bricks to a cu. ft. of wall, allowance is made for mortar.

4. What is the cost of a pile of stone 32 ft. long, 10 ft. wide, and 6 ft. high, at $4.50 per cord?

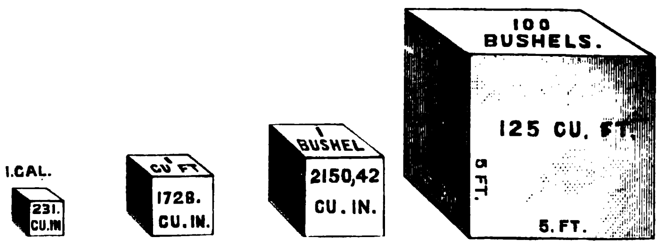
5. If a pile of wood is 64 ft. long and 4 ft. wide, how high must it be to contain 24 cords?

6. How much will the bricks for a wall 75 ft. long, 20 ft. high, and 1 ft. 6 in. thick cost, at $6.50 per M.?

7. How many perches of stone are there in a wall 6 ft. high, 2 ft. 4 in. thick, and 10 rods long?

8. How many cords of stone will be required to build a wall around a square ten-acre lot, the wall being 5 ft. high and 3 ft. thick?

BINS, TANKS, ETC.



797. *1.* Required to find the number of bushels of wheat that may be put in a granary of 5 bins, each 8 ft. long, 5 ft. wide, and 6 ft. deep.

Explanation.—Each bin contains 240 cu. ft., and the 5 bins contain 1200 cu. ft. Dividing the cu. in. in a bushel, 2150.42, by the cu. in. in a cu. ft., 1728, gives 1.24+, or approximately 1.25. Dividing 1200 by 1.25 gives 960 bushels as the capacity of the granary.

Process.

$$8 \times 5 \times 6 = 240$$
$$240 \times 5 = 1200$$
$$2150.42 \div 1728 = 1.24+$$
$$1200 \div 1.25 = 960$$

NOTE.—It is to be observed that a bushel is $\frac{1}{4}$ larger than a cu. ft., and that a bin whose capacity is 125 cu. ft. contains approximately 100 bushels.

2. Required to find the capacity in gallons of a rectangular cistern that is 8 ft. long, 7 ft. wide, and 5 ft. deep.

Explanation.—The capacity of the cistern, 280 cu. ft., reduced to inches and divided by the number of cu. in. in a gallon, 231, gives 2094.5, the contents of the cistern in gallons.

Process.

$$8 \times 7 \times 5 = 280$$
$$1728 \times 280 = 483840$$
$$483840 \div 231 = 2094.5+$$

3. Required to find the capacity in tons of a mow of hay 30 ft. long, 16 ft. wide, and 15 ft. high, 450 cu. ft. of hay weighing a ton.

Explanation.—The capacity of the mow in cu. ft. divided by 450, the number of cu. ft. of hay weighing a ton, gives 16, the contents of the mow in tons.

Process.

$$30 \times 16 \times 15 = 7200$$
$$7200 \div 450 = 16$$

NOTE.—The weight of hay depends upon the kind of hay and the size of the mow, the above estimate being a fair average for dry and well-settled hay in large mows.

4. How many tons of coal will fill a bin 10 ft. long, 7 ft. wide, and 5 ft. deep, estimating 35 cu. ft. of coal to the ton?

5. What must be the depth of a bin 10 ft. long and 8 ft. wide, to contain 400 bushels of grain? How many bushels of potatoes would the bin hold, a bushel of potatoes being heaped so as to contain $\frac{1}{4}$ more than a bushel of grain?

6. How many gallons in a tank 15 ft. long, 10 ft. wide, and 8 ft. deep? How many tons of coal in a bin of the same dimensions?

7. The length of a mow is 32 ft., and the width 18 ft.; what must be the height of the hay, that it may contain 12 tons?

8. If a bin is 15 ft. long and 7 ft. wide, how high must it be that it may contain 15 tons of coal, 35 cu. ft. of coal weighing a ton?

ANSWERS.

Art. 31.

1. 5,468.
2. 10,775.
3. 37,037.
4. 524,029.
5. 909,440.
6. 700,074.
7. 1,721,847.
8. 6,014,005.
9. 61,513,004.
10. 193,000,205.
11. 403,500,058.
12. 9,118,653,712.
13. 75,083,704,000.
14. 145,300,043,407.

Art. 36.

1. 24.
2. 29.
3. 38.
4. 45.
5. 44.
6. 49.
7. 69.
8. 94.
9. 99.
10. 108.
11. 219.
12. 400.
13. 369.
14. 449.
15. 504.
16. 509.
17. 562.
18. 598.
19. 889.
20. 1100.
21. 1400.
22. 1619.
23. 4000.
24. 50,619.

Art. 37.

1. XXIII.
2. XV.
3. XXIX.
4. XXXVII.
5. XLII.
6. XLVIII.
7. LXVII.
8. XCIII.
9. LXXXIX.
10. CXIII.
11. CIV.
12. CCL.
13. CCXXXIX.
14. CCCLXV.
15. CCCCXLIX.
16. CCCCLXXX-VII.
17. DVI.
18. DCCCCXXIII.
19. DCCCCXLIX.
20. DCCCCXCIX.
21. MXIX.
22. MCXCVIII.
23. MCCCCLXIX.
24. MCCCCXCIX.
25. MDCLIX.
26. MMXXV.
27. $\overline{\mathrm{X}}$.
28. $\overline{\mathrm{L}}$.

Art. 55.

1. 1443.
2. 2441.
3. 5410.
4. 6695.
5. 8940.
6. 13,856.
7. 18,696.
8. 24,573.
9. 400,158.

Art. 62.

1. 82.
2. 87.
3. 88.
4. 86.
5. 85.
6. 80.
7. 86.
8. 75.
9. 88.
10. 88.

Art. 64.

1. $4245.
2. $4868.
3. $4347.
4. $53.36.
5. $40.99.
6. 17,925.
7. 15,975.
8. $2372.53.
9. $1593.57.

Art. 65.

1. 926,247.
2. 131,603.
3. 80,772.
4. 29,807.
5. 285,502.
6. 454,598.
7. 2516.
8. $12,791.29.
9. $51,647.39.
10. $452,893.
11. $6510.67.

Art. 67.

1. $9851.
2. $15,525.
3. $12,800.
4. $7017.
5. 17760 trees.
6. 9169 lb.
7. $4630.
8. $70,870.
9. $1339.
10. $52,763.
11. $18,745.
12. $47,736.
13. 23,768 bbl.
14. 250,540.
15. 23,860 sq. mi.
16. $172,225.

Art. 81.

1. 4111.
2. 7302.
3. 9443.
4. 55,455.
5. 10,597.
6. 85,108.
7. 120,881.
8. 199,900.
9. 97,551.

Art. 84.

1. 80,492.
2. 143,594.
3. 114,266.
4. 29,952.
5. 562,618.
6. 46,608.
7. 284,729.
8. 233,078.
9. 315,389.
10. 38,198.
11. 366,991.
12. 88,982.
13. 714,863.
14. 168,599.
15. 21,984.
16. 1,265,499.
17. 479,989.
18. 1,997,999.
19. 898,989.
20. 2,200,151.

Art. 87.

1. 10.
2. 24.
3. 4.
4. 35.
5. 45.
6. 13.
7. 32.
8. 29.
9. 411.
10. 483.
11. 1814.
12. 125.
13. 95.
14. 378.
15. 890.
16. 276.
17. 80.
18. 526.

Art. 90.

1. 577.
2. 33 years.
3. 427.
4. 445 bu.
5. $1772.
6. $2445.
7. $6266.
8. 377.
9. $11,550.
10. $26,657.
11. $8449.
12. 18,113 miles.
13. 11,102 feet.
14. $6173.
15. 37,690 sq. mi., California.
16. $2833, gain.
17. $2075.
18. $73,031.
19. $73,685.
20. 1,889,888.

Art. 108.

1. 3456.
2. 14,586.
3. 33,296.
4. 28,400.
5. 44,154.
6. 56,000.
7. 48,056.
8. 27,612.
9. 111,000.
10. 225,954.
11. 119,325.
12. 5,297,810.
13. 7,204,491.
14. 2,149,630.
15. 3,346,368.
16. 3,008,472.
17. 5,192,982.
18. 6,372,864.

Art. 111.

1. 3432.
2. 5845.
3. 9454.
4. 18,576.
5. 26,176.
6. 39,900.
7. 725,112.
8. 627,255.
9. 678,069.
10. 1,617,972.
11. 3,907,533.
12. 3,222,552.
13. 459,075,482.
14. 326,481,129.
15. 559,549,866.
16. 586,949,784.
17. 468,563,430.
18. 341,394,697.

Art. 114.

1. 13,710.
2. 79,500.
3. 119,300.
4. 286,720.
5. 466,880.
6. 2,880,000.
7. 6,840,000.
8. 2,000,000.
9. 5,810,000.
10. 1,800,000.
11. 4,500,000.
12. 62,325,000.
13. 79,550,000.
14. 11,934,000.
15. 87,906,000.
16. 51,000,000.
17. 36,000,000.
18. 35,000,000.

Art. 115.

1. 164,640,000.
2. 104,815,200.
3. 116,610,000.
4. 103,320,000.
5. 414,375,000.
6. 52,189,200,000.
7. 115,474,560,000.
8. 149,073,000,000.
9. 339,086,250,000.
10. 296,963,820,000.

Art. 117.

1. 545,268.
2. 61,789,435.
3. 906,102.
4. 3,830,310.
5. 1360.
6. 5167.
7. 41,115.
8. 48,755.
9. 290.

10. 1260.
11. 88,618,554.
12. 29,922,300.
13. 171,443,702.
14. 1,813,375,181.

Art. 119.

1. \$57,090.
2. \$20,412.
3. \$31,374.
4. 4800 miles.
5. \$385,925.
6. \$2906.25.
7. 6675 miles.
8. \$9,043,083.
9. 1,455,520 yds.
10. \$64,575.
11. \$1181.25.
12. 779,172 hills.
13. \$16,584.15.
14. \$10,280.
15. \$1508 gain.
16. \$2676.24.
17. \$8262.98.
18. \$5275.
19. 91,289,796 mi.
20. 952 mi.
21. \$546.15 gain.

Art. 133.

1. $51,356\frac{4}{8}$.
2. $39,844\frac{6}{8}$.
3. $104,303\frac{4}{7}$.
4. 105,449.
5. 64,833.
6. $86,547\frac{1}{7}$.
7. $733,428\frac{7}{8}$.
8. $1,046,724\frac{3}{8}$.
9. $199,827\frac{7}{8}$.
10. $552,208\frac{6}{7}$.
11. $1,031,948\frac{4}{9}$.
12. $946,234\frac{1}{8}$.
13. 43,750.
14. 150.
15. $1824\frac{4}{6}$.
16. $1253\frac{4}{7}$.
17. $545\frac{6}{7}$.
18. 449.
19. $944\frac{2}{9}$.
20. $71,952\frac{5}{8}$.

Art. 138.

1. 15,236,207,439.
2. $17,321,481,709\frac{1}{4}$.
3. $11,967,254,831\frac{4}{5}$.
4. $6,082,105,310\frac{4}{6}$.
5. $12,790,974,799\frac{5}{7}$.
6. $10,384,100,030\frac{1}{6}$.
7. $6,651,372,142\frac{8}{9}$.
8. $12,926,513,562\frac{4}{5}$.
9. $42,992,018,381\frac{3}{9}$.
10. $91,229,331,230\frac{7}{8}$.
11. $46,696,417,694\frac{6}{7}$.
12. $46,674,222,291\frac{8}{9}$.
13. 50,028,071,228.
14. $27,791,680,569\frac{3}{8}$.
15. 89,666,679,000.
16. 45,096,987,375.

Art. 142.

1. $502\frac{13}{27}$.
2. $989\frac{12}{29}$.
3. $830\frac{27}{61}$.
4. $922\frac{29}{42}$.
5. $960\frac{5}{83}$.
6. $828\frac{5}{52}$.
7. $1501\frac{38}{48}$.
8. $10,255\frac{30}{81}$.
9. $9971\frac{56}{93}$.
10. $4819\frac{31}{76}$.
11. $9839\frac{8}{87}$.
12. $9726\frac{65}{98}$.
13. $5955\frac{44}{123}$
14. $2256\frac{133}{234}$.
15. $1646\frac{318}{346}$.
16. $2139\frac{98}{169}$.
17. $869\frac{232}{456}$.
18. $2040\frac{87}{387}$.
19. $1003\frac{16}{656}$.
20. $4167\frac{76}{237}$.
21. $442\frac{588}{978}$.
22. $1662\frac{193}{196}$.
23. 38,688.
24. $\$599,792\frac{575}{776}$.
25. $77,223,413\frac{80}{496}$
26. 298.
27. 848.
28. 19,609,375.
29. $4\frac{37}{98}$.
30. 3128.
31. $14\frac{50711}{67886}$.

Art. 144.

1. 87 bu.

2. $4.37.
3. $638\frac{17}{35}$ days.
4. 38 tons.
5. \$$2169\frac{35}{52}$.
6. 1238 cattle.
7. $30,575.
8. 113 days.
9. \$$136\frac{360}{365}$.
10. $75.
11. $175.
12. 760 bu.
13. $1.75.

Art. 163.

1. 91.
2. 259.
3. 103.
4. 144.
5. \$$176\frac{5}{8}$.
6. \$$455\frac{5}{12}$.
7. $4309\frac{1}{5}$.
8. \$$2970\frac{7}{10}$.
9. \$$2523\frac{1}{8}$.
10. $30,647\frac{1}{2}$.
11. \$$314,830\frac{4}{5}$.
12. $3154\frac{15}{16}$ lb.
13. $1162\frac{3}{5}$ mi.
14. $38,663\frac{2}{3}$ ft.

Art. 165.

1. $258\frac{3}{4}$ cords.
2. $7007\frac{1}{4}$ tons.
3. $233\frac{1}{6}$ yds.
4. $128\frac{1}{4}$ rods.
5. $140\frac{3}{4}$ yds.
6. $784\frac{1}{8}$ qts.
7. \$$301\frac{3}{20}$.
8. 49,831 oz.
9. \$$112\frac{3}{20}$.

Art. 173.

1. $14\frac{1}{6}$.
2. $38\frac{1}{12}$.
3. $39\frac{1}{8}$.
4. $43\frac{5}{12}$.
5. $54\frac{1}{10}$.
6. $106\frac{1}{2}$.
7. $75\frac{5}{12}$.
8. $661\frac{4}{15}$.
9. $708\frac{1}{6}$.
10. $1881\frac{1}{9}$.
11. $952\frac{1}{10}$.
12. $2289\frac{1}{2}$.

Art. 174.

1. $1279\frac{3}{4}$.
2. $767\frac{5}{12}$.
3. $242\frac{11}{12}$.
4. $856\frac{7}{10}$.
5. $834\frac{1}{9}$.
6. $1036\frac{3}{10}$.
7. $43\frac{1}{12}$.
8. $698\frac{3}{8}$.
9. $27\frac{1}{3}$.
10. $276\frac{1}{12}$.
11. $265\frac{23}{24}$.
12. 42.
13. $421\frac{1}{4}$.
14. $29\frac{3}{4}$.
15. $70\frac{13}{36}$.
16. $200\frac{4}{5}$.

Art. 176.

1. $318\frac{7}{8}$ bu.
2. $37\frac{5}{6}$ mi.
3. \$$59\frac{9}{10}$.
4. $63\frac{15}{16}$ acres.
5. $50\frac{1}{2}$ gal.
6. $19\frac{1}{12}$ years.
7. \$$41,402\frac{7}{10}$.

Art. 182.

1. 175.
2. 343.
3. $680\frac{2}{5}$.
4. $10,904\frac{3}{8}$.
5. $20,346\frac{6}{7}$.
6. $52,148\frac{1}{4}$.
7. $2,448,631\frac{7}{8}$.
8. $41,672,027\frac{1}{9}$.
9. $54,096,276\frac{2}{3}$.
10. 455,432,685.

Art. 183.

1. $907,774\frac{1}{8}$.
2. $205,000.
3. \$$23,779\frac{1}{2}$.
4. \$$571,479\frac{3}{4}$.
5. $52,389,775\frac{2}{3}$.

Art. 185.

1. $651.
2. \$$172\frac{19}{32}$.
3. \$$8160\frac{3}{4}$.

4. 149,100 sq. mi.
5. \$3000.
6. $3571\frac{7}{8}$ bu.
7. 4194 mi.
8. 3,275,000 sq. mi.
9. 136,620 ft.
10. \$$93\frac{3}{20}$.
11. \$$163\frac{4}{5}$.
12. \$$43\frac{3}{4}$.
13. $3042\frac{3}{8}$ mi.

Art. 192.

1. $23\frac{1}{13}$.
2. $9\frac{3}{19}$.
3. $20\frac{7}{13}$.
4. $11\frac{14}{101}$.
5. $26\frac{8}{101}$.
6. $7\frac{7}{24}$.
7. $10\frac{11}{45}$.
8. $36\frac{1}{80}$.
9. $11\frac{59}{204}$.
10. $41\frac{5}{108}$.
11. $45\frac{1}{3}$.
12. $159\frac{851}{896}$.
13. $639\frac{667}{1644}$.
14. $261\frac{1783}{2937}$.
15. $1879\frac{385}{581}$.

Art. 194.

1. $56\frac{7}{8}$ lb.
2. 9 miles.
3. 63,360 in.
4. \$72.
5. 80 hrs.
6. \$$5\frac{1}{3}$.
7. 360.
8. \$7672.
9. 3020 bu.
10. 800 T.; 2000 T.
11. $164\frac{1}{2}$ A.
12. 200 A.
13. \$4.
14. 80 cords.
15. $72\frac{3}{4}$ yds.

Art. 200.

1. .5.
2. .05.
3. .005.
4. .0005.
5. .0015.
6. .55.
7. .155.
8. .0155.
9. .123.
10. .505.
11. .075.
12. .0075.
13. .705.
14. .045.
15. .007.

Art. 203.

1. 127.7.
2. 45.45.
3. 45.05.
4. 36.066.
5. 1728.7.
6. 1728.17.
7. 1728.172.
8. 1728.1728.
9. 4006.006.
10. 4000.004.
11. 404.0404.
12. 4040.4004.

Art. 208.

1. \$357.13.
2. 3164.097
3. \$3559.37.
4. 379.5073.

Art. 209.

1. \$116.60.
2. \$$81.62\frac{1}{2}$.
3. \$$39.06\frac{1}{4}$.
4. 79.891.
5. 13.913.
6. 905.001.
7. 785.928.
8. \$$56.95\frac{5}{8}$.
9. \$$19.79\frac{1}{8}$.
10. 1000.011.

Art. 214.

1. 87,500.
2. 98,350.
3. 687,500.
4. 2926.5.
5. 945.6.
6. 74.56.
7. 613.27.
8. 97.5.

9. 38.755.
10. 83.1762.
11. 5.18423.
12. 3.23.

Art. 218.

1. $75.151.
2. $73.451.
3. $400.404.
4. $1000.044.
5. $45,000.50.
6. $400.05.
7. $180.06½.

Art. 222.

1. $12.75.
2. $70.80.
3. $15.025.
4. $368.05½.
5. $125.
6. $162.62½.
7. $300.125.
8. $5905.05½.
9. $1881.83¾.

Art. 223.

1. 7500 cts.
2. 8756 cts.
3. 93,625 m.
4. 1818¾ cts.
5. 10,606¼ cts.
6. 15,766⅔ cts.
7. 837,533⅓ cts.
8. 187,550 cts.
9. 936,275 cts.

Art. 226.

1. $3695.88.
2. $2204.45.
3. $23,744.57.
4. $1926.998.

Art. 227.

1. $83.23.
2. $86.79.
3. $150.44.
4. $18.19.
5. $114.245.
6. $66.67.
7. $424.82.
8. $261.692½.
9. $277.667.
10. $44.445.

Art. 229.

1. $1724.25.
2. $5028.50.
3. $4662.50.
4. $87,515.
5. $5476.
6. $1214.06¼.
7. $15,656.25.
8. $564.
9. $275.
10. $553.66.
11. $954.10.
12. $7707.37½.
13. $5470.27¾.
14. $6206.85¼.
15. $12,216.09.
16. $10,414.99½.

Art. 231.

1. 65.
2. 125.
3. 138.
4. 1460.
5. 1380.
6. 1200.
7. $79\frac{775}{1225}$.
8. $6.
9. $50.
10. 62½.
11. 49.
12. 131.
13. 829.
14. $12.50.
15. $1666\frac{750}{1125}$.
16. 125.
17. $6.15.
18. $19.20.
19. $9262.625.
20. $2886.25.
21. 6126.
22. 6380.

Art. 234.

1. $12,544; $10,584; $10,976.
2. $1715.
3. $954⅔.
4. 24,800 bu.; 16,256 bu.
5. 80 days.
6. $44,417⅗.
7. $8883.

8. $468\frac{3}{4}$ bu.
9. $780.
10. $$4138\frac{1}{3}$.

Art. 241.

1. $$182.17\frac{1}{4}$.
2. $378.60.
3. $230.94.
4. $$188.63\frac{1}{4}$.

Art. 243.

1. $1344.68.
2. $4625.98.
3. $28527.74.
4. $29838.77.

Art. 244.

1. $$7.02\frac{1}{4}$.
2. $407.235.
3. $$2195.168\frac{1}{2}$.

Art. 245.

1. $$100.30\frac{1}{4}$.
2. $$1468.10\frac{3}{4}$.
3. $$17031.41\frac{7}{12}$.

Art. 248.

1. $147.
2. $19.50.
3. 660 ft.; 482 ft.; 5280 ft.
4. 1760 yds.; 4840 yds.
5. 158400 steps.
6. 95 ft.; 5280 ft.
7. 3520 rds.
8. $$134\frac{13}{32}$.
9. 1600 rds.; 1376 rds.; 22,704 ft.
10. 973 rds.; $16,054\frac{1}{2}$ ft., 192,654 in.
11. 1661 rds.; $9135\frac{1}{2}$ yds.; $27,406\frac{1}{2}$ ft.
12. 126 ft.; 42 yds.
13. 15,840 ft.
14. 27,899 ft.; $9299\frac{2}{3}$ yds.
15. 80 lengths.
16. $1.80.
17. $25\frac{2}{3}$ yds.
18. 98 rds.
19. 276 mi.
20. 2600 ft.
21. 285,120 ft.
22. 18,000 ft.
23. $$4.14\frac{3}{4}$.
24. 2112 steps.
25. $30\frac{2}{3}$ yds.
26. $44\frac{3}{4}$ yds.
27. $$.62\frac{1}{2}$.

Art. 251.

1. 332 pints.
2. $$55.12\frac{1}{2}$.
3. $2.49.
4. $40.32.
5. $$37.18\frac{3}{4}$.
6. 64 times.
7. $$56.06\frac{1}{4}$.
8. $842.80.
9. $14.60.
10. $31.11.
11. 43 gal. 1 pt.
12. 24 bbl.
13. $$158.81\frac{1}{4}$.
14. 398 bottles.
15. 297 pints.
16. $6.75.

Art. 254.

1. 23 pks.; 184 qts. 368 pts.
2. 83 pts.
3. 991 pts.
4. 432 qts.; 54 pks.; $13\frac{1}{2}$ bu.
5. 97 pks. 6 qts 1 pt.
6. $6.
7. $11.10.
8. $76.65.
9. $$28.83\frac{1}{4}$.
10. 4100 lb.
11. $10.50.
12. $5.28.
13. $21.62.
14. 50 times.
15. $$6.37\frac{1}{2}$.

Art. 259.

1. 11,787 lb.
2. 13,206 lb.
3. 71.87\frac{1}{2}$.
4. $28.78.
5. 125,440 lb.
6. $98.35.
7. 6.03\frac{3}{4}$.
8. 38.84\frac{3}{4}$.
9. 320 lb.
10. 48 spoons.
11. 2 T. 12 cwt. 29 lb. 11 oz.
12. $138.75.
13. $1.40.
14. 476 cans.
15. 5787$\frac{1}{2}$ lb.

Art. 264.

1. $5218.40.
2. $2168.80.
3. 3101$\frac{1}{8}$ acres.
4. $1650.50.
5. $6128.75.
6. $24,719.
7. $1251.15.
8. $4858.91.
9. $10,296.45.
10. 9\frac{11}{20}$.
11. 154$\frac{19}{20}$ oz.
12. $\frac{3}{4}$.
13. 64$\frac{1}{3}$ ft.
14. $1.60.
15. 62$\frac{7}{8}$ lb.
16. $11.86.
17. 33$\frac{1}{6}$ days.
18. 126$\frac{1}{2}$ qts.
19. 200 mi.
20. 58$\frac{1}{20}$ T.
21. 1262$\frac{1}{4}$ bu.
22. 219$\frac{1}{10}$ rds.

Art. 265.

1. 5\frac{3}{10}$.
2. $\frac{11}{12}$ ft.
3. 30$\frac{1}{2}$ lb.
4. 4$\frac{2}{3}$ ft.
5. $\frac{11}{20}$.
6. $4225.
7. $2630 loss.
8. $167.
9. $3461.56.
10. 18$\frac{5}{8}$ lb.
11. 6$\frac{5}{8}$ lb.
12. 8\frac{19}{20}$.
13. $4.15.
14. $16.77.
15. 6$\frac{5}{12}$ yds.
16. $13.
17. 6$\frac{1}{8}$ ft.
18. 281$\frac{1}{6}$ yds.
19. 608$\frac{5}{8}$ lb.

Art. 266.

1. $15.12.
2. 223\frac{5}{8}$.
3. $249.90.
4. $868.40.
5. $16,310.
6. 13$\frac{1}{2}$ yds.
7. 4.45\frac{1}{2}$.
8. $18.
9. 35.03\frac{1}{8}$.
10. $1.14.
11. 4.08\frac{1}{4}$.
12. 241\frac{3}{8}$.
13. $971.10.
14. $8.85, the latter.
15. 5478\frac{1}{8}$, A· 3286\frac{7}{8}$, B.
16. $12.
17. $8119.40.
18. $262.50.
19. $1.32.
20. 484$\frac{1}{2}$ yds.
21. 360\frac{4}{7}$.
22. 135.18\frac{3}{4}$.
23. 10$\frac{3}{8}$ gal.
24. $80.
25. 40.97\frac{1}{4}$.

Art. 267.

1. 498$\frac{43305}{50922}$.
2. $4.30.
3. $9995.
4. $7951.
5. $9675.
6. 20 cts.
7. 16 cts.
8. $1.40.
9. 3$\frac{1}{2}$ wks.
10. 42 bu.
11. 16 widths; $300.

12. \$23,328.
13. 775 bu.
14. $46\frac{2}{3}$ yds.
15. 347 cattle.
16. \$3.45; \$2.70.
17. 125 cords.
18. \$625.
19. \$8000.
20. 67 lots.
21. \$1.56.
22. $20\frac{2}{3}$ hrs.
23. \$262.50.
24. $240\frac{2}{3}$ yds.
25. \$94,000.
26. $12\frac{1}{2}$ cords.
27. 1890 bu.
28. 9 days.
29. \$39.
30. \$2.50.
31. \$4.35.
32. \$6016.50.
33. 30 mi.
34. 16 hrs.

Art. 285.

1. 3, 5, 5.
2. 2, 3, 3, 5.
3. 2, 2, 3, 3, 3.
4. 2, 2, 3, 11.
5. 2, 2, 2, 2, 3, 3.
6. 2, 3, 5, 5.
7. 2, 2, 2, 2, 2, 5.
8. 3, 3, 3, 7.
9. 2, 2, 2, 3, 3, 3.
10. 2, 2, 2, 2, 3, 5.
11. 2, 2, 2, 2, 2, 2, 2, 2.
12. 2, 2, 2, 5, 7.
13. 3, 3, 5, 7.
14. 2, 2, 79.
15. 2, 2, 11, 11.
16. 2, 2, 2, 2, 2, 2, 2, 2, 5.
17. 2, 2, 2, 2, 3, 3, 13.
18. 2, 2, 3, 3, 89.
19. 2, 2, 5, 199.
20. 7, 7, 11, 13.

Art. 289.

1. 36.
2. 24.
3. 480.
4. 15.
5. 7.
6. 8.
7. $8\frac{1}{3}$.
8. 10.
9. $\frac{4}{21}$.
10. 14.
11. $10\frac{5}{12}$.
12. $6\frac{107}{128}$.
13. 18.
14. $3\frac{9}{25}$.
15. $1\frac{242}{455}$.
16. $166\frac{2}{3}$.
17. 10.
18. 4.
19. 800.

Art. 290.

1. 147 cases.
2. $18\frac{3}{4}$ bu.
3. 54 yds.
4. \$1.50.
5. 9 bbl.
6. 12 crates.
7. 16 men.
8. 16 boxes.
9. 285 crates.

Art. 297.

1. 6.
2. 15.
3. 9.
4. 14.
5. 21.
6. 24.
7. 27.
8. 35.
9. 16.
10. 39.
11. 15.
12. 18.
13. 12.
14. 24.
15. 23.
16. 37.
17. 31.
18. 29.
19. 29.
20. 87.
21. 67.
22. 53.
23. 63.

Art. 298.

1. $141; 192 horses.
2. 6 gal.
3. 15 ft.
4. 428 bags.
5. 270 boards.

Art. 306.

1. 60.
2. 105.
3. 210.
4. 420.
5. 450.
6. 900.
7. 360.
8. 270.
9. 750.
10. 1440.
11. 5040.
12. 2835.
13. 3024.
14. 24,255.

Art. 307.

1. 120 yds.
2. $720; 20 cows; 48 hogs; 15 oxen.
3. 210 days.
4. 480 acres.
5. 924 inches.
6. $2625.
7. 960 days.

Art. 322.

1. $\frac{3}{5}$.
2. $\frac{3}{5}$.
3. $\frac{2}{3}$.
4. $\frac{3}{5}$.
5. $\frac{3}{4}$.
6. $\frac{5}{7}$.
7. $\frac{3}{4}$.
8. $\frac{4}{5}$.
9. $\frac{2}{5}$.
10. $\frac{12}{13}$.
11. $\frac{5}{8}$.
12. $\frac{11}{35}$.
13. $\frac{3}{8}$.
14. $\frac{3}{7}$.
15. $\frac{5}{7}$.
16. $\frac{11}{13}$.
17. $\frac{5}{27}$.
18. $\frac{11}{12}$.
19. $\frac{7}{11}$.
20. $\frac{5}{9}$.
21. $\frac{4}{7}$.
22. $\frac{5}{6}$.
23. $\frac{5}{8}$.
24. $\frac{9}{13}$.
25. $\frac{7}{11}$.
26. $\frac{5}{8}$.
27. $\frac{27}{32}$.
28. $\frac{79}{131}$.
29. $\frac{3}{4}$.
30. $\frac{3}{5}$.
31. $\frac{2}{3}$.
32. $\frac{2}{3}$.

Art. 325.

1. $\frac{26}{3}$.
2. $\frac{48}{5}$.
3. $\frac{141}{8}$.
4. $\frac{143}{6}$.
5. $\frac{295}{9}$.
6. $\frac{459}{10}$.
7. $\frac{263}{12}$.
8. $\frac{725}{7}$.
9. $\frac{1855}{4}$.
10. $\frac{2526}{11}$.
11. $\frac{1234}{23}$.
12. $\frac{1063}{36}$.
13. $\frac{2453}{28}$.
14. $\frac{2846}{15}$.
15. $\frac{4691}{2}$.
16. $\frac{15051}{4}$.
17. $\frac{14087}{3}$.
18. $\frac{16639}{20}$.
19. $\frac{19592}{25}$.
20. $\frac{5377}{36}$.
21. $\frac{4819}{49}$.
22. $\frac{3631}{876}$.
23. $\frac{1886}{125}$.
24. $\frac{127037}{1728}$.
25. $\frac{12323}{2025}$.
26. $\frac{95083}{5280}$.
27. $\frac{35731}{100}$.
28. $\frac{85179}{1000}$.

Art. 328.

1. $15\frac{2}{3}$.
2. $16\frac{3}{5}$.
3. $27\frac{3}{7}$.
4. $208\frac{3}{4}$.

5. 108.
6. $16\frac{5}{6}$.
7. $115\frac{1}{5}$.
8. $1036\frac{1}{3}$.
9. 81.
10. $120\frac{1}{13}$.
11. 848.
12. $347\frac{7}{8}$.
13. $427\frac{1}{9}$.
14. $120\frac{3}{4}$.
15. $45\frac{2}{3}$.
16. $734\frac{3}{5}$.

Art. 331.

1. $\frac{9}{12}$.
2. $\frac{24}{30}$.
3. $\frac{25}{40}$.
4. $\frac{16}{24}$.
5. $\frac{28}{36}$.
6. $\frac{54}{63}$.
7. $\frac{35}{84}$.
8. $\frac{42}{28}$.
9. $\frac{80}{48}$.
10. $\frac{99}{135}$.
11. $\frac{13\frac{1}{3}}{20}$.
12. $\frac{7\frac{1}{2}}{12}$.
13. $\frac{22\frac{1}{2}}{10}$.
14. $\frac{26\frac{2}{3}}{32}$.

Art. 336.

1. $\frac{45}{3}$.
2. $\frac{115}{5}$.
3. $\frac{104}{8}$.
4. $\frac{133}{7}$.
5. $\frac{128}{4}$.
6. $\frac{116}{12}$.
7. $\frac{112}{20}$.
8. $\frac{180}{21}$.
9. $\frac{129}{15}$.
10. $\frac{305}{40}$.
11. $\frac{835}{30}$.
12. $\frac{675}{36}$.
13. $\frac{2268}{60}$.
14. $\frac{8}{12}$; $\frac{9}{12}$; $\frac{10}{12}$.
15. $\frac{27}{63}$; $\frac{28}{63}$; $\frac{138}{63}$.
16. $\frac{30}{48}$; $\frac{100}{48}$; $\frac{249}{48}$.
17. $\frac{48}{60}$; $\frac{54}{60}$; $\frac{55}{60}$.
18. $\frac{160}{60}$; $\frac{225}{60}$; $\frac{354}{60}$.
19. $\frac{366}{42}$; $\frac{287}{42}$; $\frac{327}{42}$.
20. $\frac{835}{45}$; $\frac{795}{45}$; $\frac{882}{45}$.

Art. 340.

1. $1\frac{29}{36}$.
2. $2\frac{1}{5}$.
3. $2\frac{5}{18}$.
4. $2\frac{3}{28}$.
5. $2\frac{19}{80}$.
6. $1\frac{43}{48}$.
7. $2\frac{23}{60}$.
8. $2\frac{11}{15}$.
9. $2\frac{5}{42}$.
10. $1\frac{83}{108}$.
11. $2\frac{23}{63}$.
12. $2\frac{8}{45}$.
13. $2\frac{1}{336}$.
14. $2\frac{82}{315}$.
15. $23\frac{19}{20}$.
16. $23\frac{5}{12}$.
17. 25.
18. $20\frac{14}{27}$.
19. $1501\frac{1}{21}$.
20. $941\frac{3}{35}$.
21. $48\frac{31}{192}$.
22. $612\frac{117}{160}$ miles.
23. $\$4429\frac{3}{10}$.
24. $1580\frac{19}{20}$ acres.

Art. 342.

1. $\frac{71}{90}$.
2. $765\frac{3}{16}$ acres.
3. $743\frac{49}{60}$ miles.
4. 1240 miles.
5. $20\frac{2}{15}$ hrs.
6. $39\frac{15}{16}$ bu.
7. $1240\frac{5}{8}$ bu.
8. $\$1152\frac{13}{20}$.
9. $100.
10. $\$151\frac{1}{2}$.
11. $697\frac{57}{80}$ acres.
12. $63\frac{3}{4}$ gal.

Art. 346.

1. $\frac{1}{18}$.
2. $\frac{1}{24}$.
3. $\frac{13}{36}$.
4. $\frac{19}{36}$.
5. $\frac{17}{48}$.
6. $\frac{13}{60}$.
7. $\frac{31}{72}$.
8. $\frac{11}{60}$.
9. $\frac{11}{42}$.

10. $\frac{5}{48}$.
11. $\frac{37}{108}$.
12. $\frac{35}{144}$.
13. $\frac{59}{156}$.
14. $\frac{67}{102}$.
15. $\frac{53}{114}$.
16. $\frac{29}{102}$.
17. $1\frac{1}{30}$.
18. $1\frac{1}{6}$.
19. $1\frac{13}{60}$.
20. $1\frac{2}{5}$.
21. 3.
22. $7\frac{23}{60}$.
23. $26\frac{131}{144}$.
24. $31\frac{31}{60}$.
25. $12\frac{3}{8}$.
26. $36\frac{1}{36}$.
27. $178\frac{43}{48}$.
28. $26\frac{253}{432}$.

Art. 348.

1. $6\frac{13}{20}$ hours.
2. $27\frac{11}{16}$ lb.
3. 149 bu.
4. $232\frac{53}{60}$ mi.
5. \$$364\frac{1}{8}$; \$$623\frac{11}{20}$.
6. \$$6388\frac{19}{20}$.
7. \$$8979\frac{1}{20}$.
8. \$$12,368\frac{1}{10}$.
9. $100\frac{489}{500}$ mi.
10. $143\frac{5}{32}$ acres.

Art. 351.

1. 10.
2. 18.
3. $40\frac{1}{2}$.
4. $97\frac{1}{2}$.
5. $130\frac{2}{3}$.
6. 1197.
7. 1548.
8. 1377.
9. $173\frac{1}{3}$.
10. $73\frac{17}{19}$.
11. $28\frac{3}{5}$.
12. $106\frac{2}{7}$.
13. $29\frac{14}{31}$.
14. $102\frac{3}{8}$.
15. 312.

Art. 354.

1. $\frac{7}{16}$.
2. $2\frac{6}{7}$.
3. $1\frac{1}{2}$.
4. $1\frac{3}{5}$.
5. 15.
6. $22\frac{1}{2}$.
7. $\frac{3}{13}$.
8. $\frac{1}{3}$.
9. $\frac{4}{21}$.
10. $107\frac{2}{3}$
11. $857\frac{1}{3}$.
12. 24.
13. $59\frac{1}{2}$.
14. 1313.
15. 6.
16. $2\frac{4}{5}$.
17. $60\frac{20}{21}$.
18. $37\frac{59}{150}$.
19. $27\frac{167}{240}$.
20. $150\frac{415}{504}$.

Art. 356.

1. \$$303\frac{1}{3}$.
2. \$80.21.
3. $71\frac{1}{2}$ yds.
4. \$$3265\frac{5}{8}$.
5. \$$3\frac{3}{5}$.
6. $270\frac{9}{16}$ bu.
7. \$103.24.
8. \$$178\frac{1}{5}$.
9. \$$757\frac{3}{20}$.
10. $478\frac{17}{30}$ mi.
11. \$$36,888\frac{1}{4}$.
12. \$$12\frac{7}{10}$.
13. $1218\frac{3}{4}$ lb.
14. \$5.25.
15. \$251.25.
16. $46\frac{3}{4}$ min. past 2 P. M.
17. \$1050.
18. \$17,294.90.
19. $53\frac{131}{160}$ mi.
20. \$$3021\frac{3}{4}$

Art. 361.

1. $13\frac{1}{2}$.
2. 16.
3. 25.
4. 27.
5. $53\frac{1}{3}$.
6. $67\frac{1}{2}$.
7. $\frac{2}{5}$.
8. $\frac{5}{24}$.
9. $\frac{11}{240}$.
10. $\frac{1}{14}$.
11. $\frac{1}{26}$.

12. $\frac{1}{96}$.
13. $\frac{7}{288}$.
14. $\frac{7}{432}$.
15. $\frac{1}{40}$.
16. $\frac{1}{20}$.
17. $\frac{1}{10}$.
18. $\frac{1}{20}$.
19. 189.
20. 40.
21. 120.
22. 135.
23. $88\frac{8}{9}$.
24. $109\frac{1}{11}$.

Art. 364.

1. $1\frac{7}{48}$.
2. $\frac{20}{33}$.
3. $1\frac{1}{6}$.
4. $1\frac{2}{5}$.
5. $1\frac{1}{56}$.
6. $\frac{23}{26}$.
7. $1\frac{1}{5}$.
8. $\frac{16}{21}$.
9. $\frac{18}{25}$.
10. $1\frac{19}{35}$.
11. 12.
12. 24.
13. $52\frac{1}{2}$.
14. 60.
15. 325.
16. 340.
17. $1\frac{1}{4}$.
18. $\frac{5}{9}$.
19. $1\frac{1}{5}$.
20. $\frac{25}{36}$.
21. $\frac{5}{14}$.
22. $\frac{2}{3}$.
23. $\frac{7}{15}$.
24. $\frac{5}{27}$.

Art. 366.

1. $\frac{9}{91}$.
2. $\frac{9}{100}$.
3. $\frac{24}{325}$.
4. $\frac{90}{391}$.
5. $23\frac{3}{4}$.
6. $8\frac{8}{9}$.
7. $\frac{7}{9}$.
8. $\frac{2}{3}$.
9. $3\frac{9}{10}$.
10. $12\frac{23}{36}$.
11. $29\frac{9}{49}$.
12. $4\frac{29}{40}$.
13. $\frac{2}{3}$.
14. $\frac{65}{108}$.
15. $1\frac{17}{64}$.
16. $1\frac{11}{25}$.
17. $\frac{8}{15}$.
18. $\frac{11}{224}$.
19. 18.
20. $1\frac{1}{175}$.

Art. 368.

1. $\frac{125}{352}$.
2. $12\frac{3}{13}$.
3. $1\frac{1}{12}$.
4. $\frac{1}{12}$.
5. $\frac{17}{32}$.
6. $47\frac{1}{4}$.
7. $\frac{8}{41}$.
8. $\frac{6}{7}$.
9. $1\frac{19}{81}$.
10. 40.
11. $1\frac{1}{2}$.
12. $\frac{2744}{8415}$.

Art. 370.

1. $1\frac{1}{3}$ bbl.; $\frac{4}{15}$ bbl
2. 16 days.
3. $15\frac{1}{3}$ days.
4. $454\frac{3}{4}$ bush.
5. \$16,459$\frac{1}{2}$, wife; \$6583$\frac{4}{5}$, son; \$1645$\frac{19}{20}$, daughter.
6. \$160.
7. \$51.84.
8. \$5.
9. 9 cords.
10. \$93$\frac{3}{4}$.
11. $4\frac{10}{17}$ hrs.
12. \$74$\frac{4}{5}$.
13. \$8.75.
14. \$20.80, butcher, \$15.60, grocer.
15. 345 bu.
16. $9\frac{1}{3}$ mi.
17. 36 yds.
18. \$8030.
19. \$14$\frac{17}{20}$.
20. \$9060.
21. \$11,960.
22. \$77.
23. \$1050, A; \$1400, B.

24. 16 days.
25. $3\frac{7}{8}$ acres.
26. $12,800.
27. $2400.
28. $4304.40.
29. $4600.
30. $55\frac{1}{2}$ mi.

Art. 371.

1. $652.
2. \$$60.09\frac{3}{8}$.
3. $68.75.
4. $775.
5. \$$316\frac{1}{24}$.
6. $32,000; $28,000.
7. 852 bu.
8. $21,000.
9. 28 yds.
10. 1400.
11. $6.
12. 260 apples.
13. $204, horse; $136, sleigh.
14. $158\frac{8}{9}$ acres; $127\frac{1}{9}$ acres.
15. 40 days.
16. \$$11.63\frac{3}{4}$.
17. 120 bbl.
18. \$$507\frac{1}{2}$.
19. $6.23.
20. 6 men.
21. \$$8\frac{1}{2}$.
22. $4080, A; $5440, B.
23. $\frac{33}{35}$.
24. $8\frac{4}{31}$ days.
25. \$$12\frac{3}{8}$.
26. $4100.
27. $13\frac{1}{3}$ days.
28. $79.20.
29. \$$64\frac{3}{8}$, father; \$$38\frac{5}{8}$, son.
30. 1680 pupils.
31. 20 cts.
32. $49,000.
33. $78.75.
34. $2.10.
35. 15 days.
36. $1046.
37. $7\frac{37}{60}$.
38. $52\frac{16}{17}$; $47\frac{1}{17}$.
39. $24,000.
40. $160.
41. 28 hrs.
42. 60 doz.
43. $1.60.
44. $3\frac{3}{8}$ mi.
45. $106.47.
46. 168.
47. 608 bu.
48. $5422\frac{1}{2}$ bu.
49. 100 weeks.
50. 14 T.
51. $330, horse; $240, buggy.
52. 1050 sheep.
53. $142.80.
54. \$$63,541\frac{1}{4}$.
55. \$$14.06\frac{1}{4}$.
56. 120 hrs.
57. \$$7999\frac{12}{25}$.

Art. 378.

1. 7500.75.
2. 990.009.
3. 5005.00005.
4. 5000.00015.
5. .05005.
6. 8,080,000-.000085.
7. 8,080,080-.000005.
8. 7,050,000,050-.000001001.
9. 3,000,100,003-.00003501.
10. 510,010,010-.0010105.
11. 8,801,725,540-.00000125305.
12. 5,320,000,070,-839.000561046346.

Art. 381.

1. .5; .8.
2. .6; .625.
3. .875; .6875.
4. .9375; .36.
5. .96; .21875.
6. .59375; .85.
7. .775; .921875.
8. .359375; .8375.

9. .5375; .936.
10. .744; .8828125.
11. .476; .75390625.
12. .825.
13. .33203125.
14. 9.45.
15. 10.24.

Art. 382.

1. $.71\frac{3}{7}$; $.83\frac{1}{3}$.
2. .8888+; .6666+.
3. .90909+; .9166+.
4. $.69\frac{3}{13}$; $.78\frac{4}{7}$.
5. .9333+; $.94\frac{2}{17}$.
6. .2777+; $.89\frac{9}{19}$.
7. 5.8666+.
8. $12.38\frac{6}{13}$.
9. $21.3\frac{2}{21}$.

Art. 385.

1. $\frac{4}{25}$.
2. $\frac{3}{4}$.
3. $\frac{24}{25}$.
4. $\frac{16}{25}$.
5. $\$\frac{13}{40}$.
6. $\$\frac{17}{25}$.
7. $\frac{27}{40}$.
8. $\frac{37}{40}$.
9. $\frac{7}{200}$.
10. $\frac{9}{400}$.
11. $\frac{21}{2500}$.
12. $\frac{49}{80}$.
13. $\frac{17}{400}$.
14. $\frac{33}{6250}$.
15. $\frac{3}{160}$.
16. $\frac{128}{3125}$.

Art. 386.

1. $\frac{7}{20}$.
2. $\frac{9}{200}$.
3. $\frac{1}{6}$.
4. $\$\frac{1}{3}$.
5. $\$\frac{2}{7}$.
6. $\$1\frac{1}{8}$.
7. $\$3\frac{5}{8}$.
8. $\$1\frac{5}{16}$.
9. $\$\frac{3}{8}$.
10. $2\frac{7}{8}$.
11. $3\frac{7}{16}$.
12. $\frac{1}{6}$.

Art. 390.

1. 2650.1996.
2. 215.6533.
3. .789852.
4. 8.44702.
5. 342.4724.
6. 462.0921.
7. .95387.
8. 45.3243.

Art. 391.

1. $1751.341.
2. 241.4413.
3. 1065.76715.
4. 389.717049.
5. 1.903811.
6. $8385.7679.

Art. 392.

1. $81.147.
2. 1.083.
3. 45.2912.
4. 1130.3364.
5. $1.9215.
6. 155.790346.
7. 1.1189.
8. .22346.
9. .9981.
10. 1.8275.
11. .34225.
12. 77.4752.

Art. 393.

1. 344 acres.
2. 29.35 inches.
3. 1323.6565 mi.
4. 62.879 mi.
5. 23.6225 inches.
6. 892.4735 tons.
7. 104.383 acres.

Art. 396.

1. 76.5625.
2. 570.3125.
3. 469.0625.
4. .942.
5. .000049.
6. 8.0325.
7. $.65205.

8. \$6.101875.
9. \$2.999166$\frac{2}{3}$.
10. \$14,892.4625.
11. \$55,732.125.
12. \$1208.2458$\frac{1}{3}$.
13. \$220.3125.
14. 979.125.
15. \$701.114$\frac{7}{12}$.
16. 149.673375.
17. 11.522275.

Art. 397.

1. 155,53125 mi.
2. \$693.140625.
3. \$73,475.453125.
4. \$6599.109375.
5. \$3534.375.
6. 1401.46875 cu. ft.; \$191.109375.
7. \$1962.56328125.
8. 24,898.99827036 mi.
9. 431.021 tons.
10. \$230.3703125.
11. 7230 children.

Art. 400.

1. 250.
2. 1040.
3. 10,500.
4. 44.
5. 218.
6. 51,472.
7. 1300.
8. 340,000.
9. 1728.
10. 9275.
11. 25,000.
12. 155.

Art. 401.

1. 2.5268+.
2. 430.2453+.
3. 1.0795+.
4. .2019+.
5. 1.5968+.
6. 33,666.6666+.
7. 9.2482+.
8. 27.6276+.
9. 28.3055+.
10. \$44.292+.
11. 148.5294+.
12. .0026+.

Art. 402.

1. 6669.205
2. 36,200.
3. 19.6519.
4. 100.
5. 4360.
6. 25.

Art. 403.

1. \$47.25.
2. \$.62$\frac{1}{2}$.
3. 28.56 mi.
4. 256 cu. ft.
5. 20.07 mi.
6. \$3.519.
7. \$9630.
8. \$2000.
9. 12.625 acres.
10. \$23.75.
11. 69.1639+ mi.
12. 3164 pounds.
13. 20,800 men.

Art. 416.

1. 3,041,340 oz.
2. 74,482,829 in.
3. \$364.65.
4. 168,179 lb.
5. \$1039.13$\frac{1}{4}$.
6. 1,009,035 lb.
7. 28,938 steps.
8. \$93,285.64.
9. 297 panels.
10. 2739.

Art. 419.

1. 273 T. 13 cwt. 9 lb. 1 oz.
2. 5802 gal 1 qt. 3 gi.
3. 1308 bu. 3 pks.
4. 151 mi. 148 rds. 5 yds. 2 ft. 7 in.
5. 521 long T. 14 cwt. 1 qr. 12 lb. 7 oz.
6. \$1225.
7. 12 bush.

8. 10 mi. 138 rds. 1 yd.

Art. 422.

1. 100 rds.
2. 300 rds.; 1650 yds.; 4950 ft.
3. 188 qts.; 1136 pts.
4. 16 cwt. 75 lb.
5. 17,360 lb.; 277,760 oz.
6. $2483.25.
7. $781.25.

Art. 425.

1. .84 T.
2. $1.92.
3. $16.71.
4. $2.82.
5. $2.56.
6. $8140.50.
7. $5.02.

Art. 427.

1. 1340 mi. 111 rds. 4 yds. 2 ft. 6 in.
2. 1199 bu. 1 pk. 1 qt.
3. 849 gal. 3 qts. 2 gi.
4. 175 T. 5 cwt. 84 lb. 6 oz.
5. 602 mi. 268 rds. 11 ft.

Art. 429.

1. 307 bu. 1 pk. 5 qts. 1 pt.
2. 41 mi. 318 rds. 4 yds. 2 in.
3. 53 T. 17 cwt. 29 lb. 6 oz.
4. 46 gal. 2 qt. 1 pt.
5. 107 rds. 3 yds. 3 in.
6. 120 bu. 6 qts.

Art. 431.

1. 407 gal. 2 qts. 1 pt.
2. 548 mi. 292 rds. 4 yd. 1 ft. 3 in.
3. 872 bu. 3 pks. 5 qts. 1 pt.
4. 1617 gal. 1 pt. 2 gi.
5. 385 T. 19 cwt. 27 lb.
6. 401 mi. 28 rds. 2 yds. 1 ft. 6 in.
7. 3 mi. 209 rds. 2 yds. 2 ft. 6 in.
8. 35 T. 15 cwt. 50 lb.
9. 353 mi. 279 rds. 2 yds. 1 ft. 6 in.

Art. 433.

1. 5 mi. 175 rds. 3 yds. 2 ft. 11 in.
2. 25 bu. 3 pks. 6 qts. 1 pt.
3. 28 T. 19 cwt. 70 lb. 10 oz.
4. 117 bu. 1 pk.
5. 98 long T. 13 cwt. 98 lb.
6. 168 gal. 3 qt. 1 pt. 2 gi.
7. 201 bu. 3 pks. 6 qts.
8. 5 gal. 3 qts. 1 pt. 3 gi.

Art. 436.

1. 7995 l.; 19805 l.; 37007 l.
2. 187.25 ch.; 98.725 ch.
3. 2000 ch.; 1500 ch.
4. 41,464 ch.; 4,146,400 l.

5. 50 ch.; 5000 l.
6. 23 mi. 57 ch. 50 l.; 62 mi. 40 ch.
7. 80,000 l.; 80,000 l.
8. 6282 l.
9. 243.9375 ch.; 3200 ch.
10. 44.573125 mi.
11. 5000 l.; 50 ch.
12. 40447.7272+ l.
13. 805,530 cm.; 8,055,300 mm.
14. 59 Mm. 3 Km. 7 Hm. 3 m. = 593,703 m.
15. 199,960 cm.

Art. 444.

1. 43,560 sq. ft.; 6,272,640 sq. in.
2. 27,878,400 sq. ft.; 3,097,600 sq. yds.
3. 19 A. 121 sq. rds. 24 sq. yds. 1 sq. ft. 108 sq. in.
4. 25,740 sq. rds.
5. .09 sq. mi.
6. 205 A. 121 sq. rds. 18 sq. yds. 1 sq. ft. $50\frac{2}{3}$ sq. in.
7. $92.
8. $380.25.
9. $33.60.
10. $2.40.
11. $6.75.
12. 17.424 city lots.
13. 64 rods.
14. $21.
15. 3.645 A.
16. 40 rds.
17. 19 A. 20 sq. rds.
18. 6.407 A.
19. $1710.
20. $\frac{1}{256}$ sq. mi.
21. 2362500 sq. l.
22. 10 A.
23. 44.14 ch.
24. $2700.
25. 135.3 sq. m.
26. 69.3 sq. m.
27. 9.045 Ha.; 22.350195 A.

Art. 447.

1. 384 cu. ft.; $288.
2. $35.
3. 80 cu. ft.
4. $$70.31\frac{1}{4}$.
5. $1575.
6. 8784 cu. ft.
7. 96 loads.
8. 315 steres; 31.5 dekasteres; 11,124.729 cu. ft.
9. 37.2 steres; 10.26348 cords
10. $$256.26\frac{1}{4}$.
11. 57,212.892 lb.

Art. 450.

1. 6123 drams.
2. 484.48+ liters
3. 58,769,500 ml.
4. $31.875.
5. 79.686+ bu.
6. $27.12.

Art. 454.

1. 114,687 gr.
2. 100 lb.
3. $49567\frac{1}{2}$ gr.
4. 3 lb. 5 oz. 9 dwt. 3 gr.
5. .7 lb.
6. .875 lb.
7. 360 gr.
8. 48 rings.
9. 1000 pieces.
10. $549.45.
11. 175 lb. Troy.
12. 31 lb. 3 oz. 10 dwt. 10 gr.
13. 33,475 gr.
14. ℔15 ℥2 ʒ3 ℈1 gr. 15.
15. 17 pounds.
16. ℔ 175.
17. $11.

18. $.3255208\frac{1}{3}$ lb.
19. ℥10 ʒ4.
20. 125.56\frac{1}{3}$.
21. $37.50.
22. 24¢ per lb.
23. 3024.45 kilos.
24. 20.1438 kilos.
25. $1802.50.
26. $80 ; $110.23.

Art. 455.

1. 20,048 far.
2. 5497 far.
3. £20 11s.
4. £.75 ; £.4.
5. £$\frac{9}{16}$; £$\frac{1}{6}$.
6. 16s. 8d.; 7s. 6d.; 14s. 8d.
7. £.79375.
8. £17 18s. 6d.; £15 12s. 6d. 2.4 far.
9. 437,320 far.; 1350d.
10. 18,225d.; 72,900 far.
11. £.625.
12. £.53125.

Art. 458.

1. 31,556,929.7 sec.
2. 86,400 sec.; 1440 min.
3. 25 days.
4. 1,063,200 half-sec.
5. 3,510,945 sec.
6. 305 days.
7. $3\frac{3}{4}$ days.
8. 15 wks. 6 da. 19 hrs. 49 min. 53 sec.
9. 24 da. 5 hrs. 22 min. 50 sec.

Art. 460.

1. 21,600′; 1,296,000″.
2. 712,133″.
3. 241° 39′ 27″.
4. $\frac{2}{3}$ C.
5. .832 C.
6. 48 days.
7. 15°.
8. 22 ft.

Art. 464.

1. 2° 57′ 15″; 11 min. 49 sec.; 48 min. 11 sec. after 11 A. M.
2. 12° 27′ 14″.
3. 8 hrs. 2 min. 17 sec.; 57 min. 43 sec. after mid-night.
4. 87° 37′ 57″ W.
5. 95° 4′ 55″ W.
6. 53 min. slow.
7. 2 hrs. 21 min. $37\frac{2}{5}$ sec.
8. 43° 9′ W.

Art. 465.

1. 437,229 gr.
2. 30 lb. 5 oz. 6 dwt. 20 gr.
3. $344.95.
4. 5 T. 5 cwt.
5. $1274.
6. $22.04.
7. 1280 times.
8. 10.725 A.
9. $5775.
10. $21.
11. $22,360.
12. 18 perch.
13. 10,368 gal.
14. 528 days.
15. 143.97 bu.; $172.764.
16. 31 ft. 3 in.
17. 39,420,000 times.
18. 188 da. 21 hrs. 20 min.
19. 1840 bu. 3 pks. 1 qt.
20. $50.625.
21. $1875.
22. $33,912.38.
23. 26.03\frac{1}{8}$.

24. 55 min. $57\frac{3}{8}$ sec. after 8 A. M.
25. 77° 30′.
26. \$250.
27. 13,824.
28. \$243.81$\frac{1}{4}$ gain.
29. \$498.78 gain.
30. \$7.41 gain.
31. \$4590.50 gain.
32. \$23,184.50 gain.
33. \$207.616.

Art. 475.

2. 251.10; 474; \$361.44.
3. \$378,000.
4. \$380.50, the latter.
5. \$55,300.
6. \$5156.624.
7. 20,361.1875.
8. 4185 yds.; 30,498 kilos; 62,520 inches.
9. \$1252.50.
10. 1505 bushels.
11. \$44,455.
12. 3685 miles.
13. \$5486.25 gain; \$21,161.25 selling price.
14. \$15 more for chestnuts.
15. \$2493 loss.
16. \$4252.50.
17. \$216.
18. \$117.60 gain.
19. \$9682.40.
20. 209,118.75 tons.
21. \$47.19.
22. \$4230.

Art. 476.

2. 123,400 bu.
3. \$28,248.
4. 1928 pages.
5. \$239.
6. 2040 pupils; 1122 girls.
8. 117 days.
9. \$7000.
10. 210,000.
11. \$2502.40 cost; \$2971.60 selling price.
12. \$12,215.
13. \$64,131.25.
14. 125 gals.
15. \$3478.75.
16. \$4200.
17. 156 pear trees; 195 peach trees; 234 plum trees; 715 apple trees.
18. \$62,000.

Art. 477.

2. $66\frac{2}{3}$%.
3. 68%.
4. $22\frac{8}{9}$%.
5. 64%.
6. 56%.
7. 45%.
8. 24%.
9. 16%.
10. 24%.
11. 20%.
12. 40%.
13. 12%.
14. 4%.
15. 20% sailed; 80% remaining.
16. 8%.
17. 36%.

Art. 478.

1. 4260.
2. 13,720.
3. 24,496.
4. 181,420.
5. 8324, smaller; 10,405, larger.
6. 2956; 3695.
7. 2880.
8. \$50.75 son; \$60.90 father.
9. \$65 for one; \$91 for other.
10. 4374 bu.
11. \$343.75.
12. 56%.
13. \$25,000.
14. 2385 bbl.

15. 35%.
16. $25,600.
17. 65%.
18. $18,000.
19. 24,453 lb.
20. 445 mi., horseback; 534 mi., rail.
21. 1350.
22. $423.
23. $75.
24. 39,632 veterans; 49,540 recruits.
25. $12,400.
26. 768 sheep.
27. $21,300.
28. $26,882.70.
29. $227.70.
30. $6.50 loss.
31. 73%.
32. $1226.25 loss.
33. $324.
34. 3300 bales.
35. $3781.25 gain.
36. $1058.10.
37. 1600 bu.
38. $14,850.
39. $275.
40. 25%, B more than A; 20%, A less than B.
41. $1375.
42. $2350.
43. $3000 cost to build; $3375 selling price.
44. $200.
45. $89\frac{2}{7}$%.
46. 10%.
47. $21\frac{7}{8}$%.
48. $16,840 1st yr.; $18,945 2d yr.
49. $14,625.
50. $30,000.
51. 25%.
52. 88%.
53. $8500.
55. $2.85; $2.75.
56. $45.
57. $216.
58. $197.51, 2d case.
59. $446.415.
60. $12.99.
61. $678.30.
62. $446.25.
63. $37.51.
64. $1736.62.
65. 56%.
66. 37%; 60%; 40%; 52%.
67. $650.
68. $625.

Art. 481.

2. $375 gain; $2250.
3. $.40; $1.20.
4. $620.
5. $13,563.
6. $655 loss; $3275.
7. $18,870 loss; $106,930.
8. $10725.
9. $12,927.60 gain; $48,837.60.
10. $162 gain; $522.
11. $14,616 gain; $6264 average.
12. $3375 gain; $12750.

Art. 482.

2. 25%.
3. 20%.
4. 8%.
5. 88%.
6. $14\frac{2}{7}$%.
7. 15%.
8. 25%.
9. 18%.
10. 15%.
11. 15%.

Art. 483.

2. $25,800.
3. $4320.
4. $15,048.
5. $67,000.

6. $13,500.
7. $3050.
8. $4200.
9. $30 gain.
10. $18,000 cost buyer; $20,000 cost builder.

Art. 487.

1. $135.
2. $8437.50.
3. $3232.512.
4. $1068.75.
5. $8926.875.

Art. 488.

1. $31,528.
2. $840.
3. $21,216.
4. $6816.
5. $3171.84.
6. $62,634.

Art. 489.

1. $43,750.
2. $97,300.
3. $456.60; $9132.
4. 1080 bbl.; $128.25.
5. 400 bales.
6. 160 lots.
7. $8440.

Art. 493.

2. $975.
3. $6.375.
4. $9712.50.
5. $2650.
6. $17.50.
7. $14,625.
8. $975.
9. $2681.25.

Art. 494.

2. $7212.
3. $235,000.
4. $109,040.
5. $9900.
6. $32,160.
7. $2356.
8. $396,000.

Art. 495.

2. 2%.
3. $\frac{3}{4}$%.
4. $3\frac{1}{2}$%.
5. $2\frac{1}{4}$%.
6. $3\frac{1}{4}$%.
7. $1\frac{7}{8}$%.

Art. 498.

2. .016; $86.
3. .018.
4. $97.75.
5. $725, A; $$1908\frac{1}{3}$, B; $$2966\frac{2}{3}$, C.

Art. 499.

2. $16.04.
3. $27.45.
4. $157.30.
5. $222.21.
6. $209.70.
7. $327.82.
8. $447.53.
9. $1511.37.
10. $13.14.
11. $14.96.
12. $21.85.
13. $56.62.
14. $231.66.
15. $578.36.
16. $1662.95.
17. $326.52.

Art. 505.

2. $62,243.75.
3. $51,312.50.
4. $18,265.63.
5. $14,843.75.
6. $44,797.50.
7. $172,357.50.
8. $145,120.
9. $404,239.50.

Art. 506.

2. 288 shares.
3. 880 bonds.
4. 76 shares.
5. 368 shares.
6. 648 bonds.

7. 123 shares.
8. 147 shares.

Art. 507.

2. $79,781.25.
3. $49,062.50.
4. $150,850.
5. $142,100.
6. $158,970.
7. $8790.
8. $94,250.
9. $93,262.50.

Art. 508.

2. 5%.
3. 5%.
4. 10%.
5. 2%, the former.
6. $\frac{24}{43}$ more %.

Art. 517.

3. $7325.50.
4. $5130.
5. $3096.
6. $24,080.
7. $480.
8. $393.75.
9. $5531.25.
10. $10,200.
11. $16,858.80.

Art. 525.

3. $64.53.
4. $124.58.
5. $67.17.
6. $153.329
7. $523.13.
8. $52.79.
9. $174.67.
10. $240.69.
11. $113.57.
12. $40.77.
13. $53.03.
14. $180.
15. $3686.94.
16. $6008.09.
17. $8565.60.
18. $946.32.
19. $892.
20. $1392.30.
21. $156.013.
22. $1097.55.
23. $3559.50.
24. $2313.44.
25. $11,057.68.
26. $18,966.75.
27. $34,784.18.

Art. 526.

2. $50.40.
3. $132.
4. $480.
5. $260.44.
6. $330.
7. $960.15.
8. $2134.52.
9. $271.70.
10. $174.58.
11. $1978.47.
12. $171.
13. $2599.30.
14. $451.77.
15. $76.36.
16. $23.50.
17. $302.12.
18. $27.49.
19. $168.88.
20. $1941.55.

Art. 527.

2. $525.
3. $562.50.
4. $1555.56.
5. $960.
6. $226.67.
7. $135.
8. $83.40.
9. $102.78.
10. $1081.25.
11. $9321.45.
12. $10,585.24.
13. $3668.02.
14. $30.86.
15. $93.33.
16. $320.25.
17. $567.77.

Art. 529.

2. $374.79.
3. $521.59.
4. $4540.
5. $4937.56.
6. $4182.31.
7. $20,259.34.
8. $1207.59.

9. $1306.90.
11. $274,900.44.
12. $692.74.
13. $819.33.
14. $5114.31.

Art. 530.

2. $1560.
3. $1750.
4. $1200.
5. $13,000.
6. $14,040.
7. $11,000.
8. $5250.

Art. 531.

2. 9%.
3. 6%.
4. 5½%.
5. 6½%.
6. 5%.
7. 4½%.
8. 6%.
9. 5%.

Art. 532.

2. 2 yrs. 3 mo.
3. 3 yrs. 7 mo. 6 da.
4. 7 yrs. 1 mo.
5. 5 yrs. 6 mo. 20 da.
6. 12 yrs. 6 mo.
7. 5 yrs. 5 mo. 10 da.
8. 3 yrs. 10 mo. 20 da.
9. 73 days.

Art. 534.

1. $3038.75.
2. $5658.80.
3. $7633.75.
4. $988.83.
5. $12,170.50.

Art. 539.

1. $1289.80.
2. $1073.78.
3. $140.14.
4. $2695.58.
5. $3617.77.
6. $5247.97.

Art. 542.

1. $711.33.
2. $477.42.
3. $1511.40.
4. $793.93.
5. $2283.59.

Art. 544.

1. $6055.68.
2. $8961.
3. $8192.86.
4. $700.37.

Art. 548.

1. $233.10.
2. $122.05.
3. $188.26.
4. $386.43.
5. $203.05.
6. $895.52.
7. $1483.34.

Art. 549.

1. $134.53.
2. $8340.
3. $22248.91.
4. $6903.69.
5. 5½%.
6. $1860.
7. $81.67.
8. $1658.30.
9. $1640.
10. $1457.18.
11. $1835.
12. 7 mo. 9 da.

Art. 553.

1. $1834.86; $165.14.
2. $3469.88.
3. $6135.75.
4. $6218.18.
5. $7714.29.
6. $371.19.
7. $18.87, the latter.
8. $25.33.
9. $3499.68.
10. $6523.08.

Art. 559.

1. $65.10; $3534.90.

2. \$7.56 ; \$1367.44.
3. \$7295.
4. \$13.21.
5. \$2541.72.
6. \$7857.08.
7. \$9730.50.
8. \$3796.75 ; \$73.75.

Art. 561.

1. \$2425.47.
2. \$938.96.
3. \$3519.09.
4. \$1888.22.
5. \$9326.84.
6. \$4452.44.
7. \$26.70.
8. \$2815.46.
9. \$8317.39.
10. \$4792.23.

Art. 570.

1. \$3677.38.
2. \$8646.55.
3. \$4309.87.
4. \$7368.34.
5. \$6455.50.
6. \$4616.80.

Art. 572.

1. \$5611.20.
2. \$5786.
3. \$8336.64.
4. \$12645.24.
5. \$7480.62.

Art. 574.

1. \$1209.68.
2. \$5733.20.
3. \$333.92.
4. \$960.
5. \$3281.49.

Art. 579.

1. In 7 mo. 2 da.
2. October 10.
3. Aug. 23, '85.
4. Jan. 29, '87.
5. April 6, '86.
6. July 31, '86.
7. May 31, '86.
8. 8 mo. 3 da.
9. May 5, '86.

Art. 580.

2. \$700 ; May 10, '87.
3. June 12, '83.
4. June 25, '79.
5. March 24, '81.
6. March 19, '85.

Art. 599.

1. 21.
2. 80.
3. 7.
4. 11.
5. 5.
6. 63.
7. 5.34.
8. \$21.25.
9. 26.
10. $2\frac{1}{8}$.
11. $\frac{3}{5}$.
12. .4.
13. 12 yds.
14. 102.
15. \$12.40.
16. $\frac{4}{5}$.
17. 12.1.
18. \$4.50.
19. 36.
20. 35.

Art. 600.

1. \$129.60.
2. 40 days.
3. 30 bbl.
4. $427\frac{1}{2}$ mi.
5. \$4278.30.
6. $511\frac{1}{2}$ mi.
7. \$372.
8. 529 mi.
9. \$7.60.
10. 40 wks.
11. 12 da.
12. 49 da.
13. 55 da.
14. \$51.25.
15. \$12,892.50.
16. 120 ft.
17. 60 boys.

18. 96 bu. rye.
19. 96 acres.

Art. 602.

1. 40 men.
2. 25 lb.
3. $96.
4. $$37\frac{1}{2}$.
5. $$66\frac{2}{3}$.
6. $$916\frac{2}{3}$.
7. $$28.12\frac{1}{2}$.
8. $$72.91\frac{2}{3}$.
9. $1104.
10. 8370 lb.

Art. 604.

1. $396.
2. $67\frac{1}{2}$ bu.
3. $1856.25.
4. $123.48.
5. $528.75.
6. 62 perches.
7. $1807.07.
8. 20 days.
9. 144 men.
10. 40 wks.
11. 4 days.
12. $250.
13. 15 days.

Art. 610.

1. $1600, A; $1800, B.
2. $3000, A; $3600, B; $5000, C.
3. $4301, A; $6919, B; $7480, C.
4. $60, 1st; $68, 2d; $76, 3d.
5. $1600, A; $2000, B; $2400, C.

Art. 612.

1. $1800, 1st; $3200, 2d; $4500, 3d.
2. $4000, A; $6000, B; $8400, C.
3. $3796.61, A; $5491.53, B; $2711.86, C.
4. $2720, M; $1800, N; $2240, R.
5. $150, A; $270, B.

Art. 620.

1. 1; 4; 9; 16; 25; 36; 49; 64; 81; 100; 121; 144.
2. 625; 576; 4225; 5625; 4096; 15,625; 40,000.
3. 20.25; 56.25; 69.7225; 95.0625; 1406.25.
4. $\frac{4}{9}$; $\frac{9}{16}$; $\frac{25}{81}$; $\frac{144}{625}$; $1\frac{17}{64}$; $\frac{64}{169}$; $\frac{49}{144}$; $3\frac{1}{16}$; $3\frac{6}{25}$; .0036.
5. 1; 8; 27; 64; 125; 216; 343; 512; 729; 1000; 1331; 1728.
6. 8000; 15,625; 27,000; 42.875; .421875; .000125; .000000125.
7. 421.875; 3164.0625; 1.030301; 1.04060401.
8. $\frac{81}{256}$; $\frac{256}{625}$; $5\frac{1}{16}$; $197\frac{193}{256}$
9. 40,353,607; 6561.
10. 3125; 19,683; 823,543; 262,144.
11. 2000; 135,000; 124,416.

Art. 630.

1. 45.

2. 64.
3. 86.
4. 53.
5. 95.
6. 87.
7. 67.
8. 77.
9. 125.
10. 108.
11. 427.
12. 565.
13. 509.
14. 4028.
15. 5991.
16. 60.95.
17. 65.2752+.
18. 5.53404+.
19. 2.9483+.
20. 20.7288+.
21. .065.
22. .125.
23. .205.
24. 12.7358+.
25. $\frac{35}{48}$; $1\frac{8}{67}$; $\frac{165}{361}$; $\frac{301}{405}$; $\frac{72}{93}$.

Art. 635.

1. 18.
2. 19.
3. 21.
4. 31.
5. 24.
6. 26.
7. 32.
8. 37
9. 46.
10. 42.
11. 5.9.
12. 8.5.
13. .72.
14. 215.
15. 16.5.
16. 2.05.
17. 968.
18. 962.
19. 8.05.
20. 12.07.
21. 2.098.
22. 1.259.
23. 1.442.
24. $\frac{8}{14}$; $\frac{13}{16}$.
25. 56.
26. 3.

Art. 636.

1. $875.
2. 3525 l.
3. 64 ft.
4. 25 ft.
5. 45 ft. by 90 ft.
6. 24 ft.
7. 18 ft.
8. $735.
9. 25 ft. wide,
   25 ft. high,
   75 ft. long.
10. 24 in.
11. $30.
12. 24 in.
13. 12 in.

Art. 646.

2. 84.
3. 103.07+ ft.
4. 32.87+ ft.
5. 26.907+ mi.
6. 144.056+ ft.
7. 13.228+ ft.
8. 39.05+ ft.
9. 14.764+ rds
10. 166.43+ ft.

Art. 649.

2. 1050 sq. rds.
3. $131\frac{1}{2}$ acres.
4. 285 acres.
5. $410\frac{2}{3}$ sq. ft.
6. 1350 sq. rds.

Art. 651.

2. 384 sq. rds.
3. 2400 sq. rds.
4. $2347.50.
5. 60 acres.

Art. 658.

2. 5400 sq. ft
3. 7.03 acres.
4. 328.125 acres.
5. 29600 sq. ft.
6. $3085.50.

Art. 660.

2. 6.777+ acres.
3. 70.3 acres.

4. \$184.32.
5. 5.5125 acres.

Art. 662.

2. 18.487+ acres.
3. 120.652+acres.
4. \$17056.70.

Art. 664.

2. 1560 sq. rds.
3. 19.5 acres.

Art. 666.

2. 200 rds.
3. 565.488 rds.
4. 477.463+ ft.
5. 376.992 ft.

Art. 668.

3. 20106.24 sq. ft.
4. Square larger; 3090.24 sq. rds.
5. 117.75+ ft.
6. 6361.74 sq. yds.
7. 57.08+ rds.
8. 5.917+ ft.
9. \$27.31.

Art. 674.

2. 1280 sq. ft.
3. 960 sq. ft.; 1088 sq. ft.
4. \$60.

Art. 676.

2. 5196.15 cu. in.
3. \$1002.375.
4. 12656.25 lb.
5. 128 more cu. ft.

Art. 679.

2. 1429.428 sq.in.; 1694.893+ sq. in.
3. 21.2058 sq. ft.
4. 49.5783 sq. ft.

Art. 681.

2. 7050.24 gal.
3. 35.343 cu. ft.
4. 2150.42+ cu. in.
5. 4.71+ cu. in. more.
6. 11061.05 lb.
7. 4.08 gal.

Art. 684.

2. 2400 sq. in.
3. 3801.18+ sq.ft.
4. 1125 sq. ft.
5. $41\frac{1}{12}$ sq. ft.

Art. 686.

2. 23,256.25 cu.ft.
3. 1732.05+cu. yds.
4. 135.554+cu ft.
5. 1,400,000 lb.

Art. 689.

2. 2261.952 sq. in.
3. 1884.96 sq. in.
4. 163.625 sq. ft.

Art. 691.

2. 4523.904 cu. in.
3. 81430.272 cu. in.
4. 215.985 cu. ft.

Art. 694.

2. 1256.64 sq. ft.
3. \$23.09.

Art. 696.

2. 33.5104 cu. ft
3. 1181.736 lb.

Art. 698.

1. 76.5 gal.
2. 122.4 gal.
3. 394.944 gal.

Art. 700.

4. \$42.50.
5. 5.0625 acres.
6. $\$8\frac{1}{8}$.
7. 15 rods.

Art. 701.

3. 207.72+ lb.
4. \$27.
5. \$6.25.
6. \$250.
7. 1280 tons.
8. 128 lb.

Art. 702.

1. 588 sq. ft.
2. 169.705+ rods.
3. 47.69+ ft.
4. 71.62+ hhd.
5. 7.106+ mi.
6. 38.4846 sq. ft.
7. 94.248 cu. ft.
8. 256 cu. ft.
9. $1312.50.
10. 125.664 acres.
11. 16 lots.
12. 1728 spheres.
13. 462.85+ bu.
14. 35.343 cu. ft.
15. 1093.715+ sq. ft.
16. 10,800 bricks.
17. 13,445.38 times.
18. 4416 sq. ft.
19. 1 rd. wide.
20. 320 rds.
21. 2744 cu. ft.
22. 32.31+ ft.
23. $1.46.
24. $7786\frac{2}{3}$ cu. ft.

Art. 714.

1. $\frac{1}{12}$.
2. $\frac{2}{63}$.
3. $\frac{2}{35}$.
4. $\frac{3}{10}$.
5. $\frac{7}{20}$.
6. $\frac{5}{12}$.
7. $\frac{4}{45}$.
8. $\frac{7}{24}$.
9. $\frac{1}{12}$.
10. $\frac{2}{45}$.
11. $\frac{1}{15}$.
12. $\frac{5}{54}$.
13. $\frac{3}{28}$.
14. $\frac{7}{20}$.
15. $\frac{11}{42}$.
16. $1\frac{1}{8}$.

Art. 715.

1. 98 demijohns.
2. $\frac{9}{16}$ lb.
3. $18\frac{3}{4}$ yds.
4. $\frac{1}{16}$ lb.
5. 51 kegs.
6. $5\frac{5}{8}$ gal.
7. $\$\frac{9}{10}$.

Art. 718.

1. $1\frac{4}{5}$.
2. 4.
3. 24.
4. $31\frac{1}{2}$.
5. 18.
6. $58\frac{1}{3}$.
7. 147.
8. $96\frac{1}{4}$.
9. 30.
10. $23\frac{1}{3}$.
11. 70.
12. 8.
13. 120.
14. $5343\frac{1}{4}$.
15. $73{,}300\frac{2}{5}$.
16. $52{,}950\frac{4}{7}$.

Art. 719.

1. $84\frac{1}{3}$ mi.
2. $\$33\frac{3}{4}$.
3. 56.
4. 110 yds.
5. $127\frac{1}{2}$ bu.

Art. 727.

1. $\frac{2}{3}$, $\frac{1}{3}$, $\frac{31}{111}$, $\frac{4}{7}$.
2. $\frac{35}{111}$, $\frac{122}{1111}$, $\frac{17}{37}$, $\frac{187}{999}$, $\frac{55}{111}$.
3. $\frac{13}{33}$, $\frac{9}{11}$, $\frac{13}{37}$, $\frac{465}{1111}$, $\frac{4987}{15873}$.

Art. 729.

1. $\frac{37}{110}$, $\frac{5}{11}$, $\frac{7}{11}$, $\frac{151}{180}$, $\frac{139}{1110}$, $\frac{74}{99}$.
2. $\frac{41}{18}$, $\frac{127}{30}$, $\frac{163}{90}$, $\frac{1223}{990}$.
3. $\frac{386}{165}$, $\frac{517}{90}$, $\frac{8939}{1100}$, $\frac{359}{1100}$, $\frac{19}{100}$.

Art. 731.

1. $.5\dot{3}53535\dot{5}$.
   $.7\dot{5}95959\dot{9}$.
   $.5\dot{6}85685\dot{5}$.
   $.4\dot{8}19819\dot{9}$.
2. $.5\dot{6}15615\dot{5}$.
   $.8\dot{1}91919\dot{9}$.
   $.7\dot{3}73737\dot{7}$.
3. $.3\dot{3}33\dot{3}$.
   $.5\dot{5}55\dot{5}$.
   $.1\dot{9}19\dot{1}$.
   $.5\dot{3}21\dot{7}$.

4. $.3179\dot{1}79179\dot{9}$.
$.4381\dot{2}4678\dot{1}$.
$.1032\dot{4}7474\dot{7}$.

Art. 733.

1. $199.\dot{3}8220\dot{0}$.
2. $43.\dot{3}69906\dot{2}$.
3. $42.5\dot{2}94\dot{8}$.
4. $15.\dot{2}47346445$-
$36\dot{6}$.
5. $11.\dot{2}30867\dot{2}$.

Art. 743.

1. 320 farms.
2. $360.
3. $420.
4. $240.
5. $100.
6. $560.

Art. 746.

2. 32 sq. ft. 5′ 6″.
3. 147 cu. ft. 9′ 4″.
4. 158 cu. ft. 9′ 6″ 8‴.
5. 17 sq. ft. 1′ 4″.

Art. 750.

1. 241 sheets.
2. 12 books.
3. 100 bu.
4. 250 bu.
5. 64 gal.
6. 2650.725 lb.
7. 24897.6 mi.
8. 525 lb.
9. 175 lb. Troy.
10. 18 tons.
11. 970.32 + bu.
12. 66 in.
13. 300 perches.
14. 462.85 + bu.

Art. 752.

1. 4 mo. 15 da.; 4 mo. 15 da.; 136 da.
2. 4 yrs. 8 mo. 13 da.; 4 yrs. 8 mo. 12 da.; 4 yrs. 257 da.
3. 11 mo. 12 da.; 11 mo. 12 da.; 347 da.
4. 3 mo. 15 da.; 3 mo. 16 da.; 107 da.
5. 2 yrs. 6 mo. 4 da.; 2 yrs. 6 mo. 3 da.; 2 yrs. 187 da.

Art. 754.

1. $283.91.
2. $390.
3. $1286.46.
4. $75.65.
5. $17.28.
6. $57.51.
7. $435.68.
8. $521.91.
9. $1099.16.
10. $4448.58.
11. $1907.74.
12. $4751.46.
13. $4054.91.
14. $774.88.
15. $11,872.80.
16. $725.65.

Art. 758.

1. $81.97.
2. $1308.38.
3. $42.67.
4. $714.67.
5. $584.93.
6. $1811.22.
7. $7034.44.
8. $4758.43.
9. $198.98.
10. $1820.37.

Art. 763.

4. $5166.
5. $4212.81.
6. $4966.80.
7. $2133.62.
8. $3835.88.

Art. 765.

3. $1597.03.
4. 210 bu. wheat

5. $390.
6. $416.12.

Art. 768.

2. $.75. 3. $.32.
4. $.09.
5. $3.29.

Art. 770.

2. 1, 1, 2, 1, or 2, 5, 5, 4,
3. 1, 1, 2.
5. 8, 1, 1, 7, or 4, 2, 7, 1.
6. 7, 2, 3, 8, or 1, 7, 4, 3.
7. 2, 3, 1, 1, or 2, 1, 9, 3.
8. 2, 5, 2, 1, or 6, 3, 2, 5.
9. 5, 2, 3, 4, or 1, 10, 4, 3.
10. 24, 20, 28, 16, or 22, 15, 44, 7.

Art. 775.

1. 64. 4. 152.
2. 43. 5. $105.
3. 15. 6. $39.
7. $17\frac{1}{2}$ in.

Art. 777.

1. 1455. 2. 340.
3. 3441.
4. $$383\frac{1}{3}$.

Art. 782.

1. 128. 3. 2.
2. 3. 4. 81,920.

Art. 784.

1. 765. 2. 5465.
3. 1330.
4. 5022.
5. 660.
6. 39,060.

Art. 791.

2. $15,240.
3. $22,950.
4. 15,700.

Art. 792.

2. $12,917.97.
3. $5416.33.
4. $9919.79.
5. $15,093.48.

Art. 793.

2. $240.
3. $141\frac{1}{3}$ ft.
4. 131.5 ft.
5. $20.
6. 37.5 ft.
7. 24 ft.
8. 33 b'ds; $12.41.
9. 12 boards.
11. 300 ft.
12. $8.96.
13. 675 ft. 14. $32.
15. 240, 20.
16. $154.05.
17. 16 ft.
18. $175.

Art. 794.

2. $9.96.
4. 5 yds.
5. 288 boards.
6. $135.90.
7. $8.88. 8. $65

Art. 795.

4. $343.50.
5. 28 rolls.
6. $60.
7. $349.25.
8. $43.07.

Art. 796.

4. $67.50.
5. 12 ft.
6. $292.50.
7. $92\frac{2}{5}$ perches.
8. $309\frac{3}{8}$ cords.

Art. 797.

4. 10 tons.
5. $6\frac{1}{4}$ ft.; 320.
6. 8976.6+, $34\frac{4}{7}$.
7. $9\frac{3}{8}$ ft. 8. 5 ft.